



Femtoscscopy in heavy-ion collision experiments at various μ_B

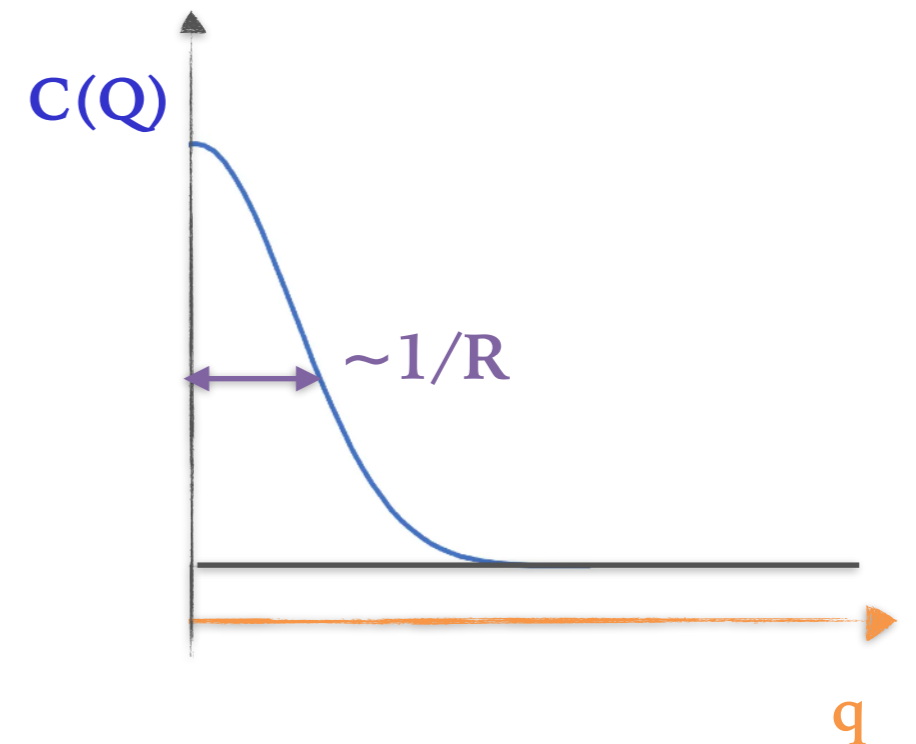
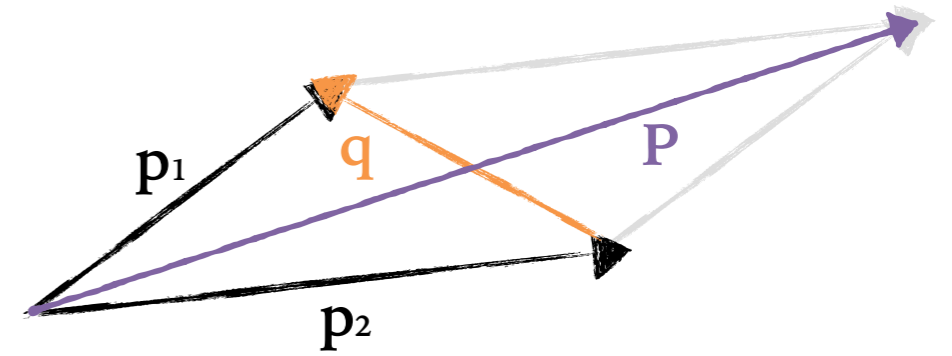
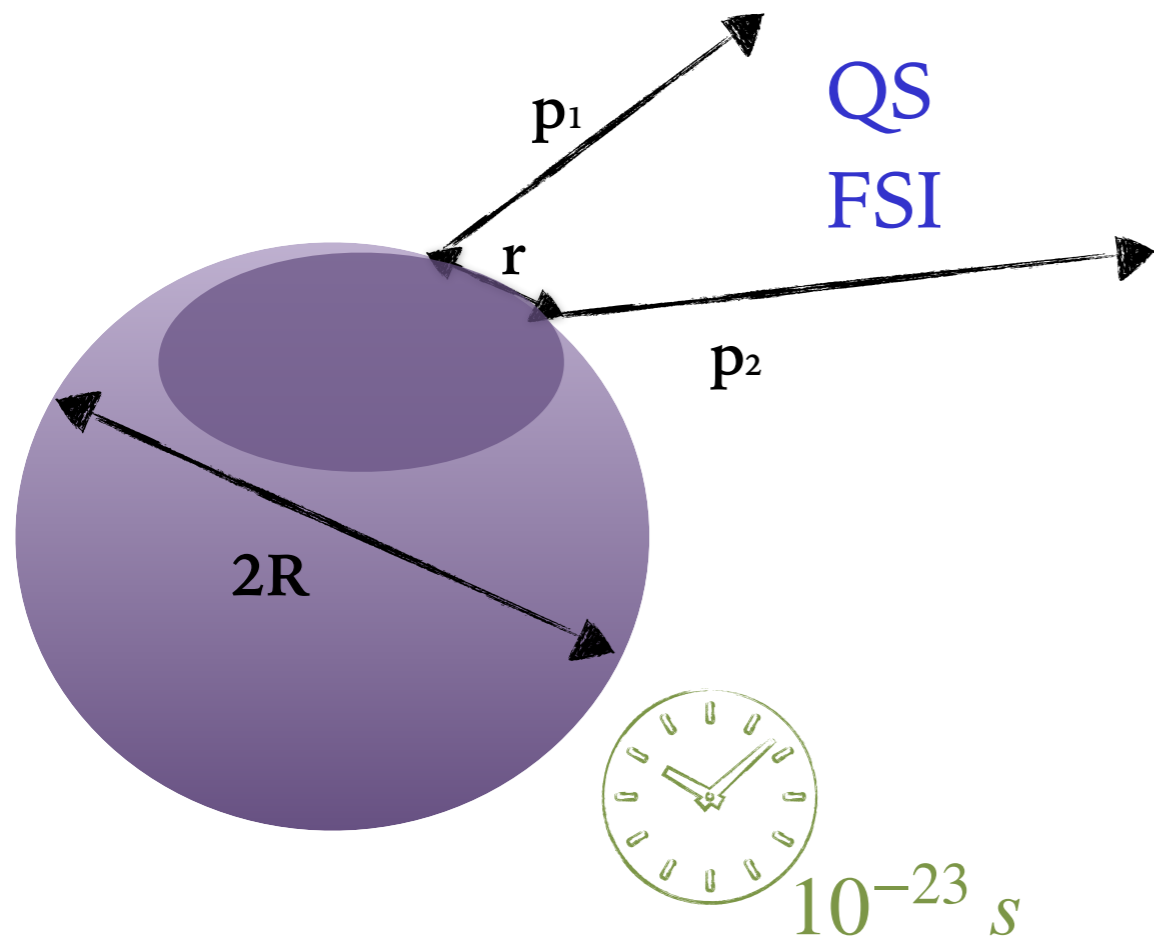
Hanna Zbroszczyk

Introduction

Scanning QCD phase diagram:

- neutron stars
- neutron star mergers
- phase transitions

Summary



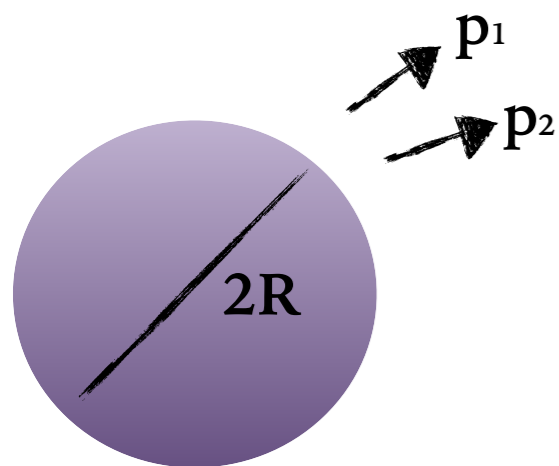
Femtoscscopy

... the method to probe **geometric** and **dynamic** properties of the source
(emission region, range of correlations-interactions, phase-space cloud, ...)

Femtoscscopy does not measure the whole source, but **homogeneity length**.

Classic femtoscopy

Femtoscscopy (originating from HBT):
the method to probe **geometric** and **dynamic** properties of the source



Space-time properties ($10^{-15}m$, $10^{-23}s$) determined thanks to two-particle correlations:

Quantum Statistics (Fermi-Dirac, Bose-Einstein);
Final State Interactions (Coulomb, strong)

$$C(k^*, r^*) = \int \overset{\text{determined}}{S(r^*)} \overset{\text{assumed}}{|\Psi(k^*, r^*)|^2} d^3r^* = \overset{\text{measured}}{\frac{Sgnl(k^*)}{Bckg(k^*)}}$$

$S(r^*)$ - source function

$\Psi(k^*, r^*)$ - two-particle wave function (includes e.g. FSI interactions)

$\frac{Sgnl(k^*)}{Bckg(k^*)}$ - correlation function

k^* - momentum of the first particle in the Pair Rest Frame reference



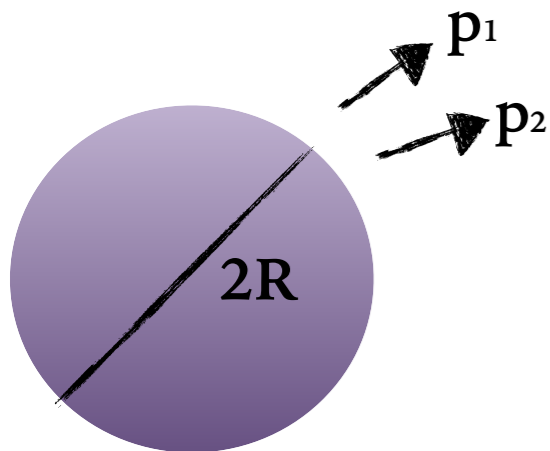
Gateway to study interactions

If we assume we know the **source function**, measured **correlations** are used to determine **interactions in the final state**.

Space-time properties ($10^{-15}m, 10^{-23}s$) determined thanks to two-particle correlations:

Quantum Statistics (Fermi-Dirac, Bose-Einstein);

Final State Interactions (Coulomb, strong)



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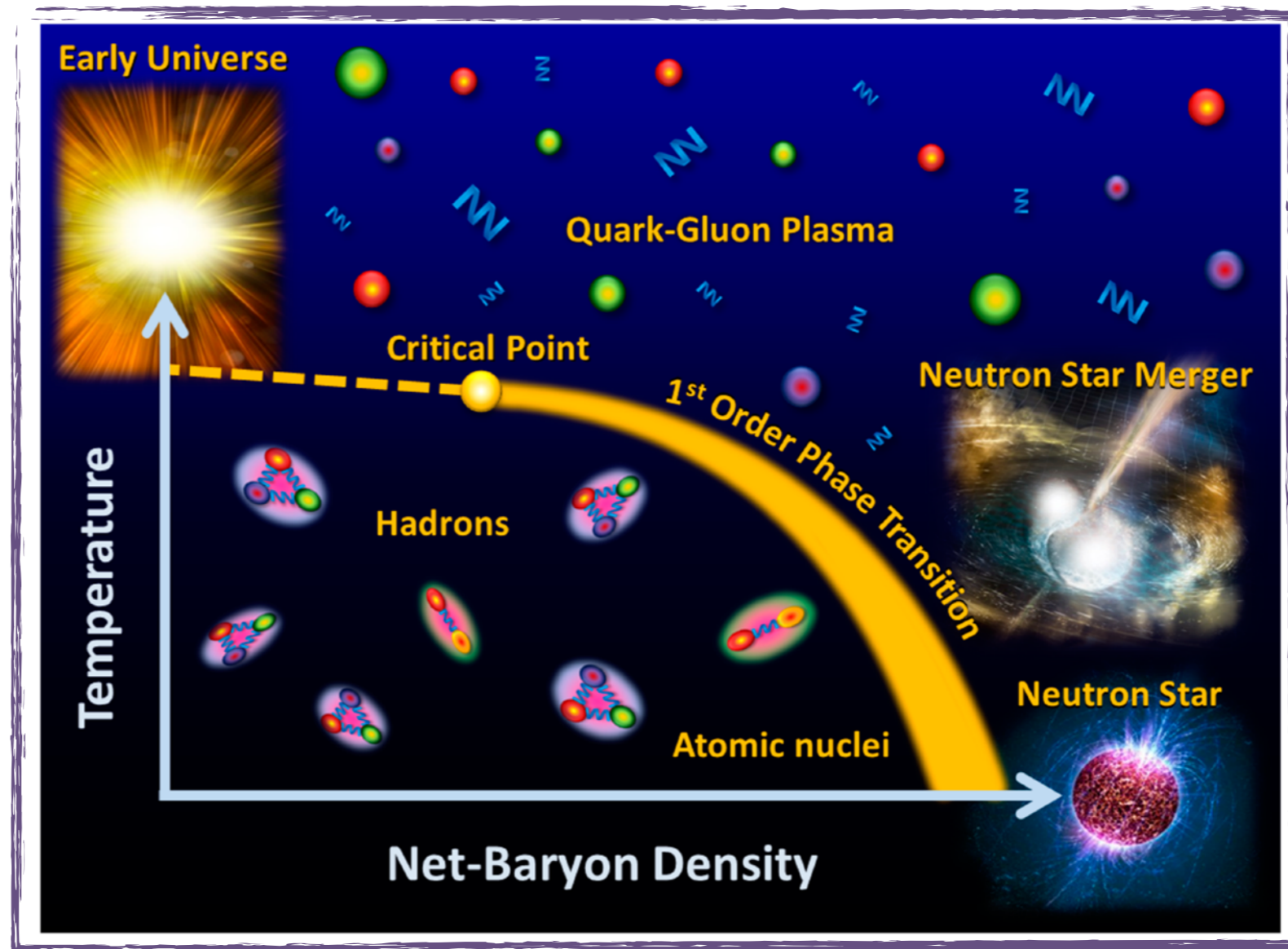
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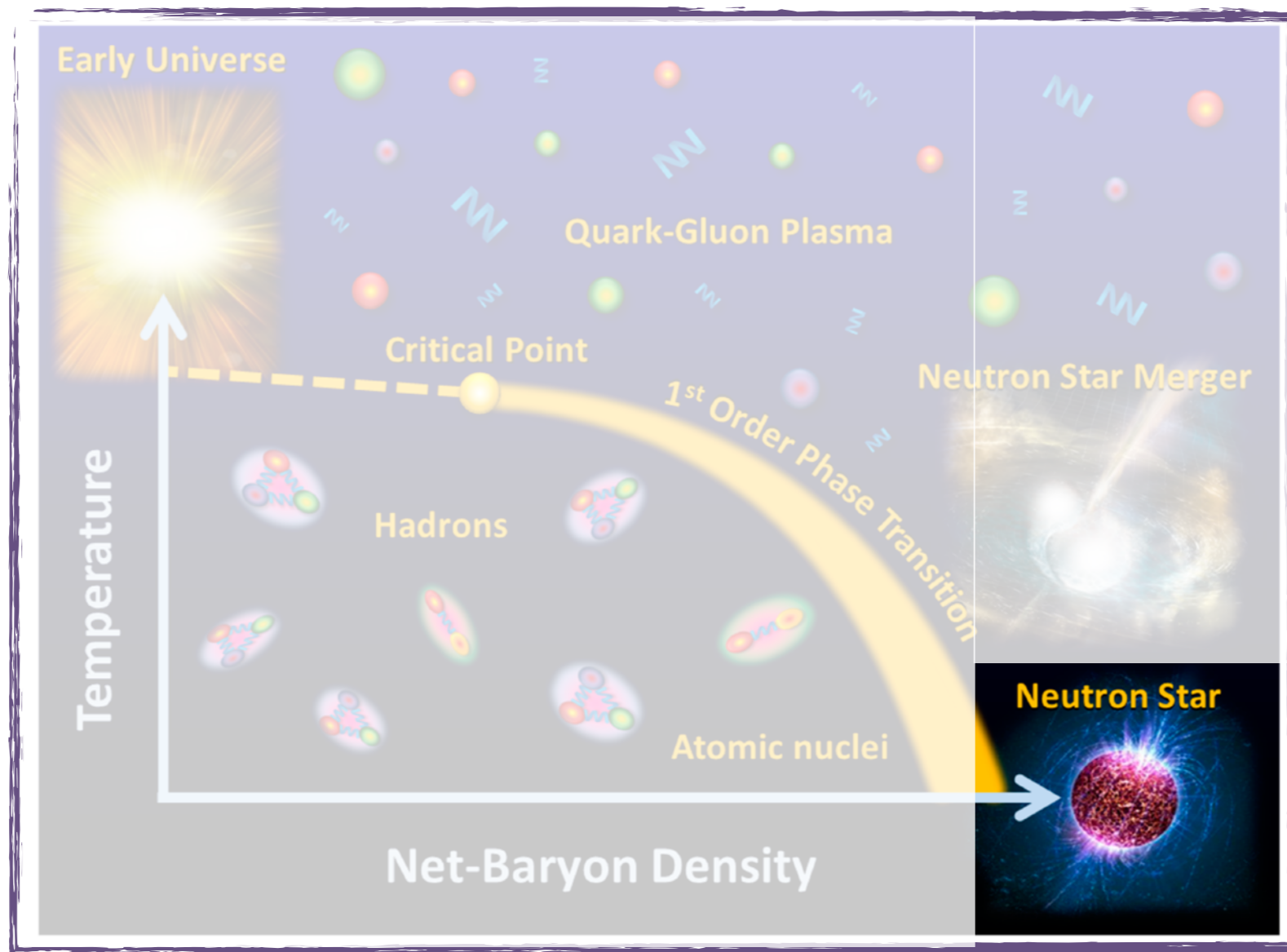
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Scanning various μ_B

- .. to study strongly interacting matter
- .. to explore unknown QCD territory

Neutron stars

Exploration of unknown QCD territory: high μ_B



Temperature
 $T < 10 \text{ MeV}$

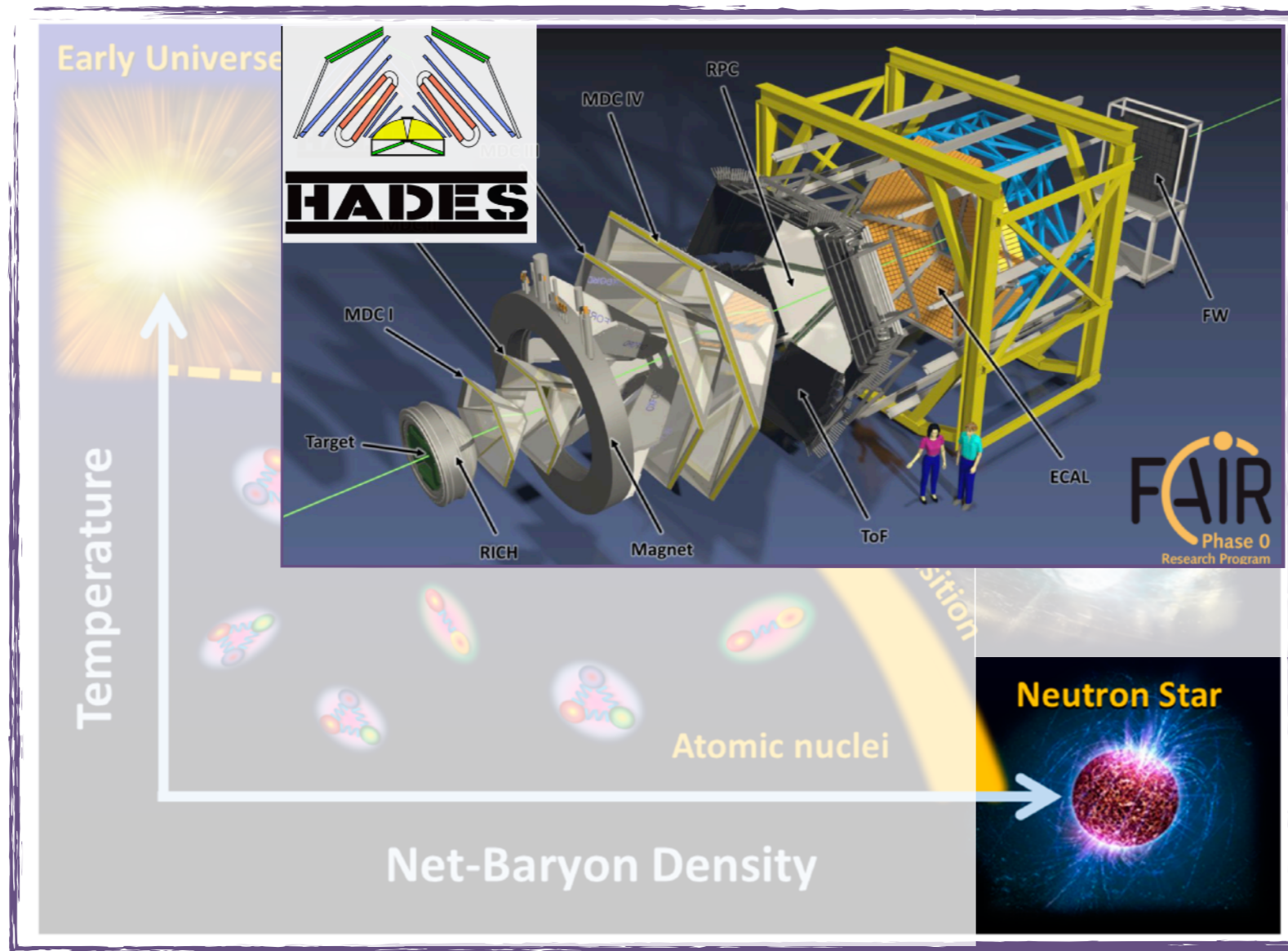
Density
 $n < 10n_0$

Lifetime
 $t \sim \text{long}$

<https://www.researchgate.net>

Neutron stars

Exploration of unknown QCD territory: high μ_B



Temperature
 $T < 10 \text{ MeV}$

Density
 $n < 3n_0$

Lifetime
 $t \sim \text{long}$

<https://www.researchgate.net>

Neutron star puzzle

- **Hyperons:** expected in the core of neutron stars; conversion of N into Y energetically favorable.
- Appearance of Y: The relieve of Fermi pressure → **softer EoS** → mass reduction (incompatible with observation).

The solution requires a mechanism that could provide the **additional pressure** at high densities needed to make the EoS stiffer.

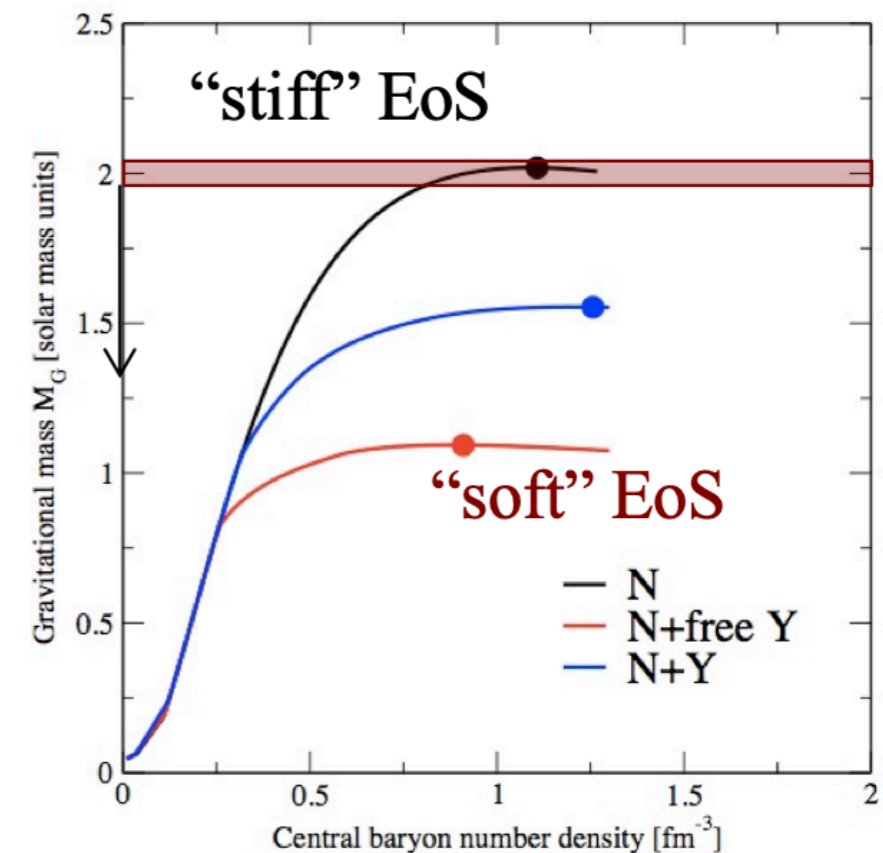
A few possible mechanisms, one of them:

Two-body YN & YY interactions

The existence of **hypernuclei** (confirmed by attractive YN interaction) → indicates the possibility to bind Y to N.

The measurement of the YN and YY interactions leads to important implications for the possible formation of **YN or YY bound states**.

$$\begin{aligned} M_{\text{NS}} &\approx 1 \div 2 M_{\odot} \\ R &\approx 10\text{-}12 \text{ km} \\ \rho &\approx 3 \div 5 \rho_0 \end{aligned}$$



$$\rho_0 \approx 2.8 \times 10^{14} \text{ g/cm}^3$$

Lednicky-Lyuboshitz model ($p\Lambda$ example)

The normalized pair separation distribution (source function) $S(r^*)$ is assumed to be Gaussian,

$$S(r^*) = (2\sqrt{\pi}r_0)^{-3} e^{-\frac{r^{*2}}{4r_0^2}},$$

Ref : Lednicky, Richard & Lyuboshits, V.L.. (1982). Sov. J. Nucl. Phys. (Engl. Transl.); (United States). 35:5.

The correlated function can be calculated analytically by averaging Ψ^s over the total spin S and the distribution of the relative distances $S(r^*)$

$$C(k^*) = 1 + \sum_S \rho_s \left[\frac{1}{2} \left| \frac{f^S(k^*)}{r_0} \right|^2 \left(1 - \frac{d_0^S}{2\sqrt{\pi}r_0} \right) + \frac{2\Re f^S(k^*)}{\sqrt{\pi}r_0} F_1(Qr_0) - \frac{\Im f^S(k^*)}{r_0} F_2(Qr_0) \right]$$

$$\text{with } F_1(z) = \int_0^z dx e^{x^2 - z^2} / z \text{ and } F_2(z) = (1 - e^{-z^2}) / z$$

Decomposition for spin channels :

$$C(k^*) = \frac{1}{4} (1 + \lambda C(k^*, s = 0)) + \frac{3}{4} (1 + \lambda C(k^*, s = 1))$$

f_0 and d_0 - parameters of strong interaction.

Theoretical correlation function (k^*) depends on: R , f_0 and d_0 .

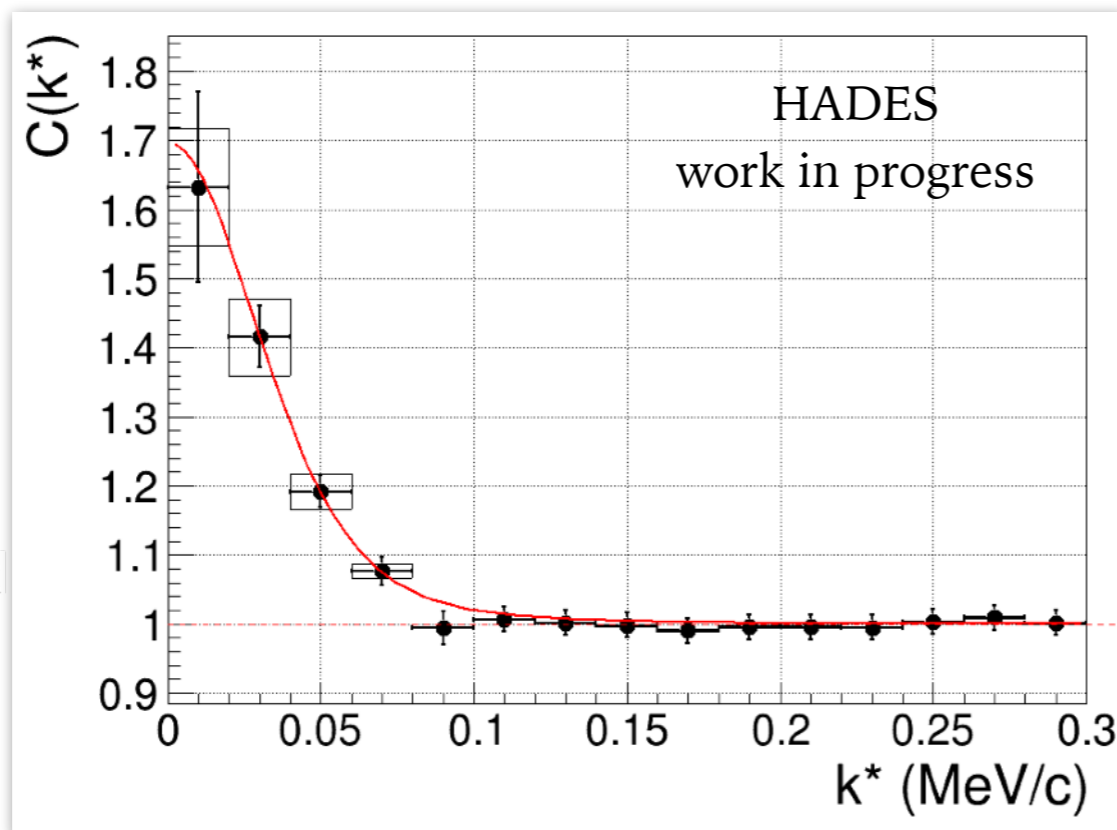
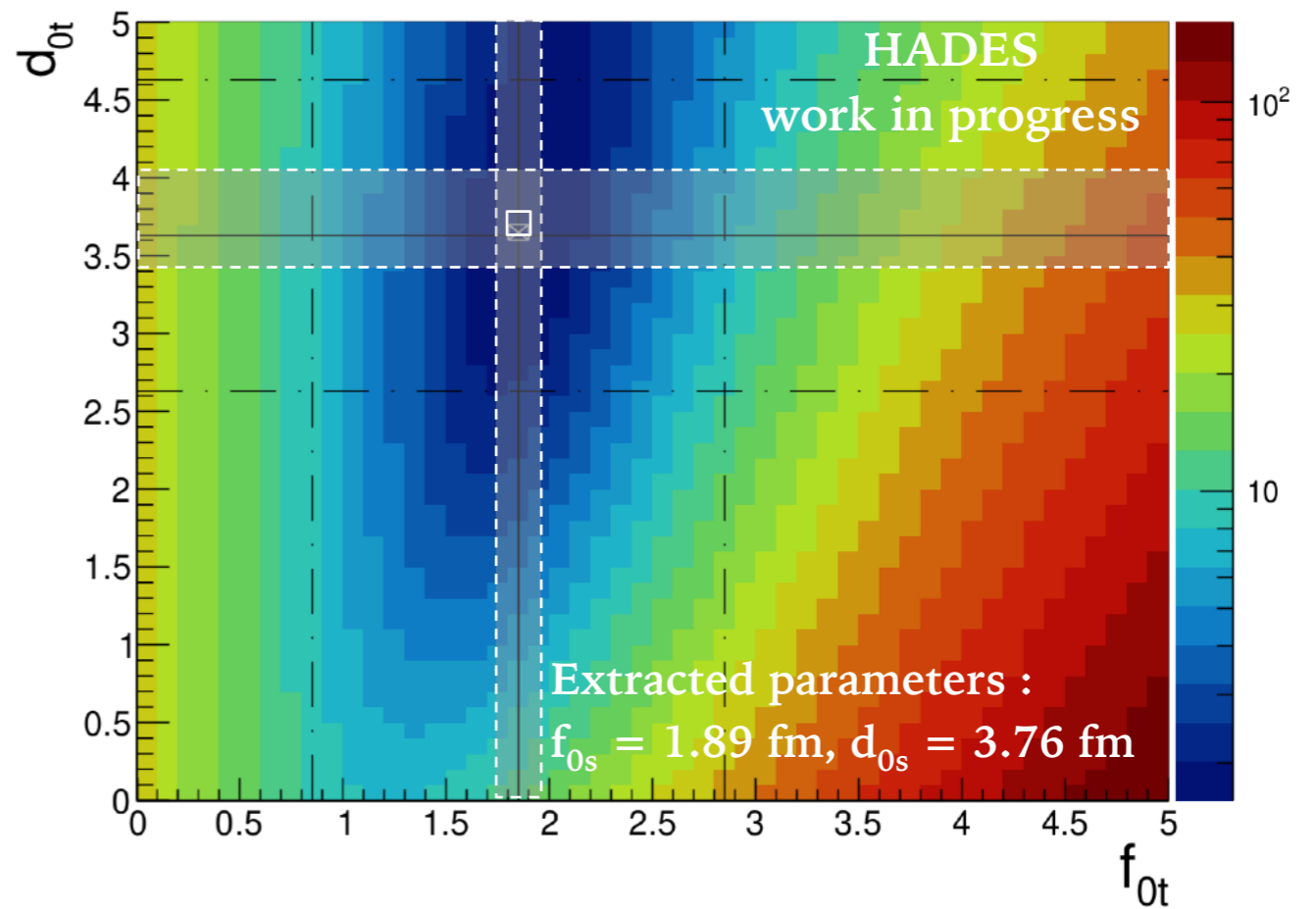
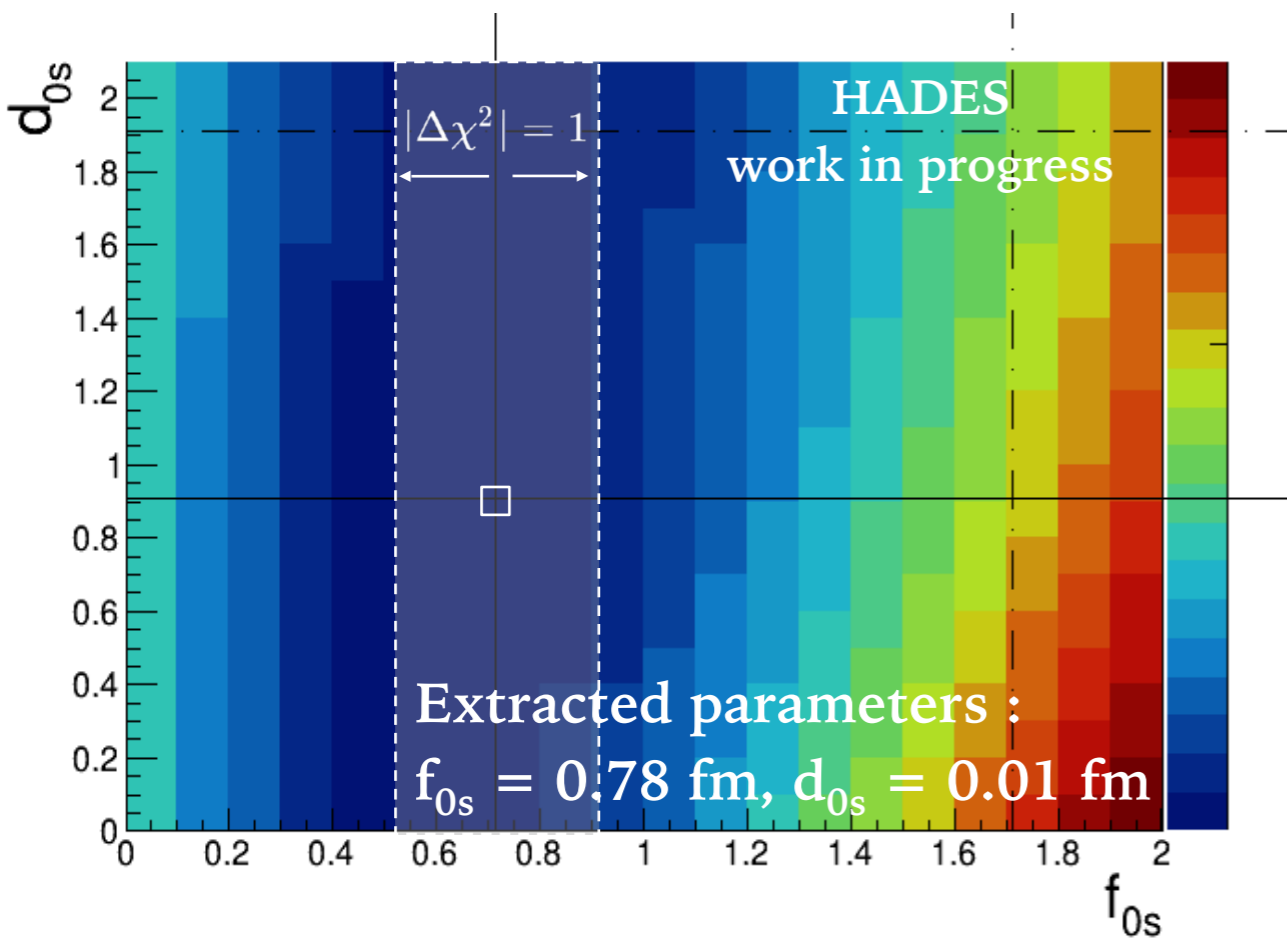
f_0 - the scattering length, determines low-energy scattering.

The elastic cross section, σ_e , (at low energies) determined by

the scattering length, $\lim_{k \rightarrow 0} \sigma_e = 4\pi f_0^2$

d_0 - the effective range, corresponds to the range of the potential (simplified scenario - the square well potential.

YN ($p\Lambda$) correlations at HADES



$$\lambda = 0.74$$

$$f_{0s} = 0.80^{+0.39}_{-0.32}$$

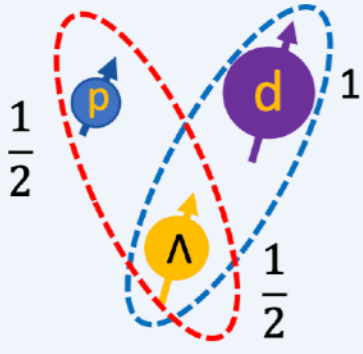
$$d_{0s} = 0.01$$

$$f_{0t} = 1.89^{+0.10}_{-0.09}$$

$$d_{0t} = 3.76^{+0.27}_{-0.25}$$

$$r = 2.24^{+0.12}_{-0.11}$$

YN correlations at STAR ($\sqrt{s_{NN}} = 3$ GeV)



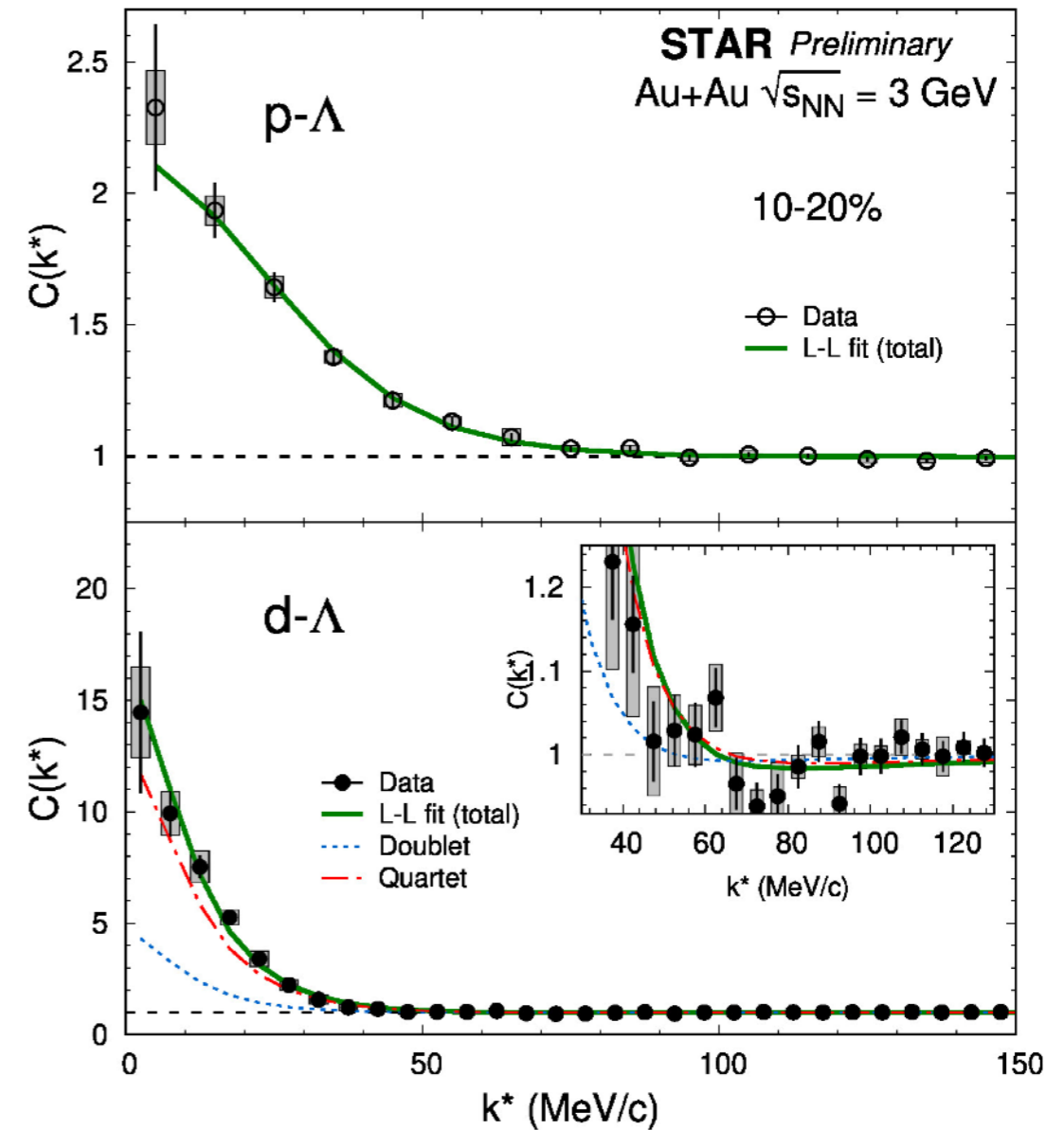
The diagram shows two nucleons, a proton (p) and a neutron (n), with their spins represented by arrows. They are coupled to form a deuteron (d) with a total spin of 1. The spin of the deuteron is shown as a yellow arrow pointing up. The spin of the nucleons is shown as blue arrows pointing up and down. The deuteron is shown as a yellow circle with a blue arrow pointing up.

Singlet State	1S_0	(S)
Triplet State	3S_1	(T)
Doublet State	$^2S_{1/2}$	(D)
Quartet State	$^4S_{3/2}$	(Q)

p- Λ : $|\psi(r, k)|^2 \rightarrow \frac{1}{4} |\psi_0(r, k)|^2 + \frac{3}{4} |\psi_1(r, k)|^2$

d- Λ : $|\psi(r, k)|^2 \rightarrow \frac{1}{3} |\psi_{1/2}(r, k)|^2 + \frac{2}{3} |\psi_{3/2}(r, k)|^2$

- ❖ Different spin states with different f_0 and d_0 parameters
- ❖ **p- Λ correlation:** current statistics is not enough to separate two spin states \rightarrow spin-averaged fit
- ❖ **d- Λ correlation:** very different f_0 for (D) and (Q) are predicted \rightarrow **Spin-separated fit**

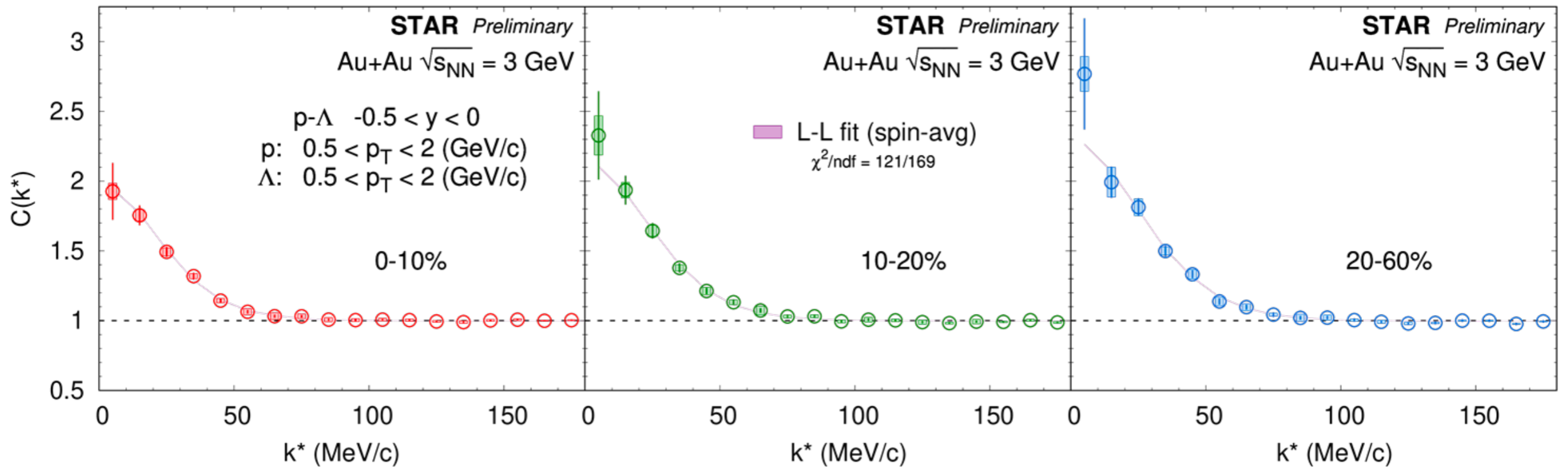


Different spin states with different FSI parameters

p- Λ correlation: currently spin-averaged fit

d- Λ correlation: spin-separated fit

YN ($p\Lambda$) correlations at STAR

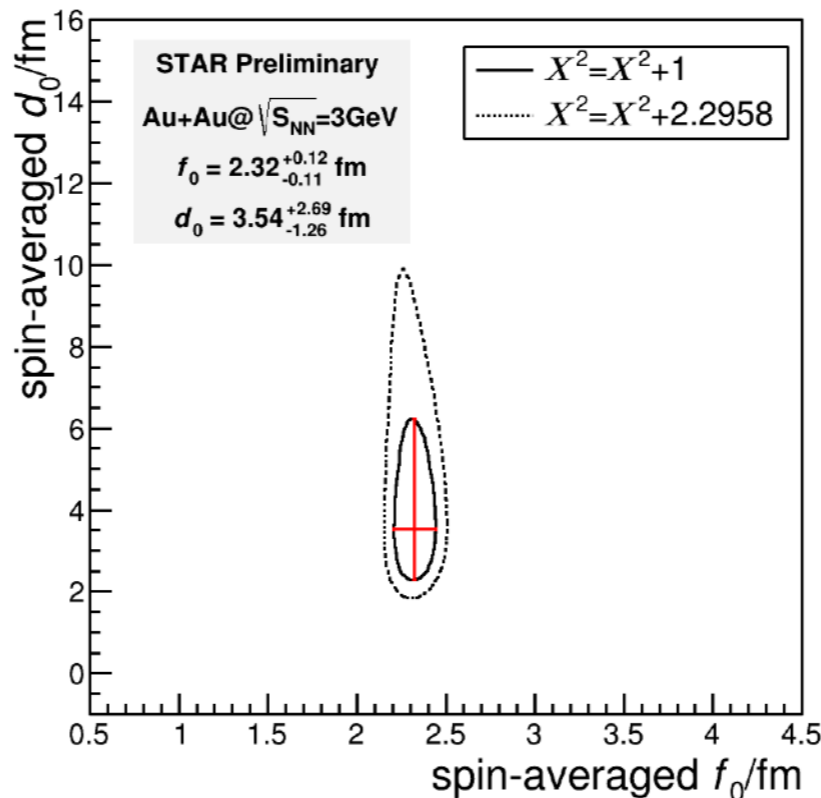


- Simultaneous fit to data in different centralities and rapidities;
- Source size R and parameters of SI: f_0 and d_0 with Lednicky-Lyuboshitz approach;
- Spin-averaged scattering length and effective range:

$$f_0 = 2.32_{-0.11}^{+0.12} \text{ fm}$$

$$d_0 = 3.5_{-1.3}^{+2.7} \text{ fm}.$$

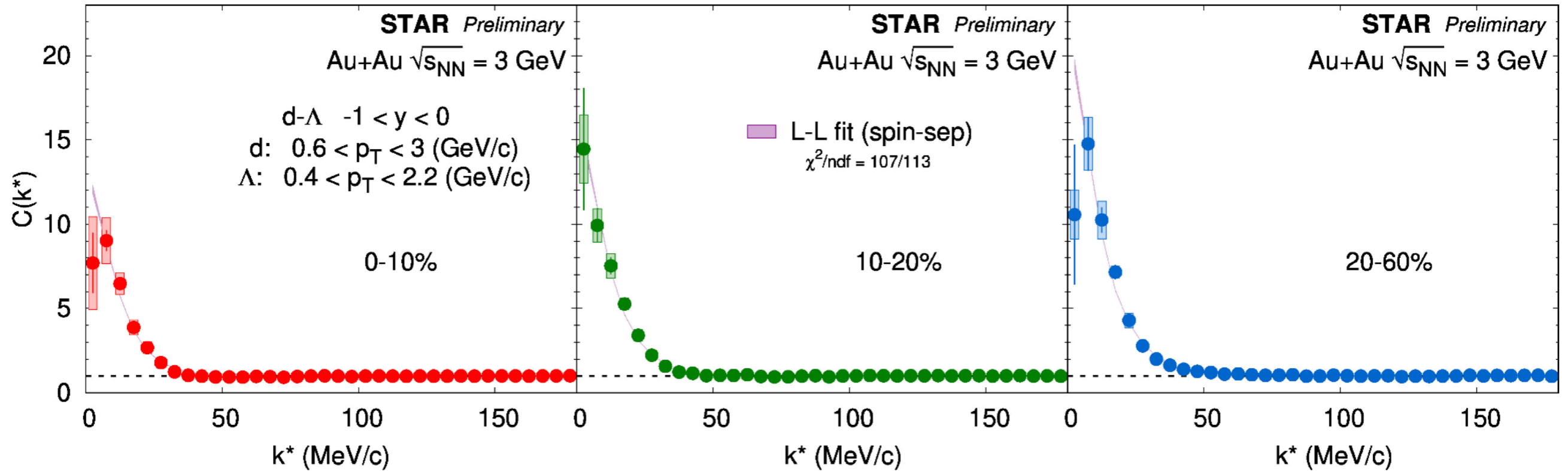
χ^2 contour of spin-averaged d_0 and f_0 for $p\Lambda$ ($-1 < y < 0$)



Source size extracted from the source assuming Gaussian shape;

Separation of emission source from the parameters of the final state interaction;

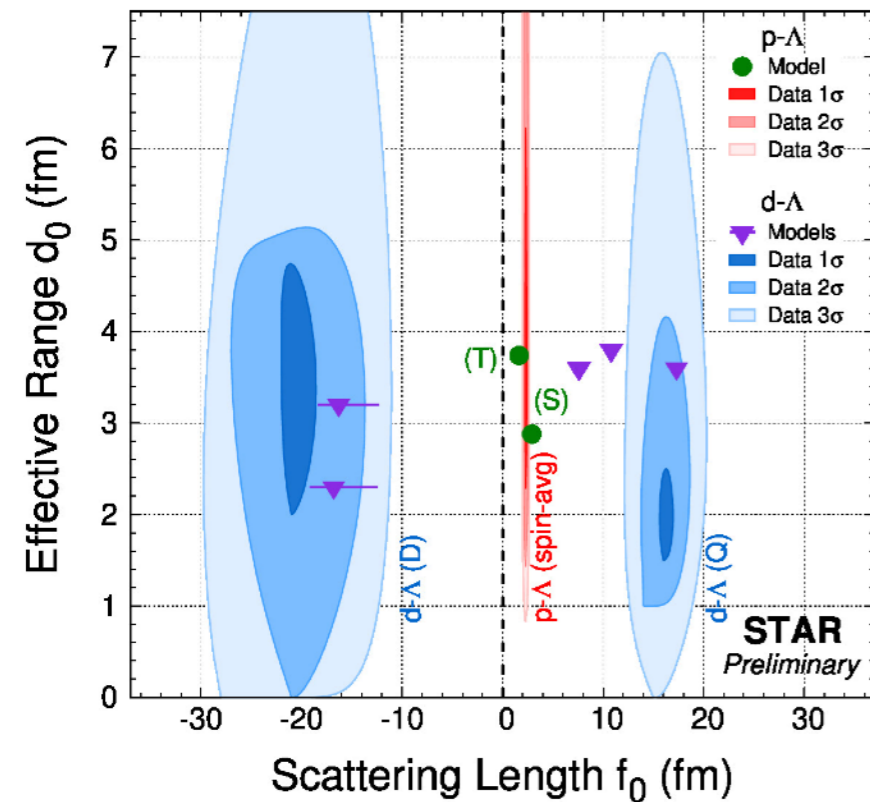
YN ($d\Lambda$) correlations at STAR



- Simultaneous fit to data in different centralities and rapidities;
- Source size R and parameters of SI: f_0 and d_0 with Lednicky-Lyuboshitz approach;
- Spin-separated scattering length and effective range:

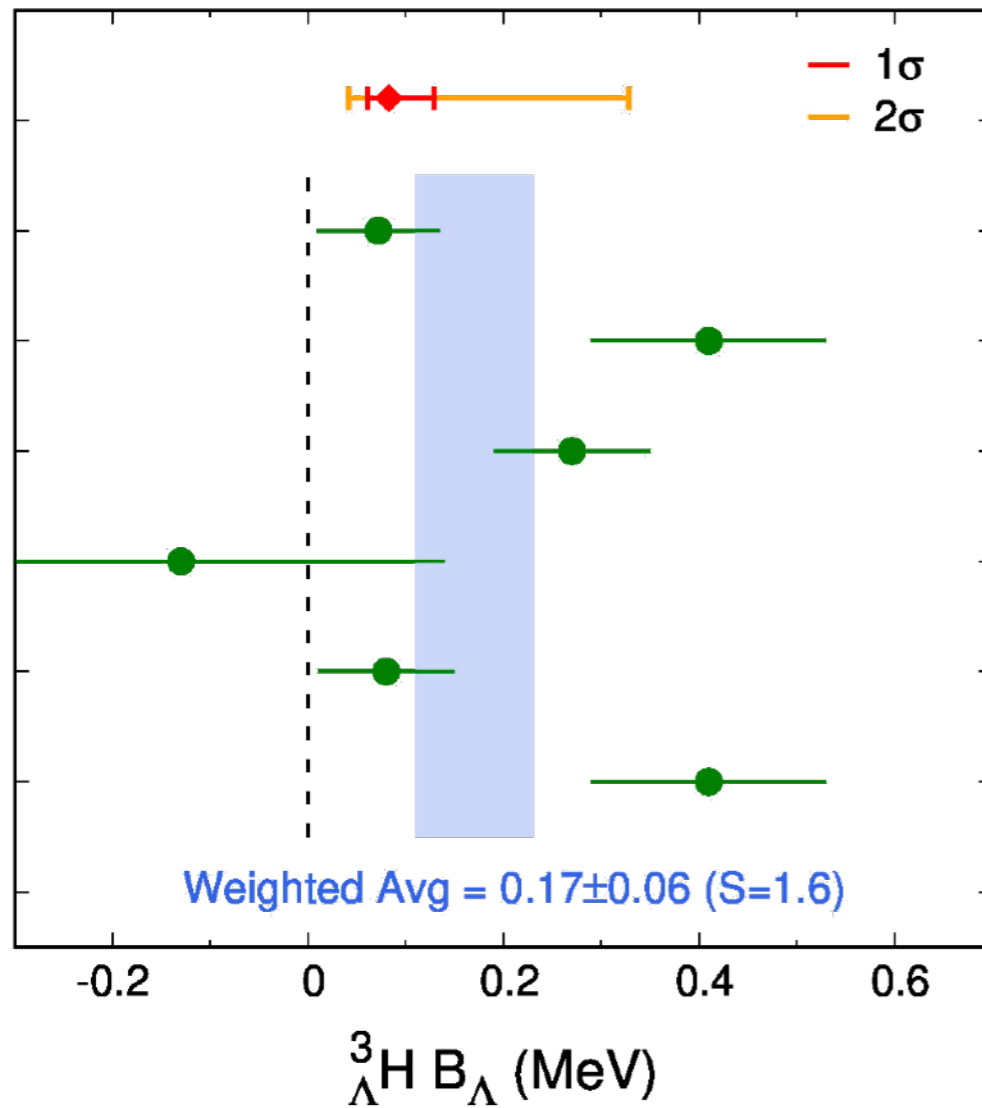
$$f_0(D) = 20^{+3}_{-3} \text{ fm}; \quad d_0(D) = 3^{+2}_{-1} \text{ fm};$$

$$f_0(Q) = 16^{+2}_{-1} \text{ fm}; \quad d_0(Q) = 2^{+1}_{-1} \text{ fm}.$$



H. W. Hammer, Nucl. Phys. A 705 (2002) 173
A. Cobis, et al. J. Phys. G 23 (1997) 401
J. Haidenbauer, Phys.Rev.C 102 (2020) 3, 034001
M. Schäfer, et al. Phys.Lett.B 808 (2020) 135614
G. Alexander, et al. Phys. Rev. 173 (1968) 1452
J. Haidenbauer, et al. Nucl. Phys. A 915 (2013) 24
F. Wang, et al. Phys.Rev.Lett. 83 (1999) 3138

YN ($d\Lambda$) correlations at STAR, binding energy



Estimated from
STAR Preliminary
d- Λ Correlation

ALICE 2022

STAR 2020

NPB52 1973

PRD1 1970

NPB4 1968

NPB1 1967

Bethe formula from Effective Range Expansion (ERE) parameters $f_0(D)$ and $d_0(D)$.

$$\frac{1}{-f_0} = \gamma - \frac{1}{2}d_0\gamma^2$$

- ❖ $B_\Lambda = \frac{\gamma^2}{2\mu_{d\Lambda}}$
- ❖ $\mu_{d\Lambda}$: reduced mass
- ❖ γ : binding momentum

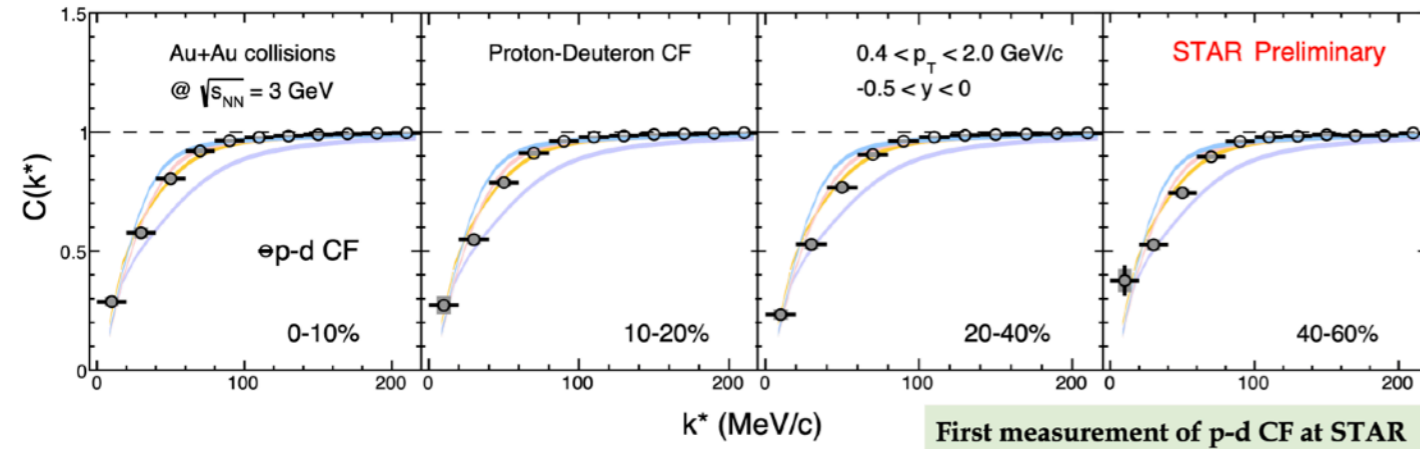
❖ ${}^3_\Lambda\text{H } B_\Lambda = [0.04, 0.33] \text{ (MeV) @ 95\% CL}$

Consistent with the world average

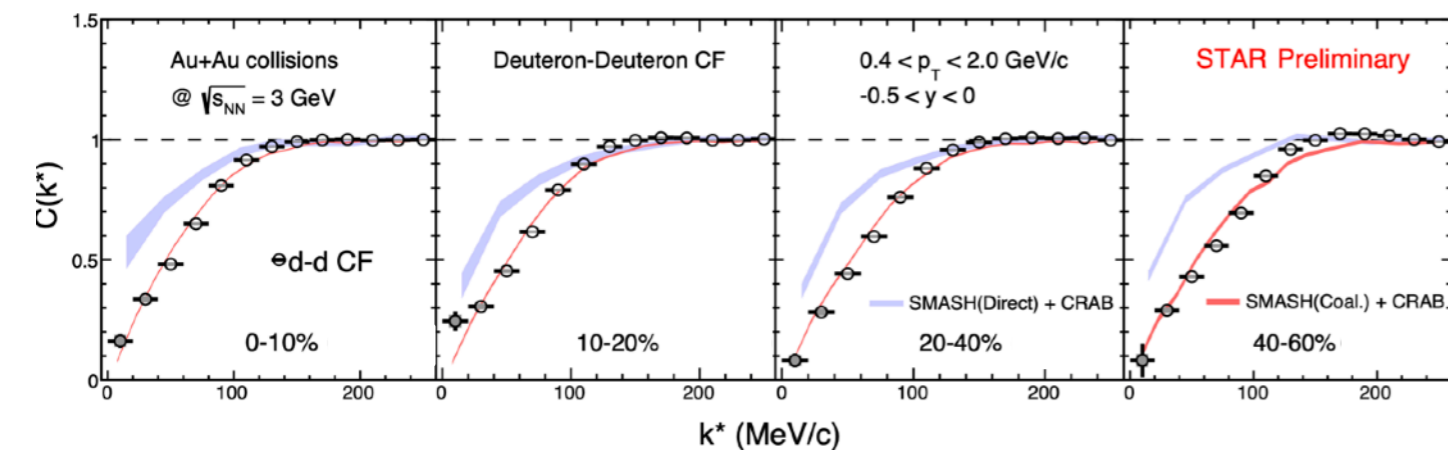
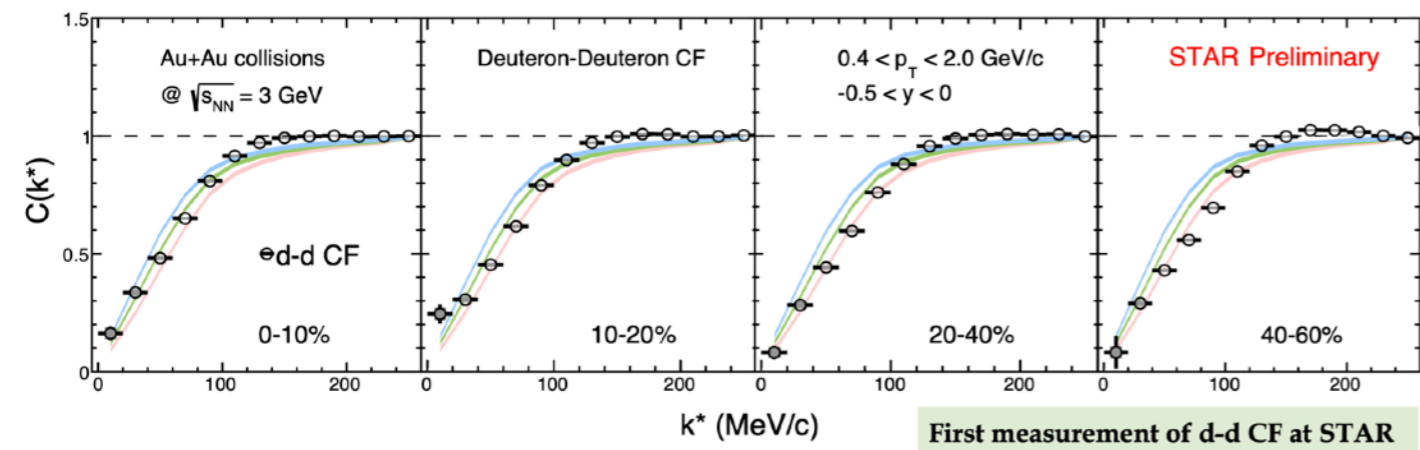
❖ A new way to constrain the ${}^3_\Lambda\text{H}$ structure

Light nuclei production at STAR

- First measurement of proton-deuteron and deuteron-deuteron correlation functions from STAR



- Proton-deuteron and deuteron-deuteron correlations qualitatively described by theory;



- Deuteron-deuteron correlations described better by the model including coalescence. Light nuclei are likely to be formed via coalescence.

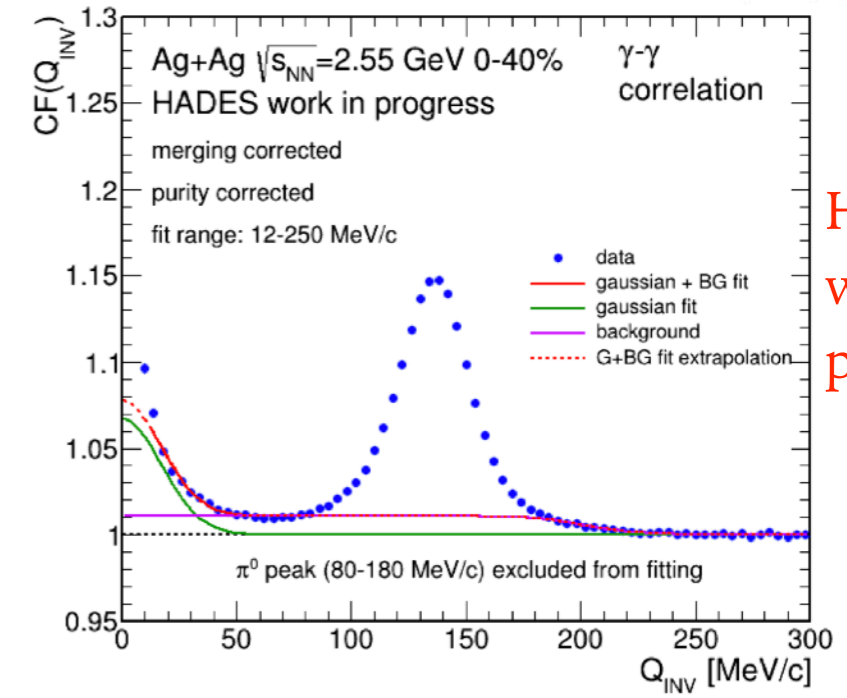
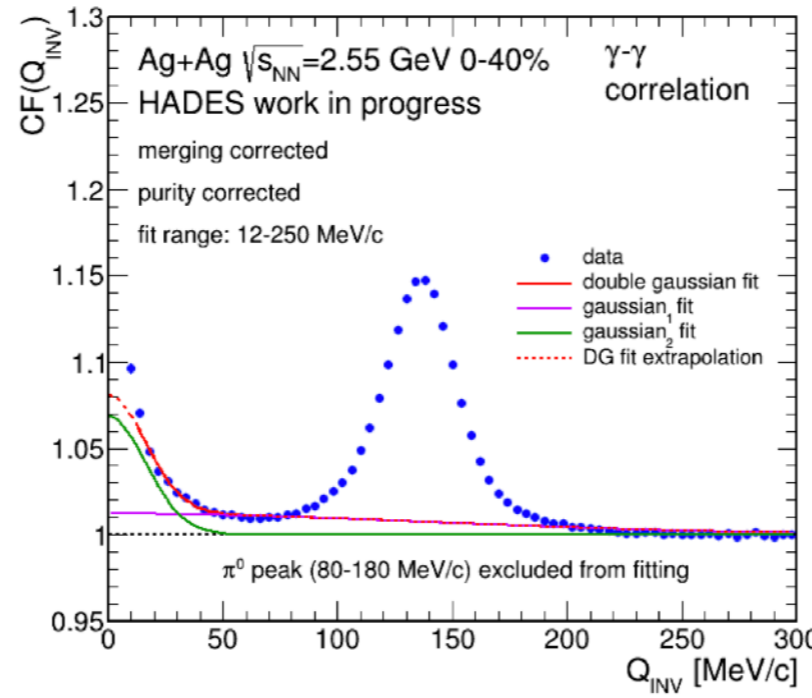
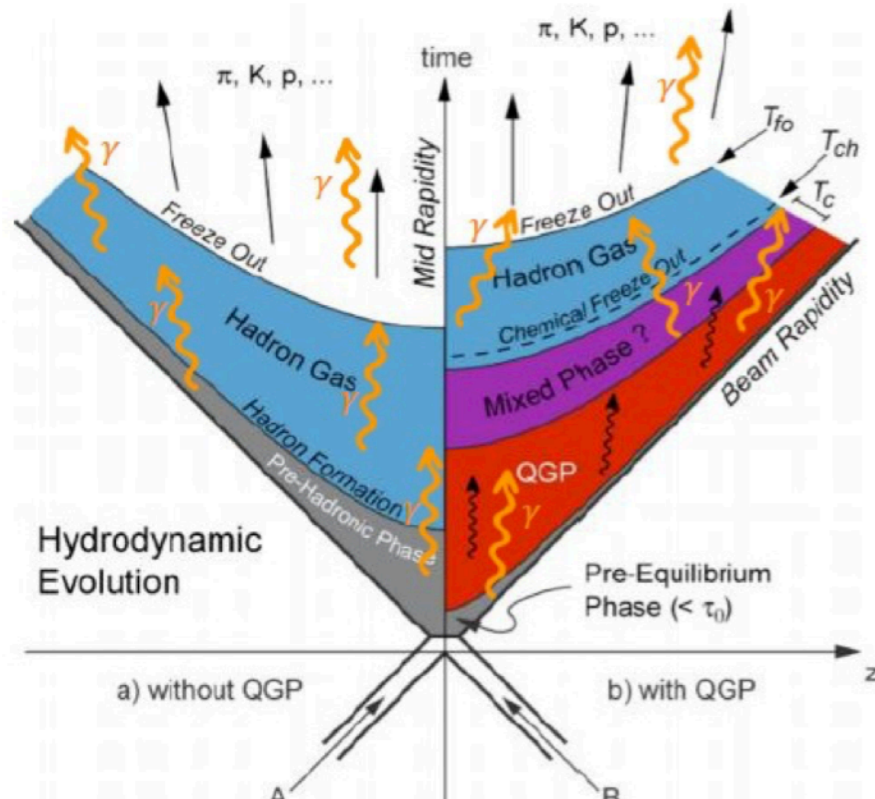
E-M probes at HADES

What if:

Non-interacting bosons → assuming 2 sources
(double gaussian or gaussian + some background)

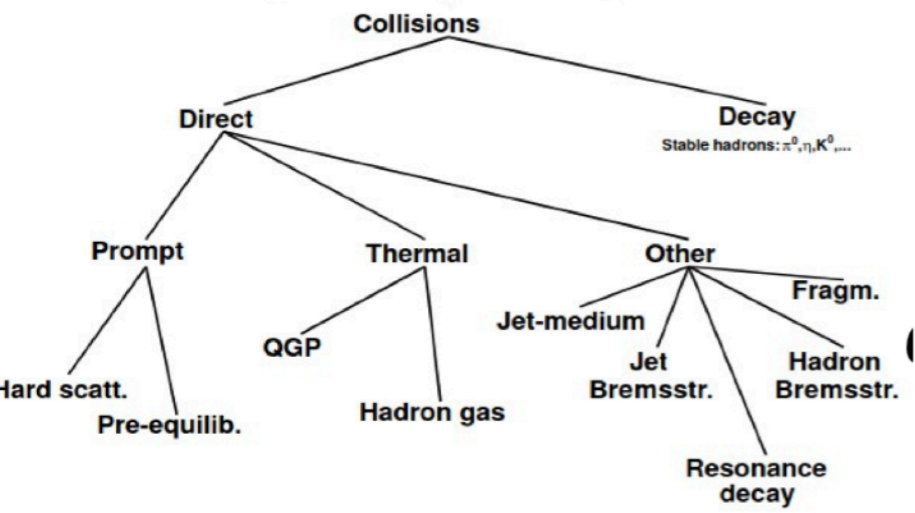
$$CF(Q_{inv}) = 1 + \lambda_1 e^{-Q_{inv}^2 \cdot R_1^2} + \lambda_2 e^{-Q_{inv}^2 \cdot R_2^2}$$

$$CF(Q_{inv}) = 1 + \lambda e^{-Q_{inv}^2 \cdot R^2} + \frac{a_0}{(1 + (a_1 \cdot Q_{inv})^{a_2})}$$



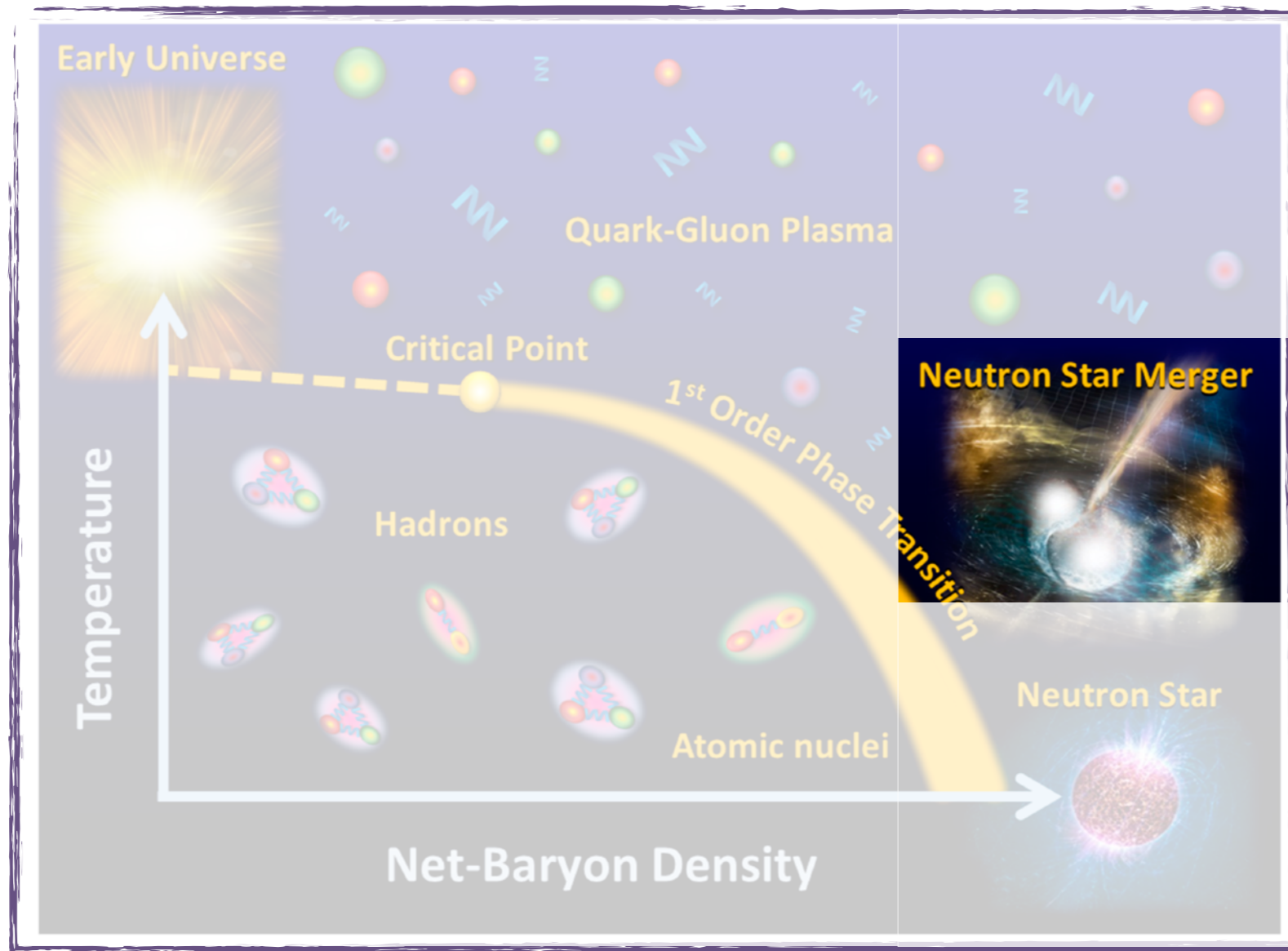
HADES work in progress

E-M probes signal various stages of the system's evolution;
 Information about early stages (inaccessible for hadrons);
 Access before freeze-out era;
 A way to hunt for direct photons.



Neutron star mergers

Exploration of unknown QCD territory: still high μ_B



Temperature
 $T < 50 \text{ MeV}$

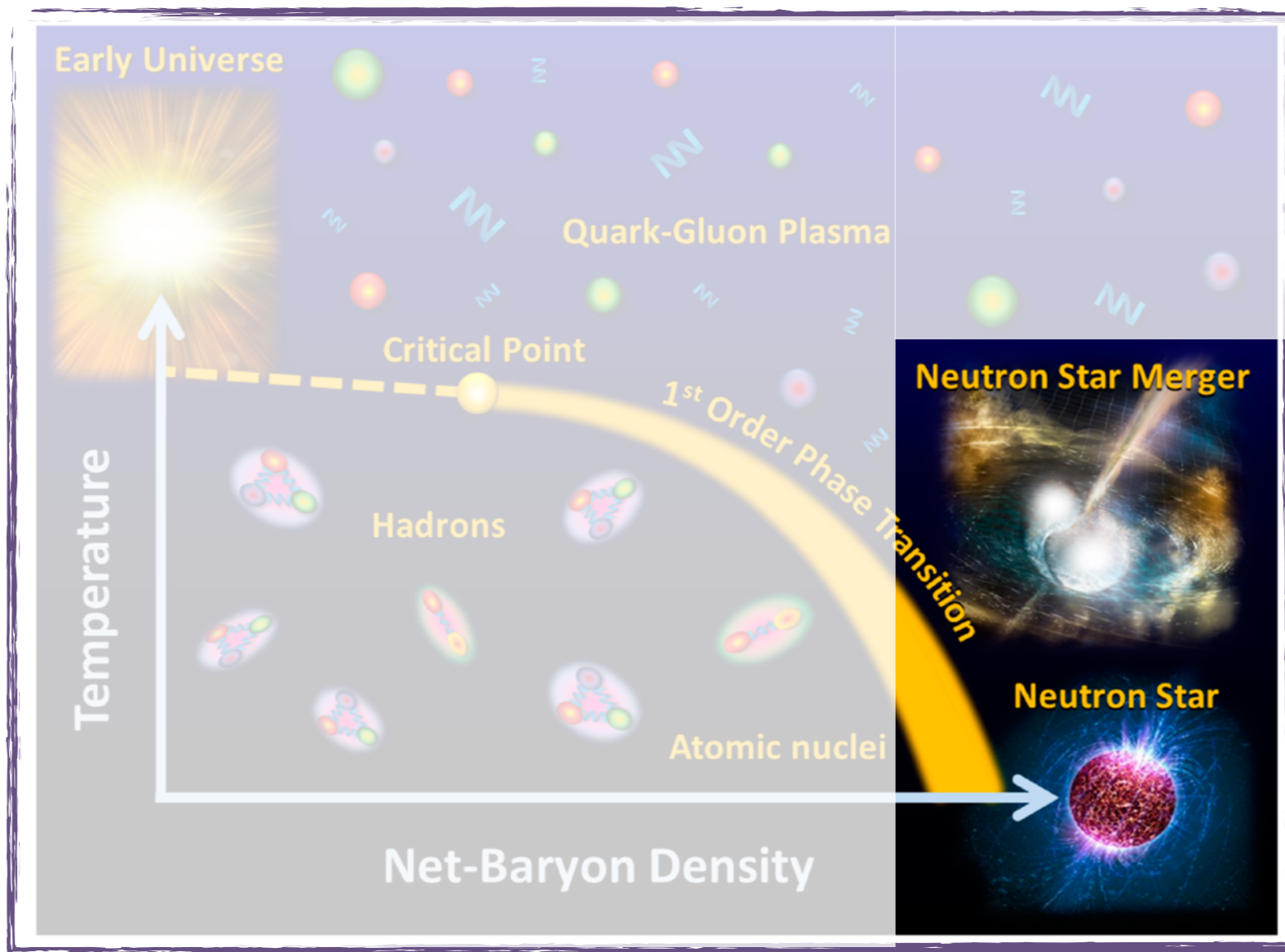
Density
 $n < 2 - 6n_0$

Reaction time
 $t \sim 10 \text{ ms}$
(GW170817)

<https://www.researchgate.net>

Neutron star mergers

CBM and HADES future



<https://www.researchgate.net>

Temperature
 $T < 50 \text{ MeV}$

Density
 $n < 2 - 6n_0$

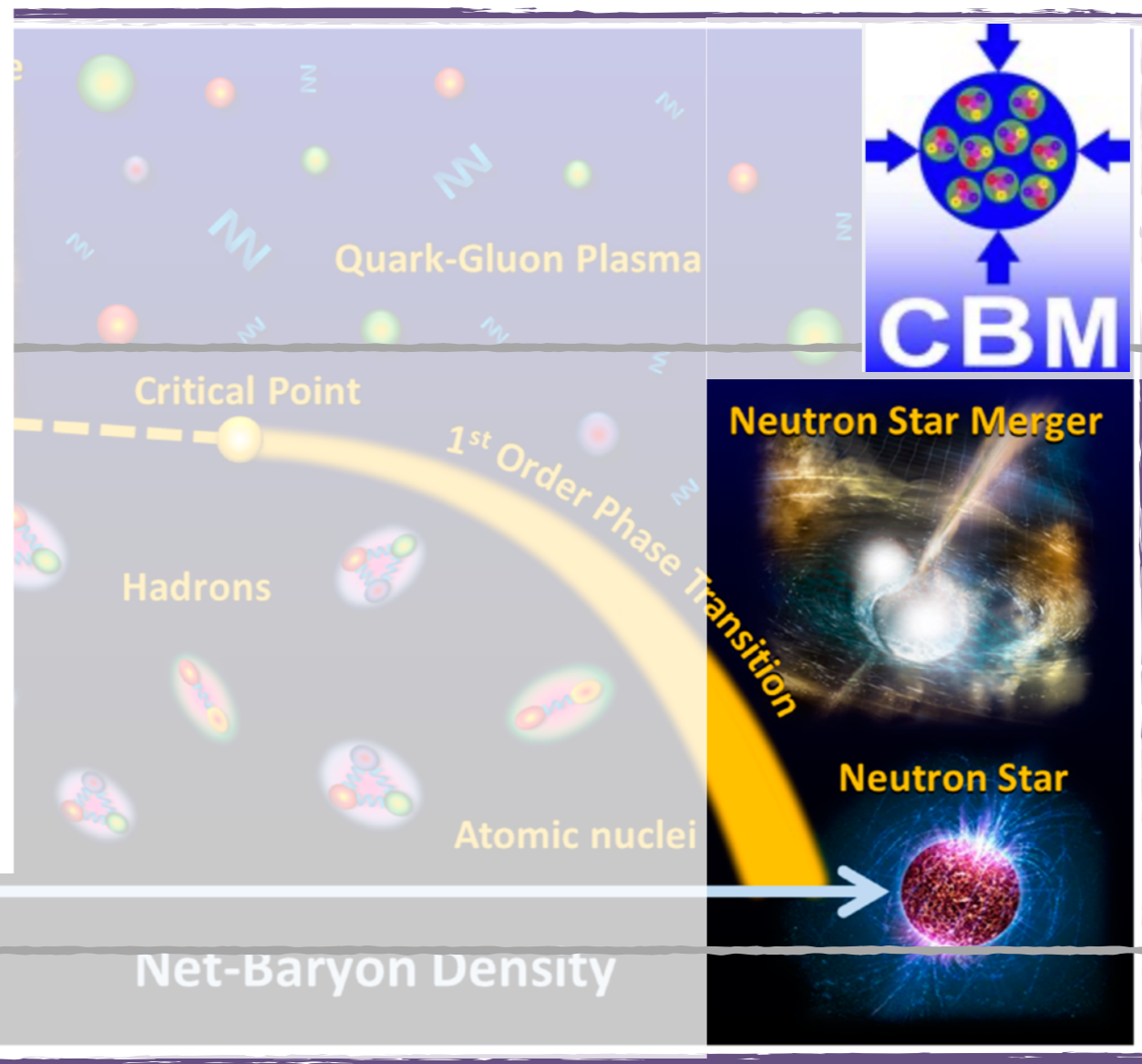
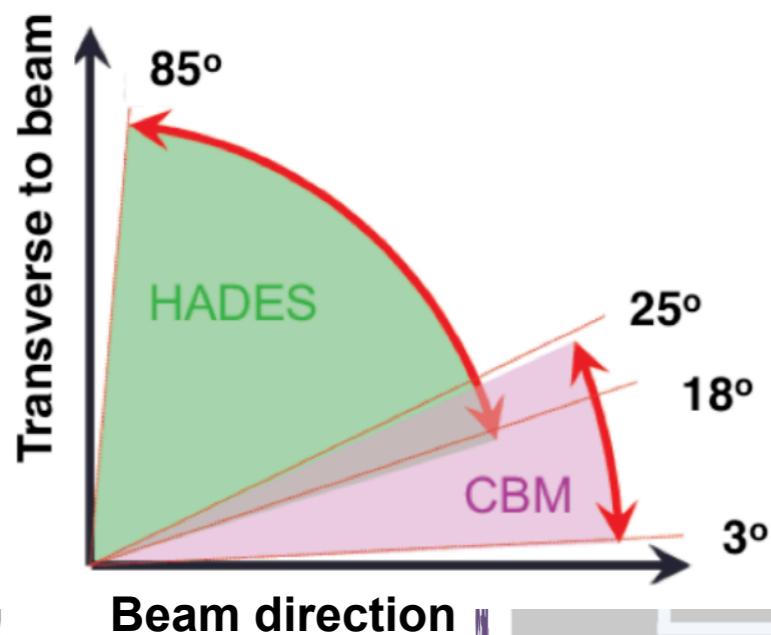
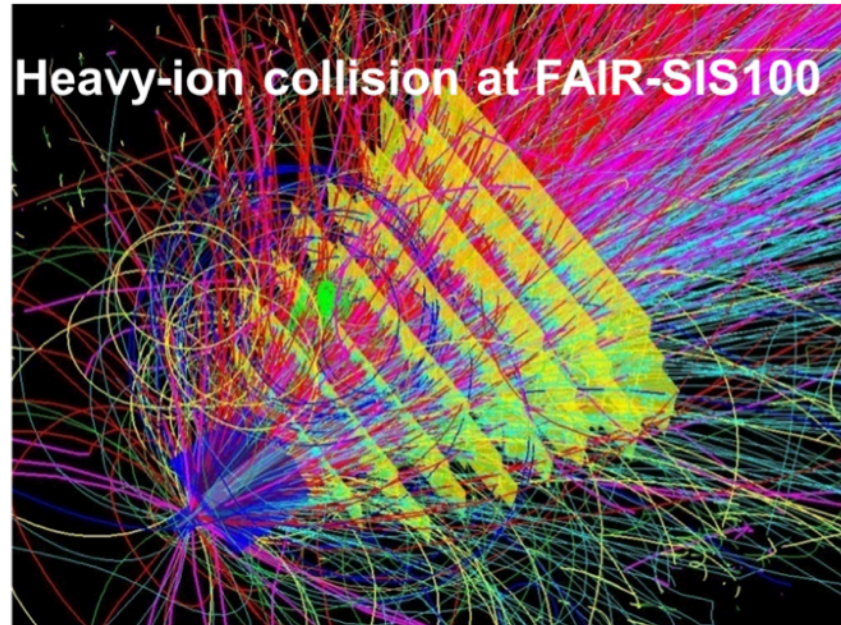
Reaction time
 $t \sim 10 \text{ ms}$
 (GW170817)

Temperature
 $T < 10 \text{ MeV}$

Density
 $n < 10n_0$

Lifetime
 $t \sim \text{infinity}$

CBM and HADES future



Temperature
 $T < 50 \text{ MeV}$

Density
 $n < 2 - 6n_0$

Reaction time
 $t \sim 10 \text{ ms}$
(GW170817)

Temperature
 $T < 10 \text{ MeV}$

Density
 $n < 10n_0$

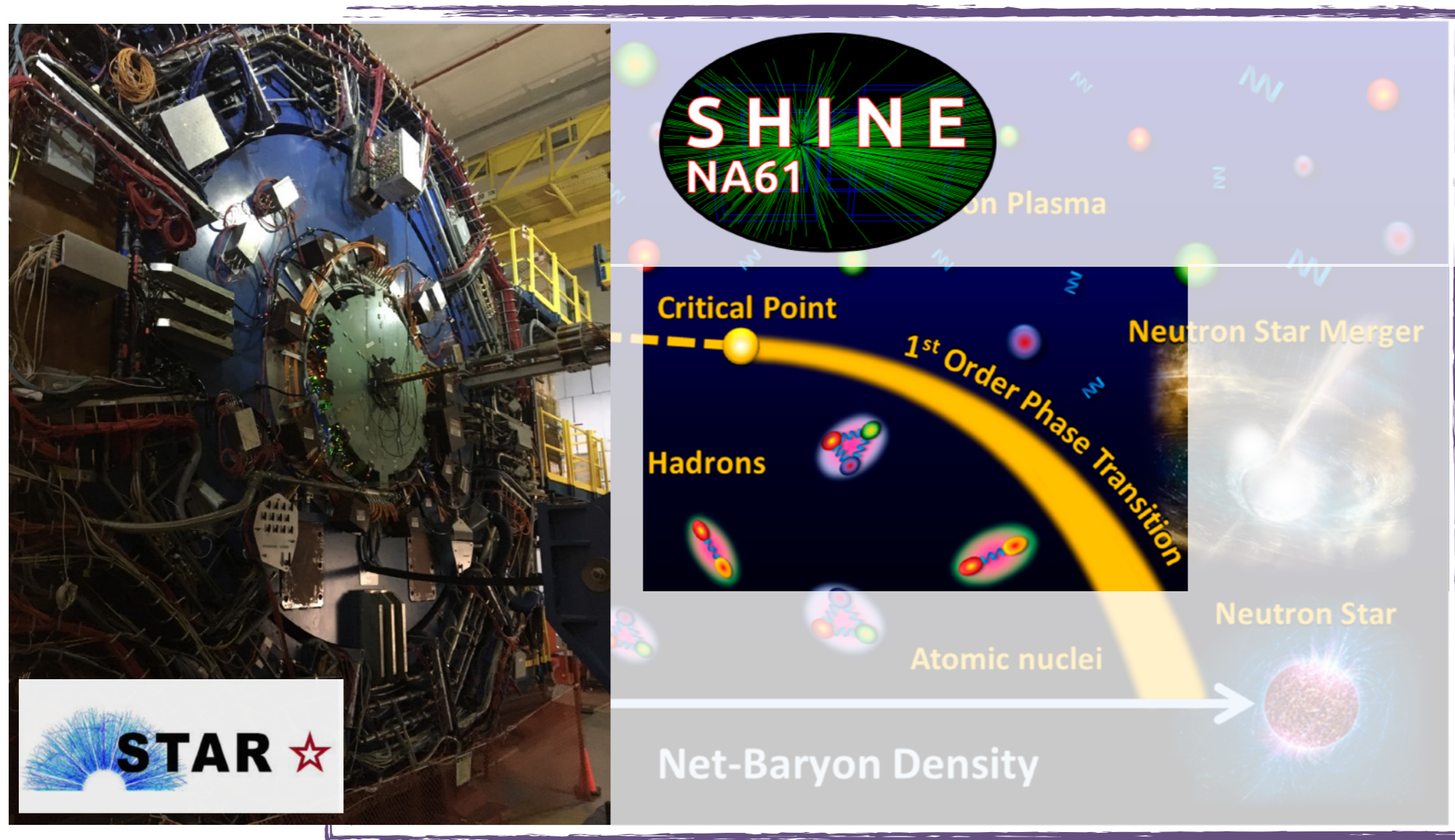
Lifetime
 $t \sim \text{infinity}$

<https://www.researchgate.net>

SIS-100:	Temperature $T < 120 \text{ MeV}$	Density $n < 8n_0$	Reaction time $t \sim 10^{-23} \text{ s}$
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Phase transitions

Exploration of unknown QCD territory: moderate μ_B

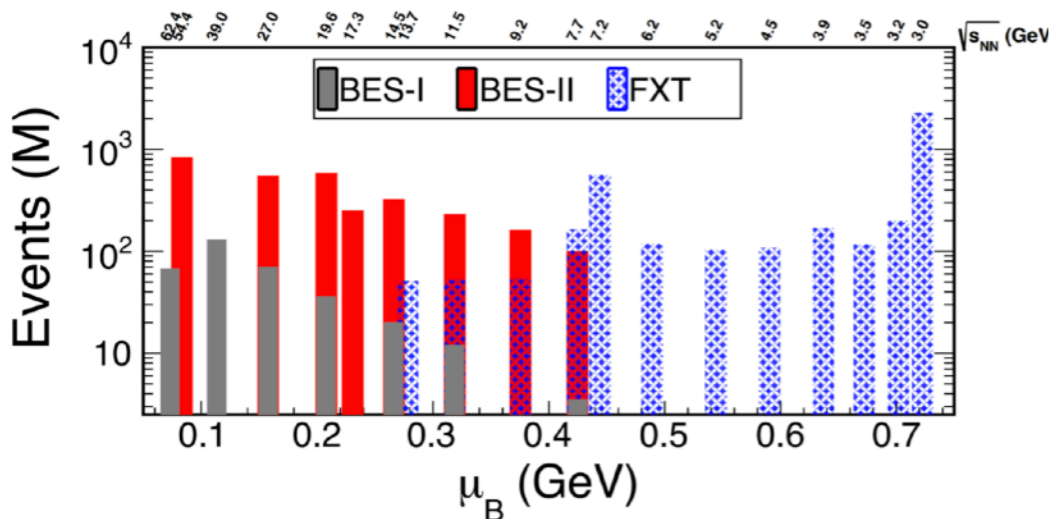
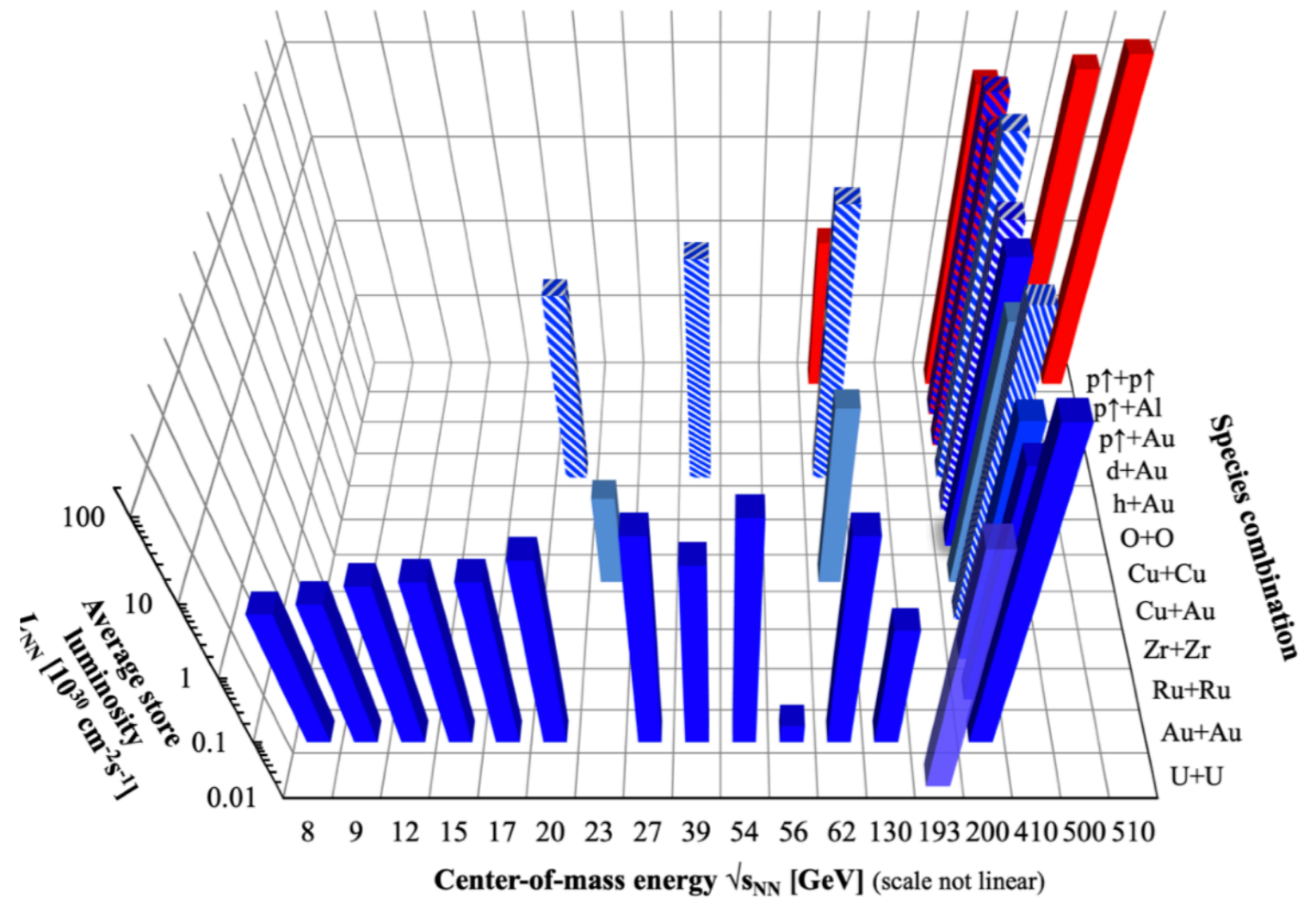
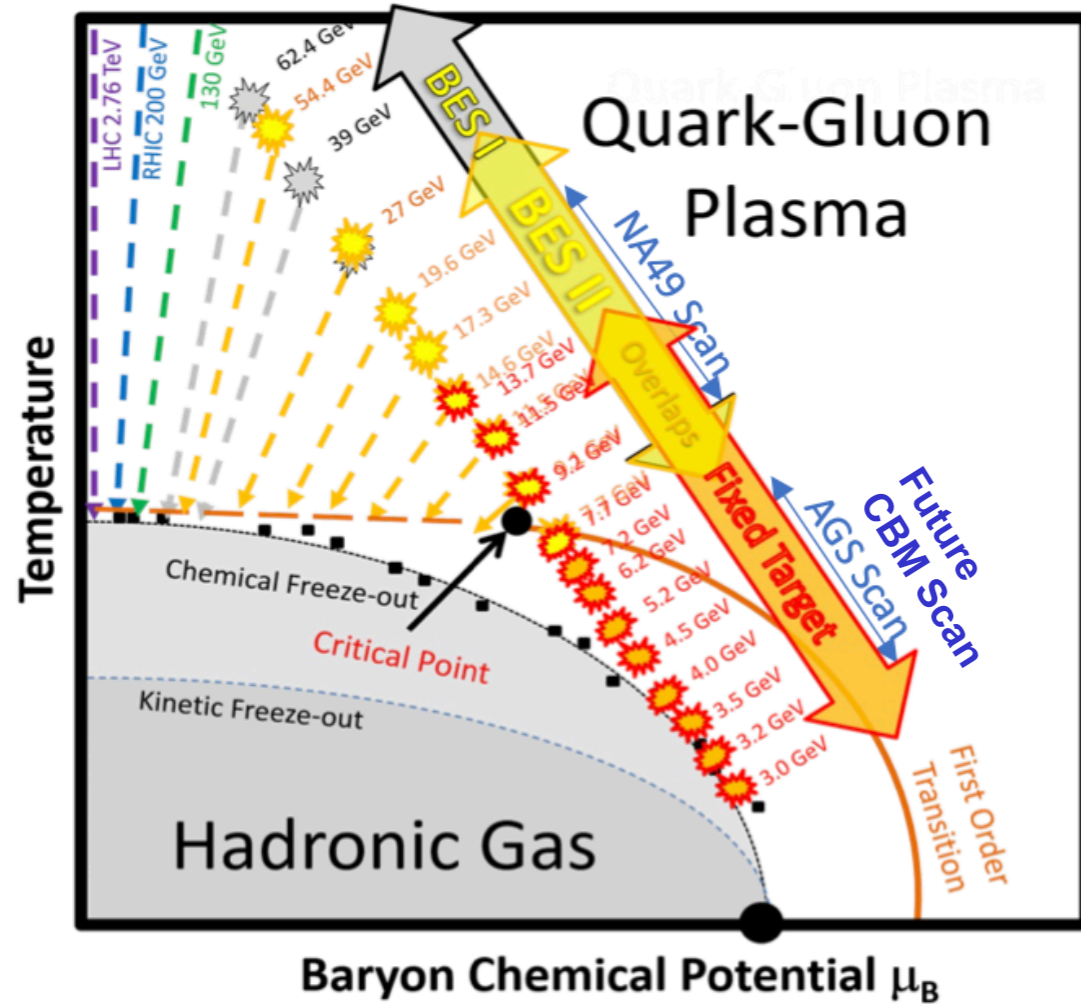


Searching for phase transitions signatures

<https://www.researchgate.net>

Beam Energy Scan at STAR

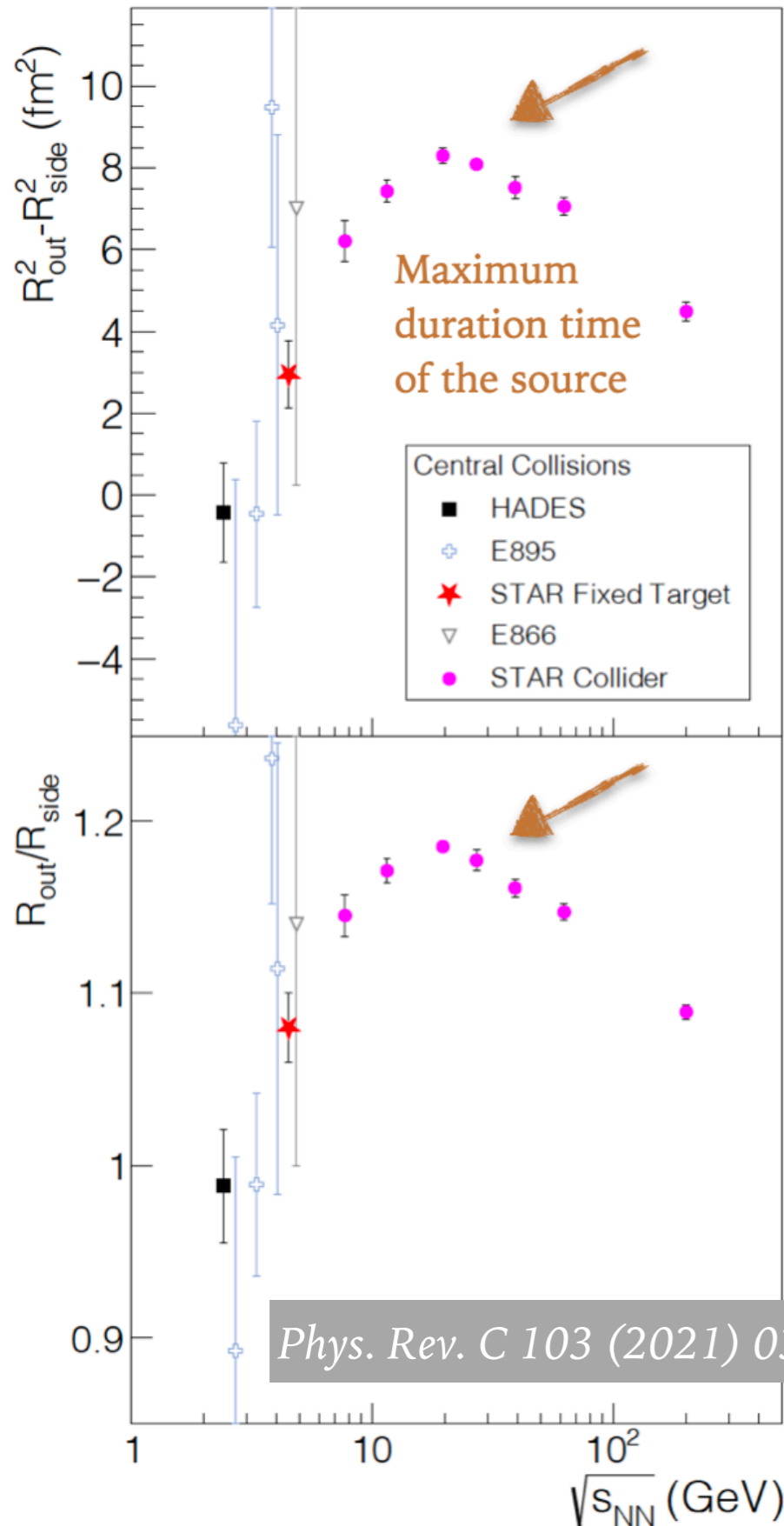
RHIC energies, species combinations and luminosities (Run-1 to 22)



- Beam Energy Scan: Au+Au 3-62.4 GeV**
1. Search for turn-off of QGP signatures
 2. Search for signals of the first-order phase transition
 3. Search for QCD critical point

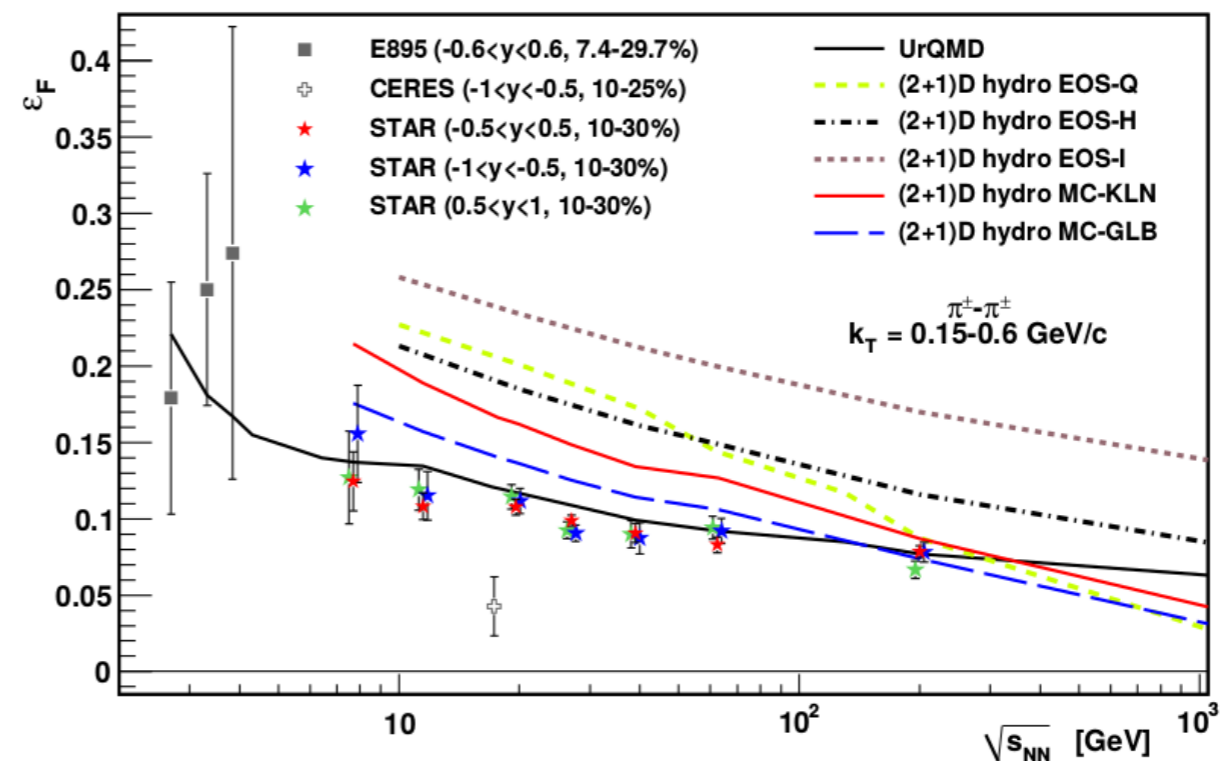
Fixed-Target Program: Au+Au: 3.0-13.7 GeV

Looking for phase transition

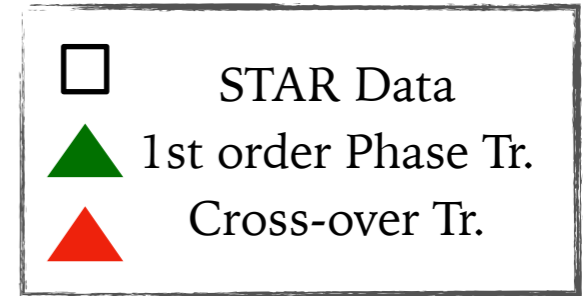
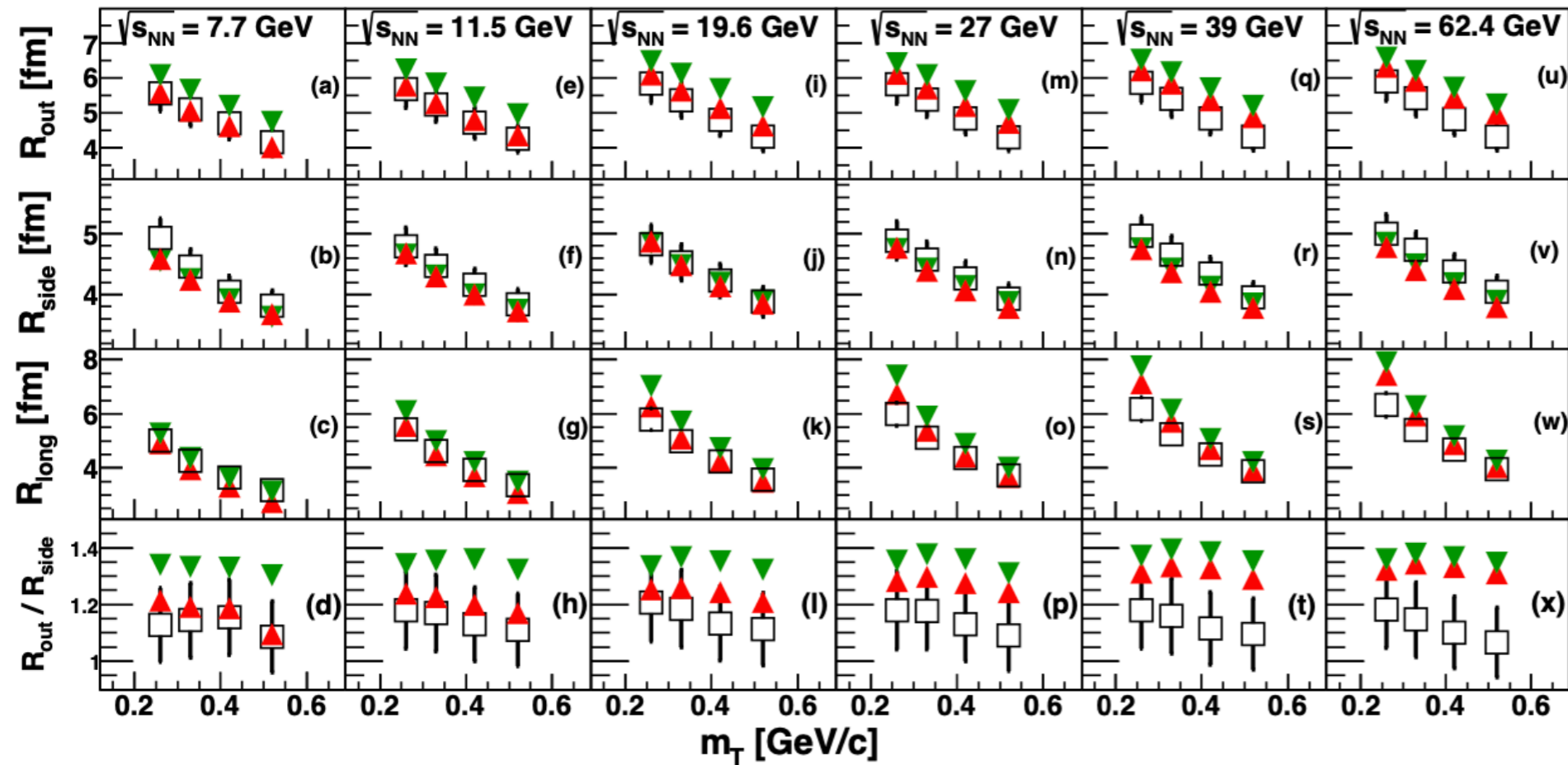


- R_{side} spatial source evolution in the transverse direction
- R_{out} related to spatial and time components
- R_{out}/R_{side} signature of phase transition
- $R_{out}^2 - R_{side}^2 = \Delta\tau^2 \beta_t^2$; $\Delta\tau$ – emission time
- R_{long} temperature of kinetic freeze-out and source lifetime

System evolves faster in the reaction plane



How to measure phase transition?



vHLEE (3+1)-D viscous hydrodynamics: Iu. Karpenko, P. Huovinen, H. Petersen, M. Bleicher; Phys.Rev. C 91, 064901 (2015), arXiv:1502.01978, 1509.3751



HadronGas + Bag Model → 1st order PT ; P.F. Kolb, et al, PR C 62, 054909 (2000)

Chiral EoS → crossover PT (XPT); J. Steinheimer, et al, J. Phys. G 38, 035001 (2011)

vHLEE + UrQMD model verify sensitivity of HBT measurements to the first-order phase transition

Phys. Rev. C 96 (2017) no.2, 024911

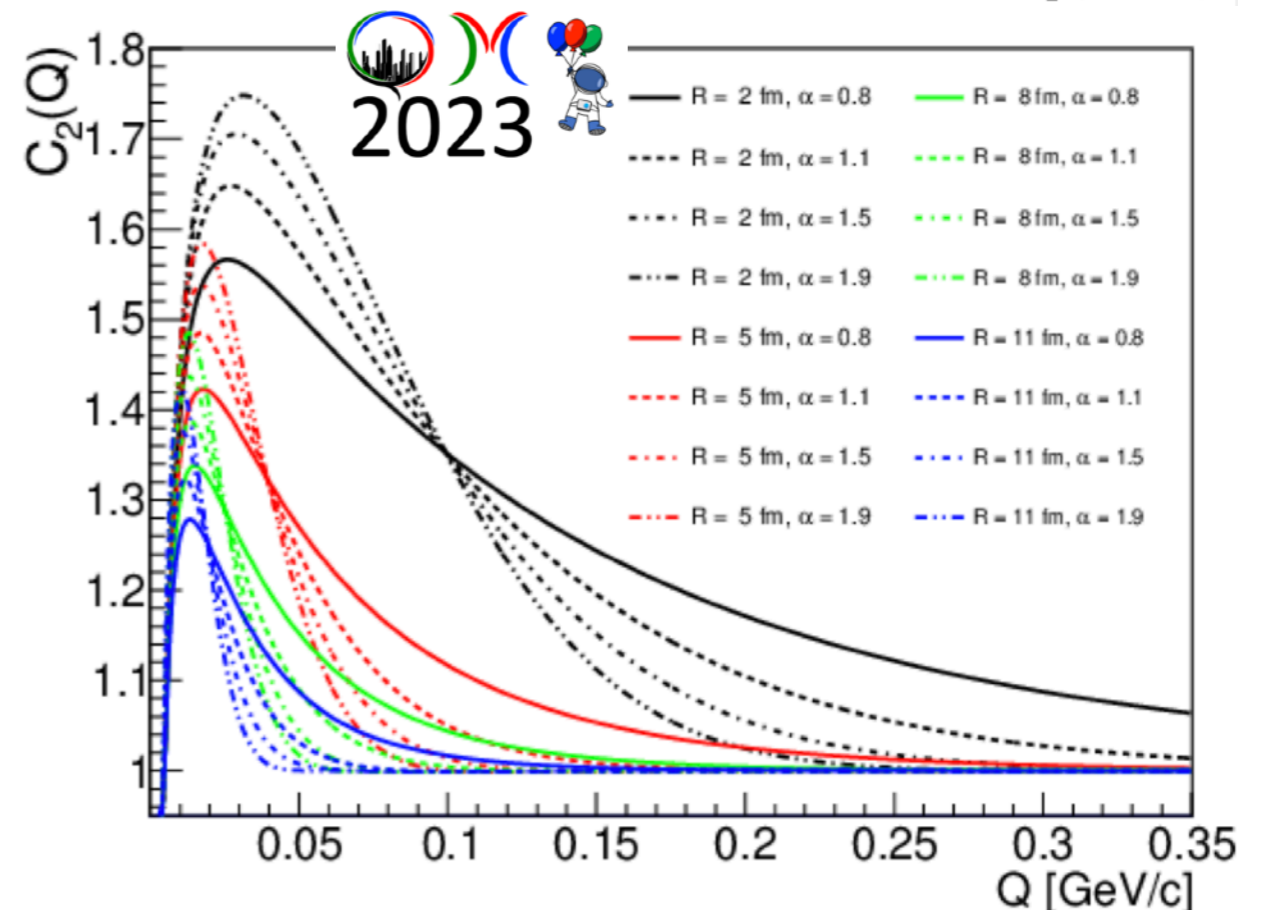
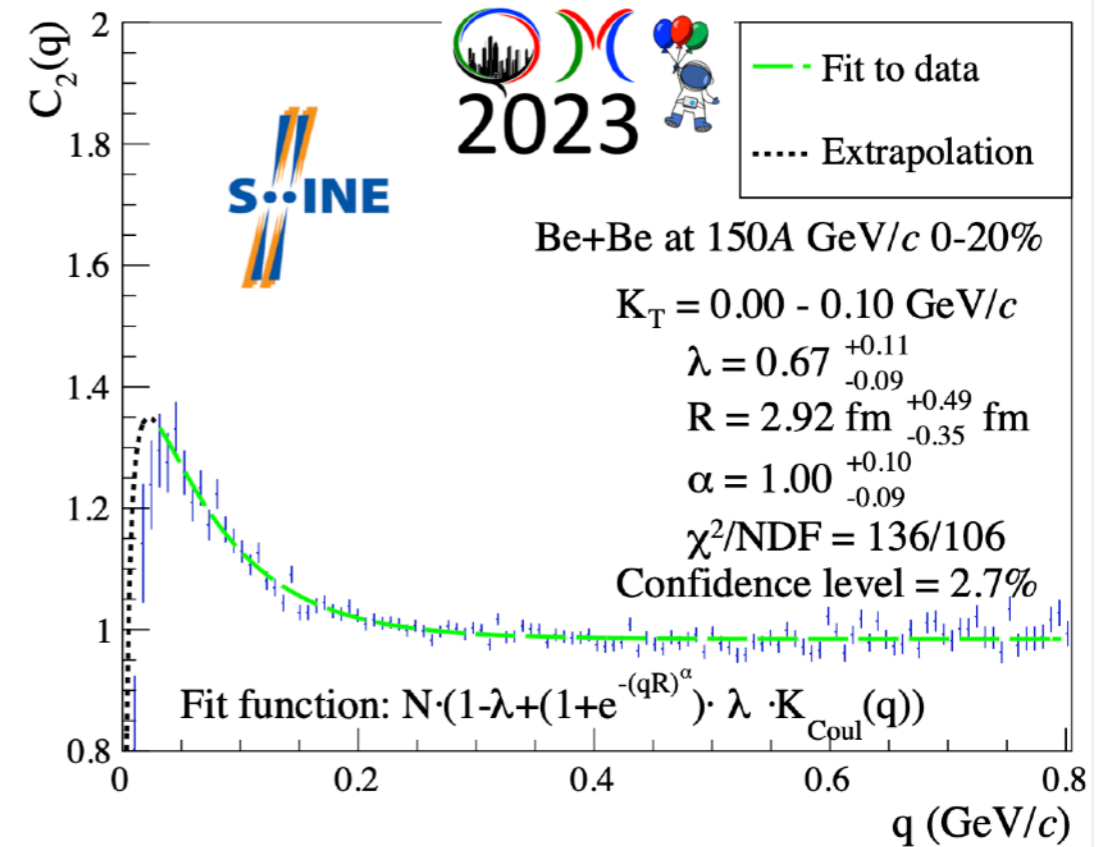
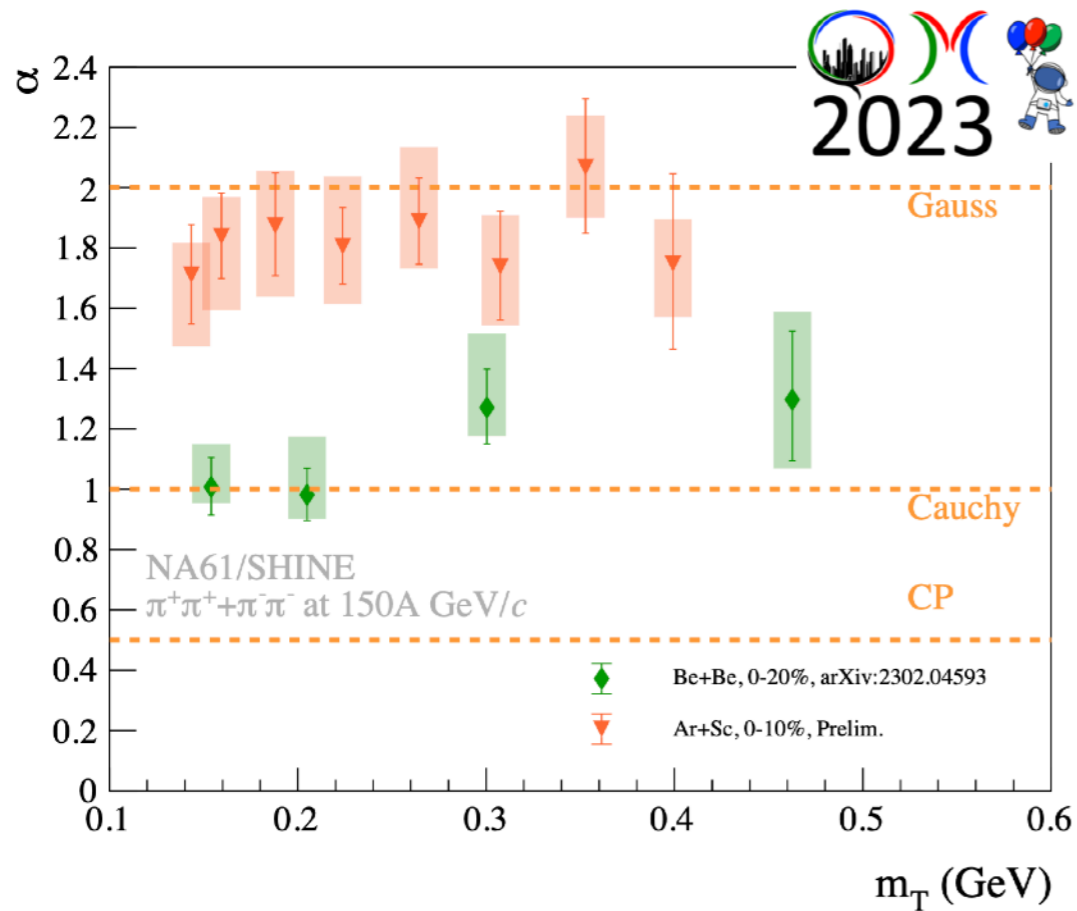
How to measure a phase transition? CP?

Levy-stability index

$\alpha = 2$ - Gauss

$\alpha = 1$ - Cauchy

$\alpha = 0.5$ - expected near the CP

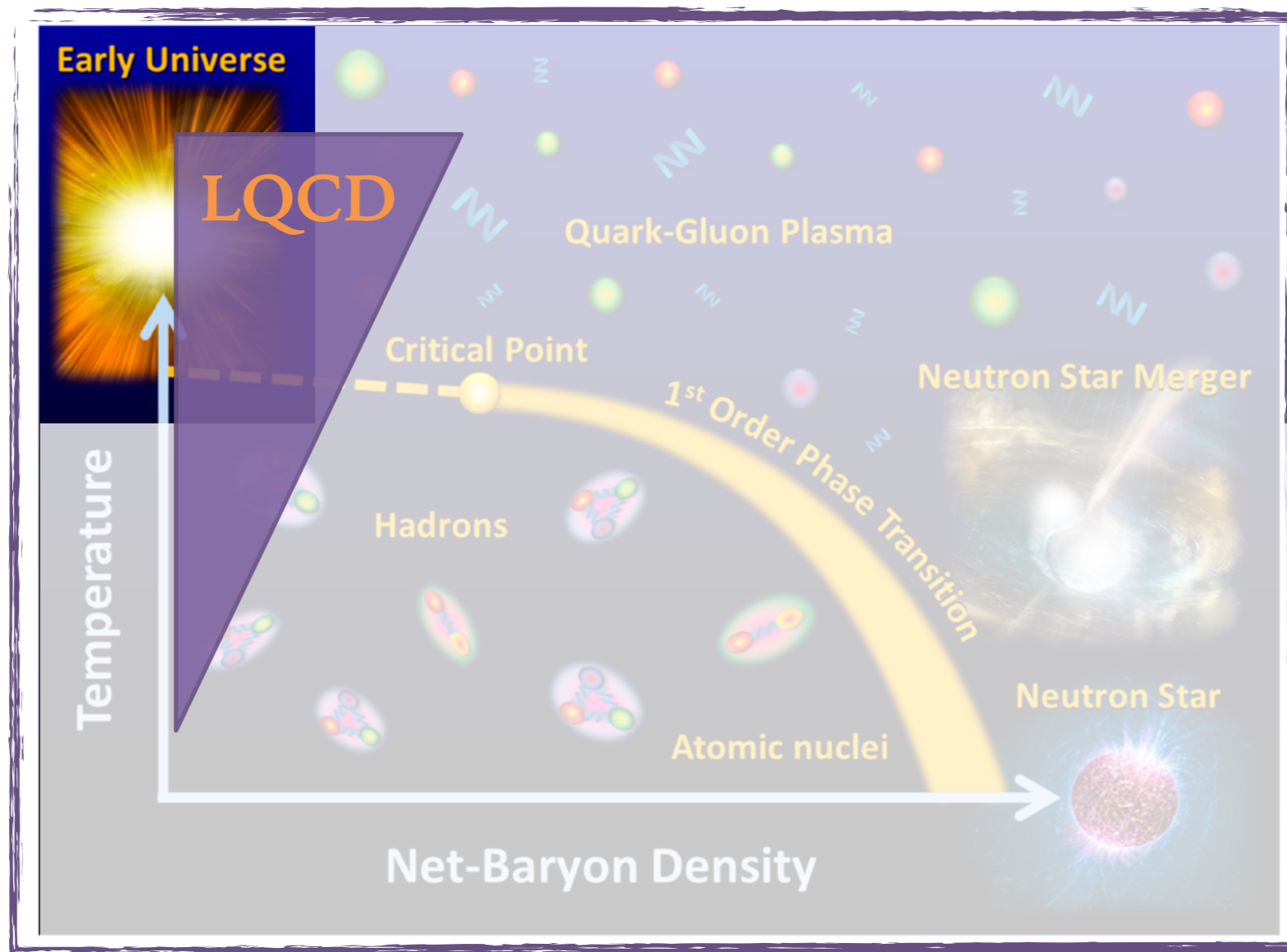


Levy-stability index

Far away from the CP

Early Universe

Exploration of ~~unknown~~ from lattice QCD: vanishing μ_B

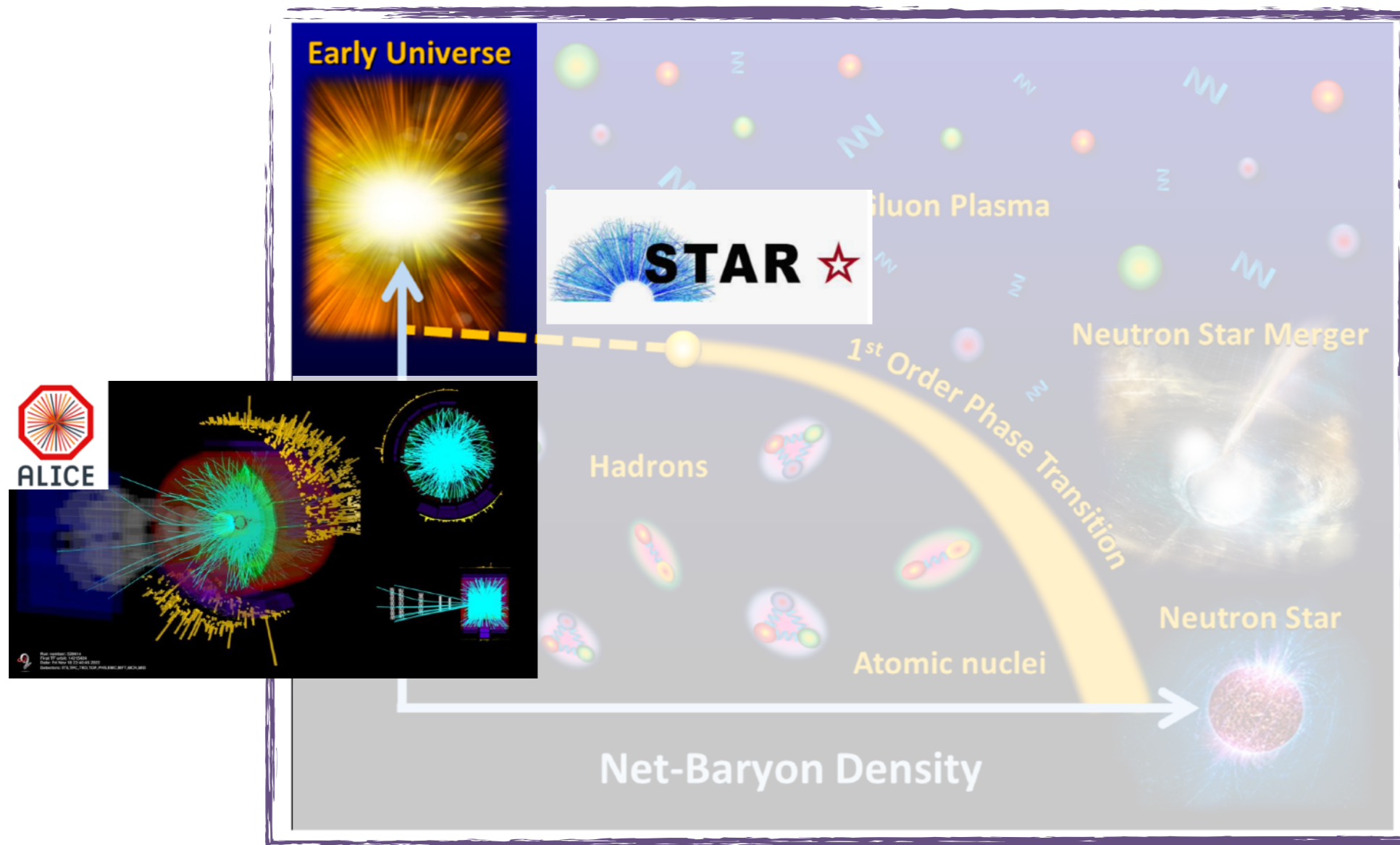


Studies of
Early
Universe

<https://www.researchgate.net>

Early Universe

Exploration of ~~unknown~~ from lattice QCD: vanishing μ_B

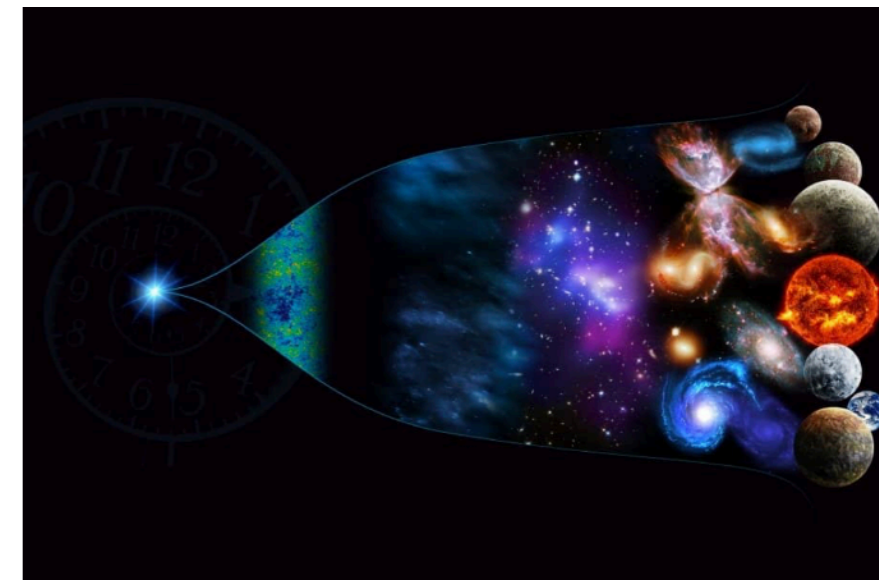


Studies of
Early
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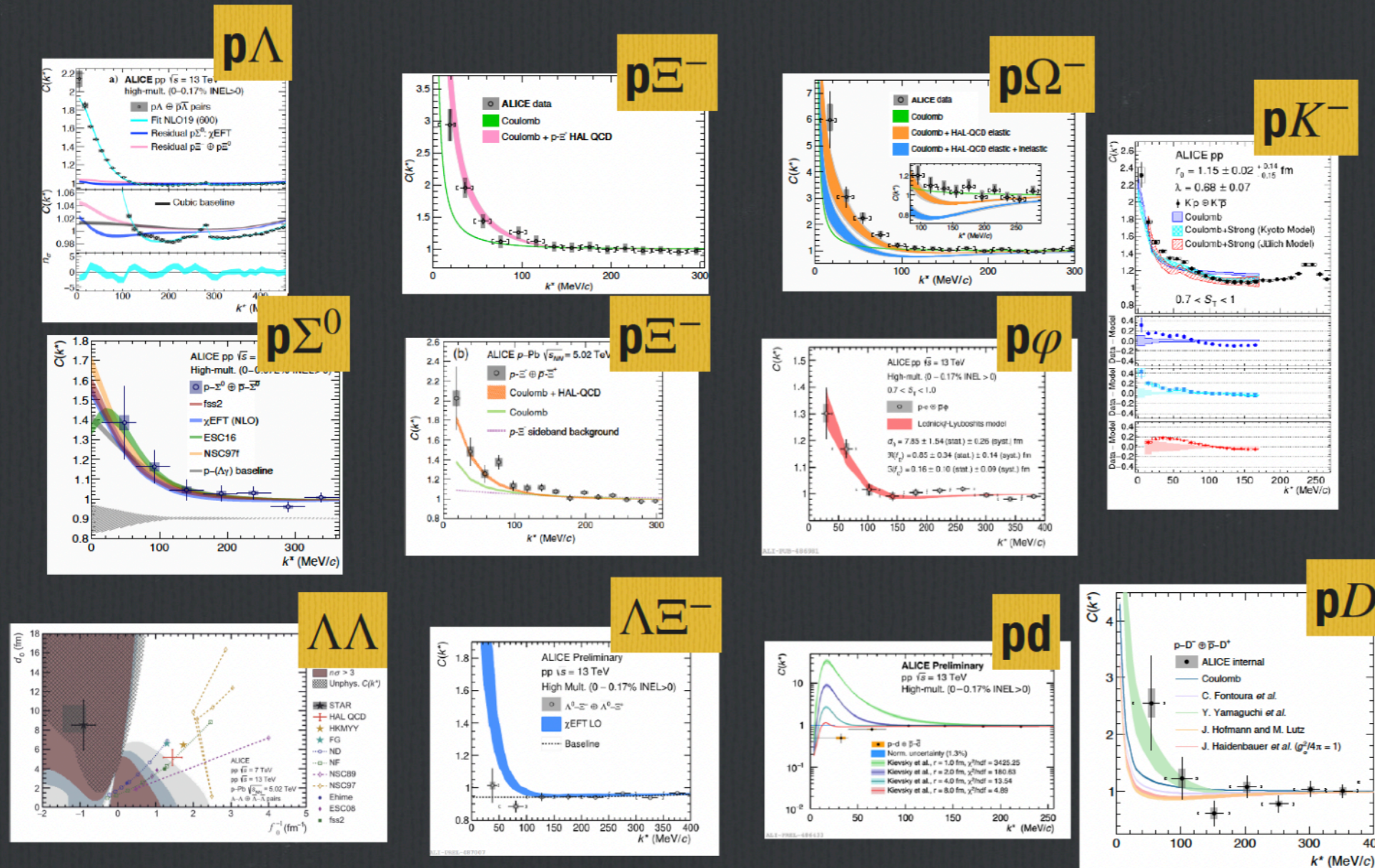
<https://www.researchgate.net>

Early Universe, vanishing μ_B

- Probe the condition \sim Early Universe,
- Very high abundances of particles produced,
- Statistical heaven to study exotic particles,
- Studies of heavy-flavor,
- Incredible laboratory of anti-matter.



<https://insidetheperimeter.ca/the-universe-before-atoms/>



CATS: EPJA 78 (2018)
 Projector: EPJC 82 (2022)
 Review 1: Prog.Part.Nucl.Phys. 112 (2020)
 Review 2: Ann.Rev.Nucl.Part.Sci. 71 (2021)
 p - ϕ bound state: arXiv:2212.12690
 p - K : PRL 124 (2020) 092301
 p - K : PLB 822 (2021), EPJC (2022)
 p - p , p - Λ , Λ - Λ : PRC 99 (2019) 024001
 Λ - Λ : PLB 797 (2019) 134822
 p - Ξ : PRL 123 (2019)
 p - Ξ , p - Ω : Nature 588 (2020) 232–238
 p - Σ^0 : PLB 805 (2020) 135419
 p - ϕ : PRL 127 (2021)
 p - \bar{p} , Λ - $\bar{\Lambda}$, p - $\bar{\Lambda}$: PLB 829 (2022)
 p - Λ : PLB 832 (2022) 137272
 Λ - Ξ : PLB 137223 (2022)
 D - p : PRD 106, 052010 (2022)
 ppp , $pp\Lambda$: arXiv:2206.03344

Summary

Collisions of (heavy) ions give us access to:

High T, low μ_B

- Cross-over to QGP \rightarrow Investigations of properties of QGP
- LQCD: no CP indication for $\mu_B/T < 3$

Lower T, high μ_B

- Phase structure?
 - first-order phase transition?
 - CP?
 - New phases of QCD?
- Characterization of dense matter,
- EOS?
- Properties of hadrons?



Collisions of (heavy) ions give us access to: High T, low μ_B

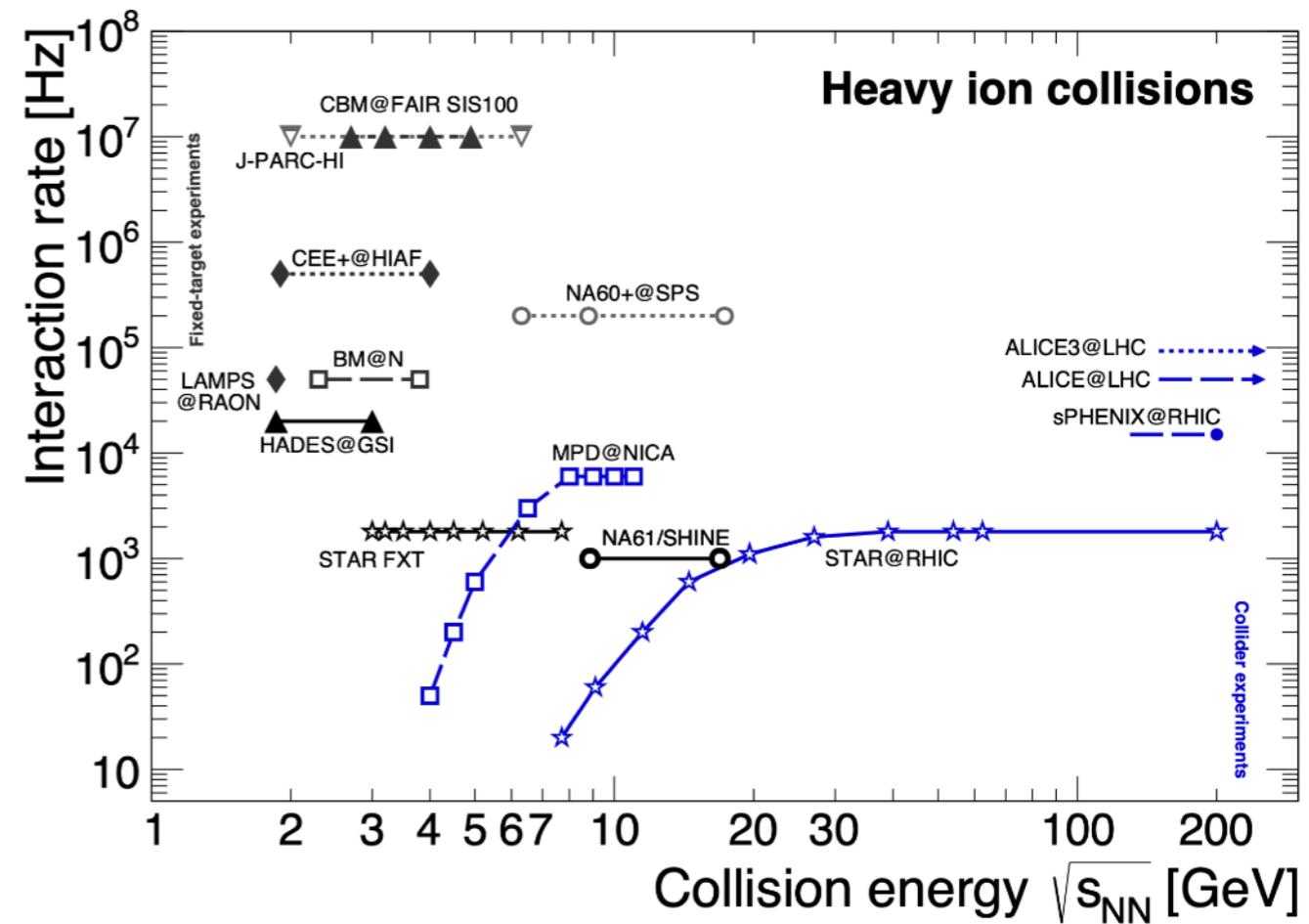
- Cross-over to QGP \rightarrow Investigations of properties of QGP
- LQCD: no CP indication for $\mu_B/T < 3$



Lower T, high μ_B

- Phase structure?
 - first-order phase transition?
 - CP?
 - New phases - $f_0(1370)$?
- Characterization
- EOS?
- Properties of hadrons

Thank you



NSM and HIC

Top row: simulation of neutron stars mergers

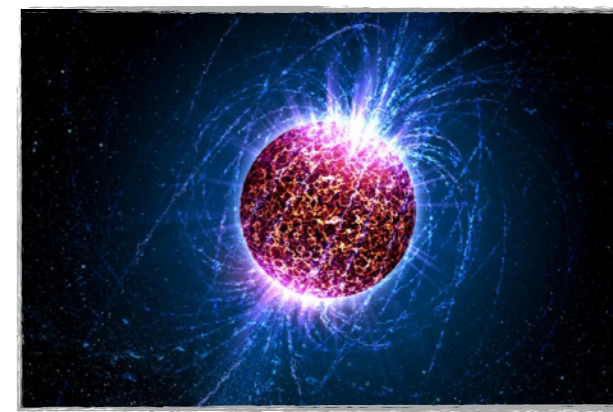
2 neutron stars of $1.35 M_{\odot}$ each,

merging into a single object ($2R \sim 10 \text{ km}$, $n \simeq 5n_0$, $T \leq 20 \text{ MeV}$).

Overlap region: $t \simeq 20 \text{ ms}$, $n \simeq 2n_0$, $T \simeq 75 \text{ MeV}$

▲ - max. temperature

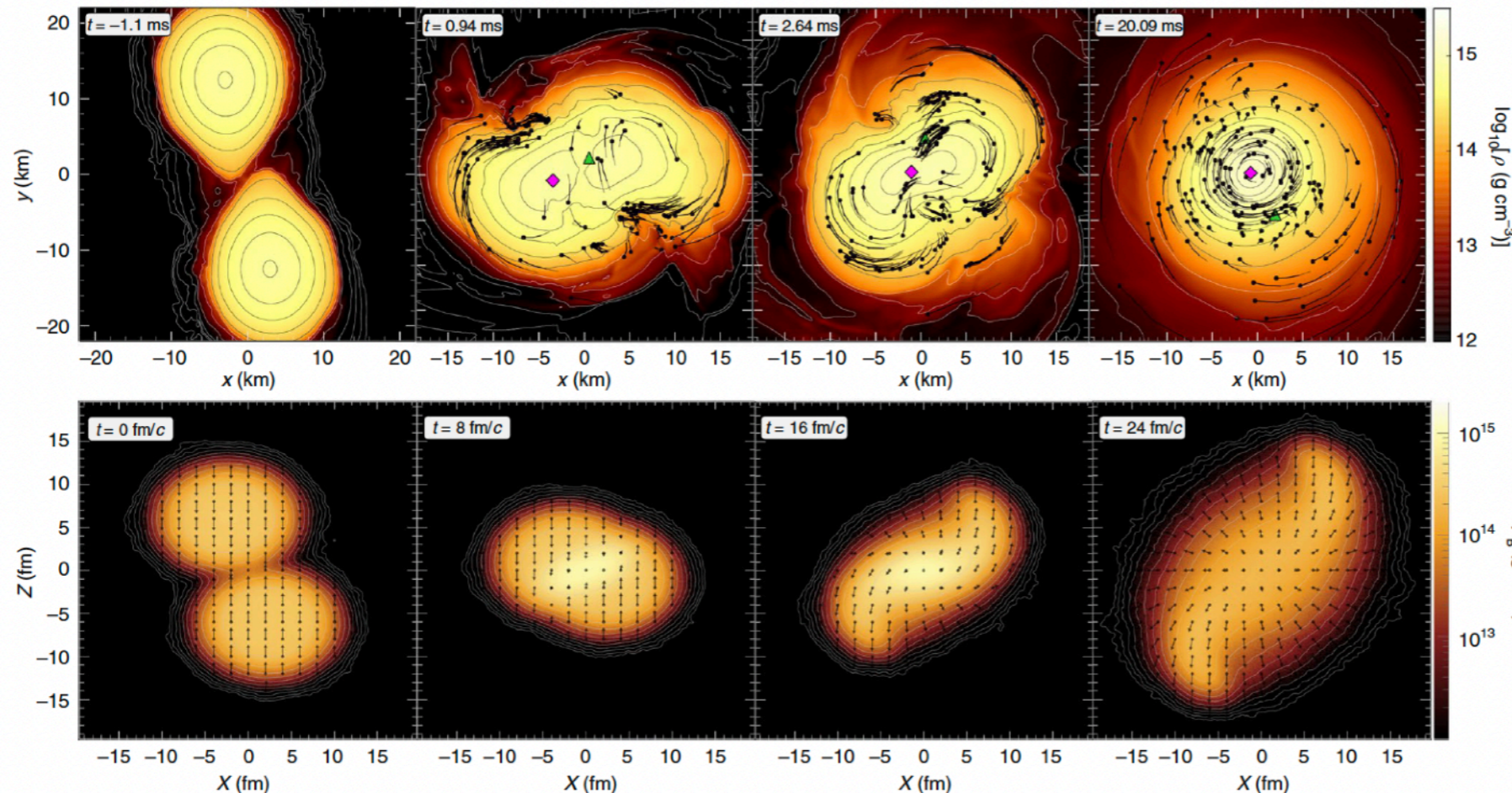
◆ - max. density



Bottom row: non-central-collision Au+Au at $\sqrt{s_{NN}} = 2.42 \text{ GeV}$

$n \simeq 3n_0$, $T \simeq 80 \text{ MeV}$

HADES, Nature Phys. 15, 1040–1045 (2019)



Similar densities and temperatures are achieved.

Space and time scales are vastly different (km - NS, fm - HIC).

The collision events differ in duration by 20 orders of magnitude.

Lednický-Lyuboshitz model

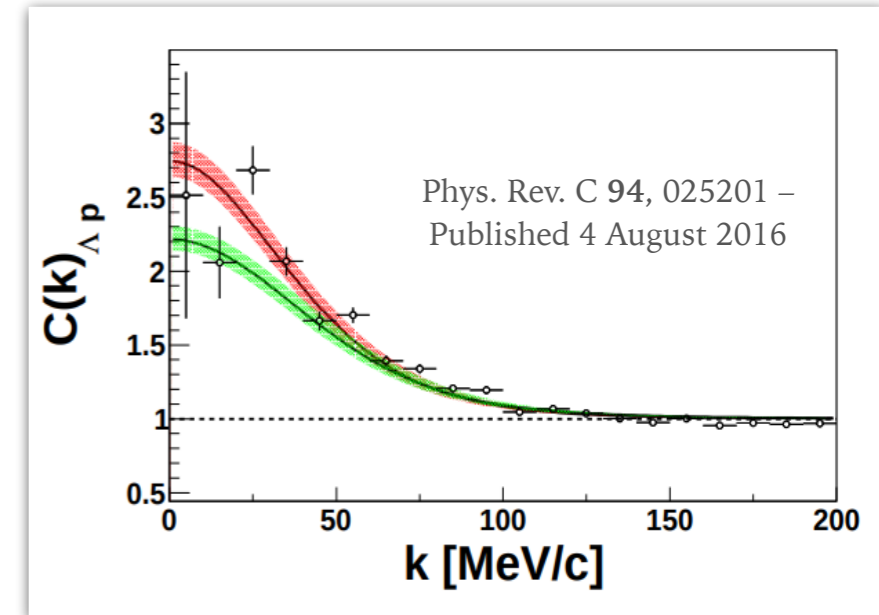
Model	$f_0^{S=0}$ (fm)	$f_0^{S=1}$ (fm)	$d_0^{S=0}$ (fm)	$d_0^{S=1}$ (fm)	n_σ	
ND [77]	1.77	2.06	3.78	3.18	1.1	
NF [78]	2.18	1.93	3.19	3.358	1.1	
NSC89 [79]	2.73	1.48	2.87	3.04	0.9	
NSC97 [80]	a	0.71	2.18	5.86	2.76	1.0
	b	0.9	2.13	4.92	2.84	1.0
	c	1.2	2.08	4.11	2.92	1.0
	d	1.71	1.95	3.46	3.08	1.0
	e	2.1	1.86	3.19	3.19	1.1
	f	2.51	1.75	3.03	3.32	1.0
ESC08 [81]	2.7	1.65	2.97	3.63	0.9	
χ EFT	LO [25]	1.91	1.23	1.4	2.13	1.8
	NLO [26]	2.91	1.54	2.78	2.72	1.5
Jülich	A [82]	1.56	1.59	1.43	3.16	1.0
	J04 [83]	2.56	1.66	2.75	2.93	1.4
	J04c [83]	2.66	1.57	2.67	3.08	1.1

S. Acharya *et al.* Phys. Rev. C 99, 024001 – Published 13 Feb 2019

<https://doi.org/10.1103/PhysRevC.99.024001>

parameter scan boundaries : f_0 [0.01, 5.0], d_{0s} [0.01, 2.0] and d_{0t} [0.01, 5.0]

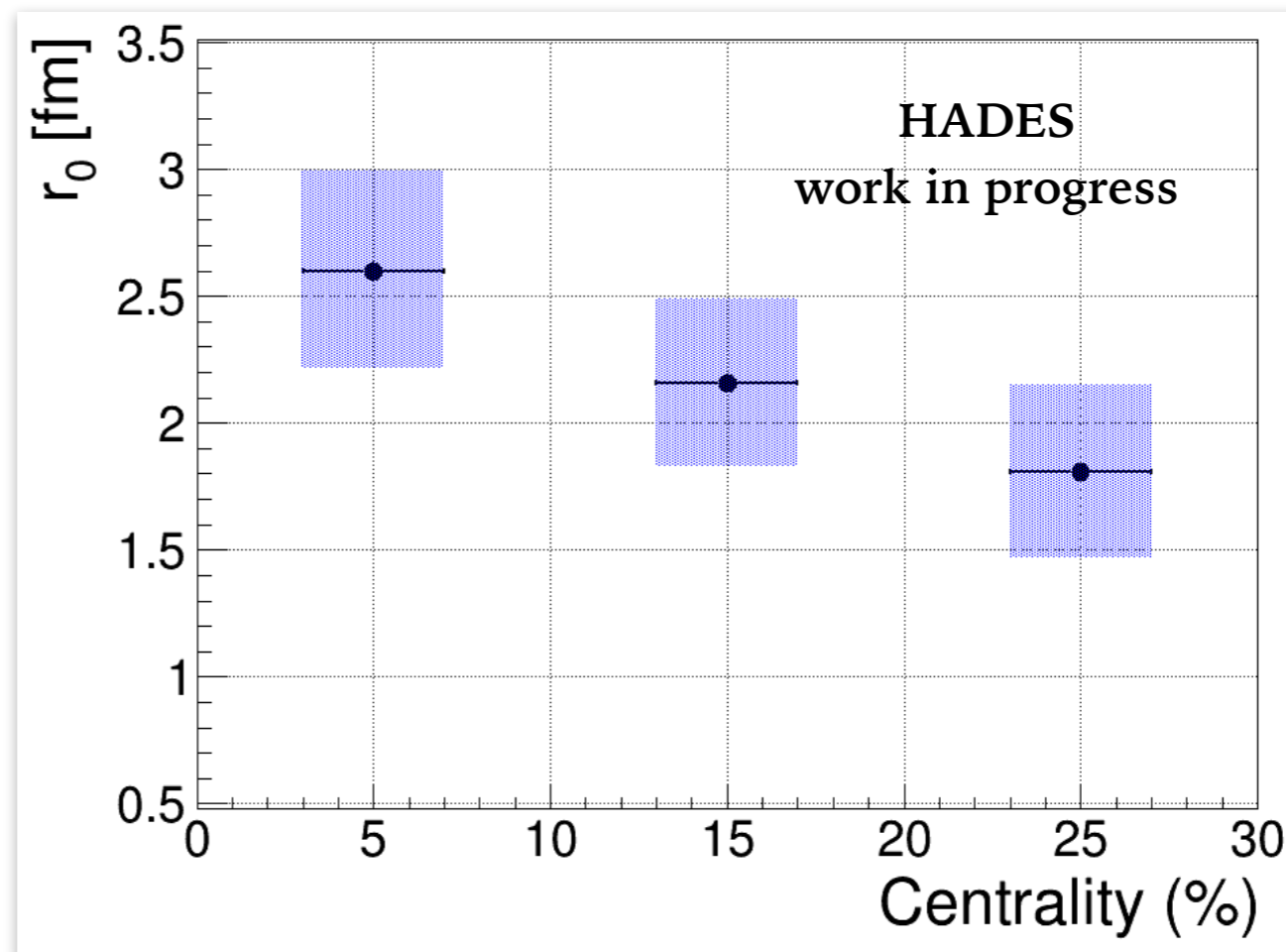
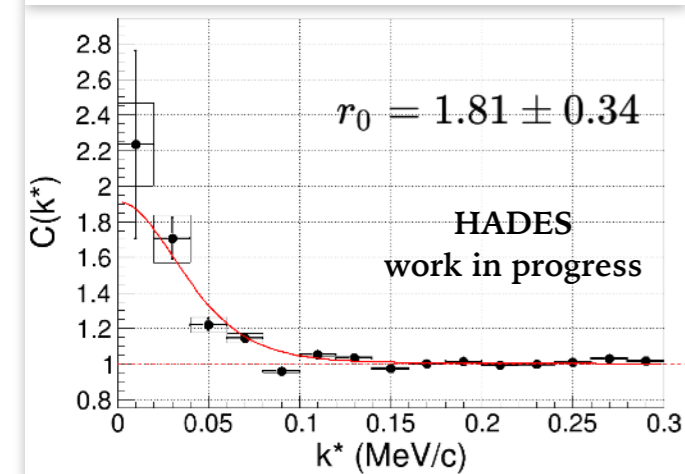
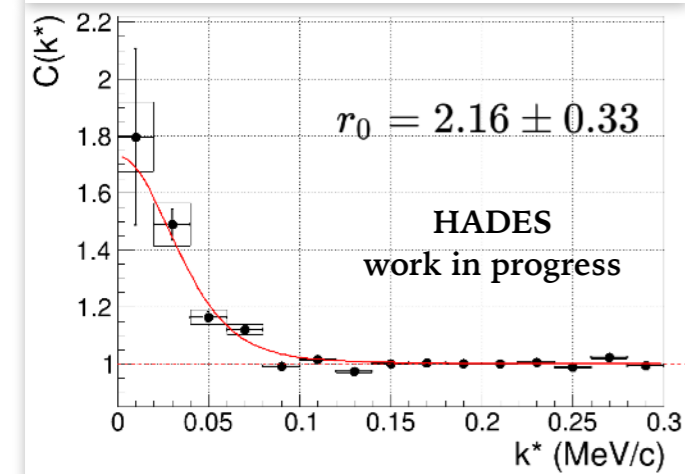
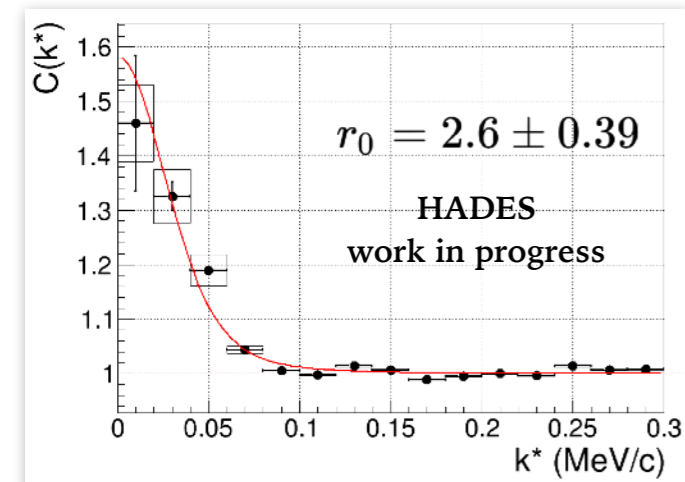
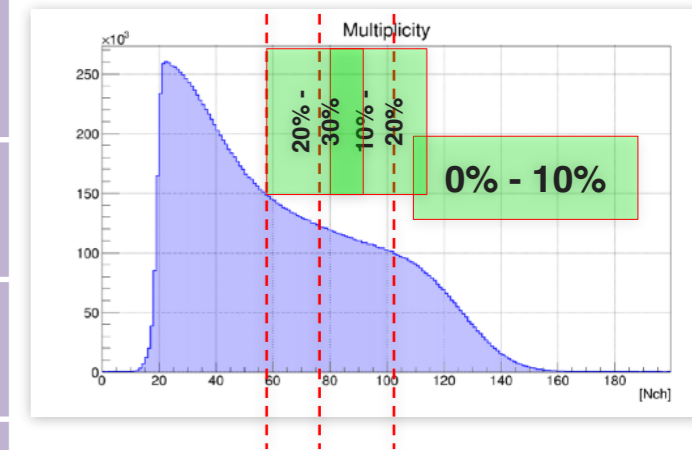
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Parameters	p-Nb (LO)	p-Nb (NLO)
f_{0s}	1.91 fm	2.91 fm
d_{0s}	1.40 fm	2.78 fm
f_{0t}	1.23 fm	1.54 fm
d_{0t}	2.13 fm	2.72 fm
r_0	1.71 ± 0.10	1.62 ± 0.02

Results: centrality dependence

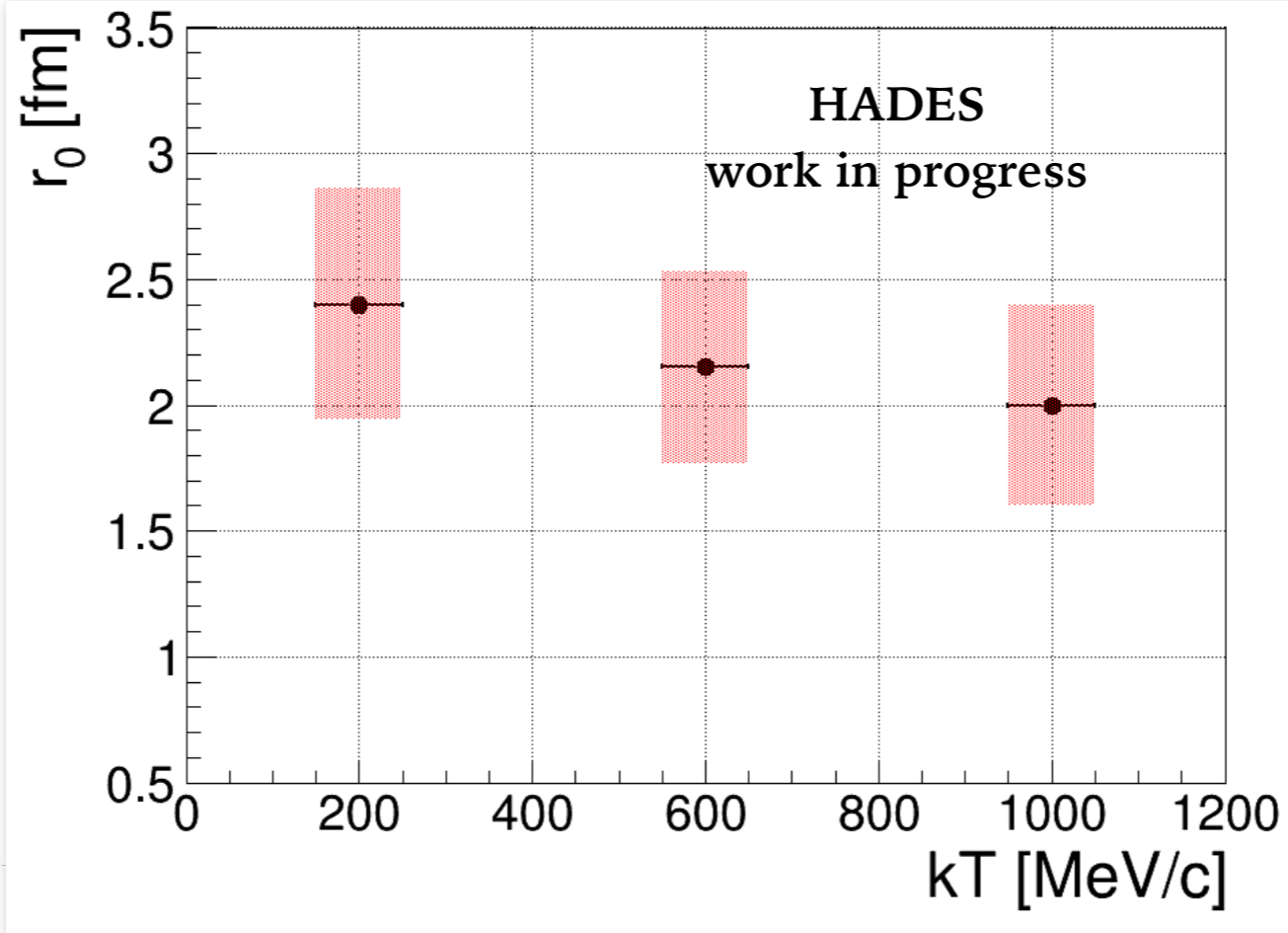
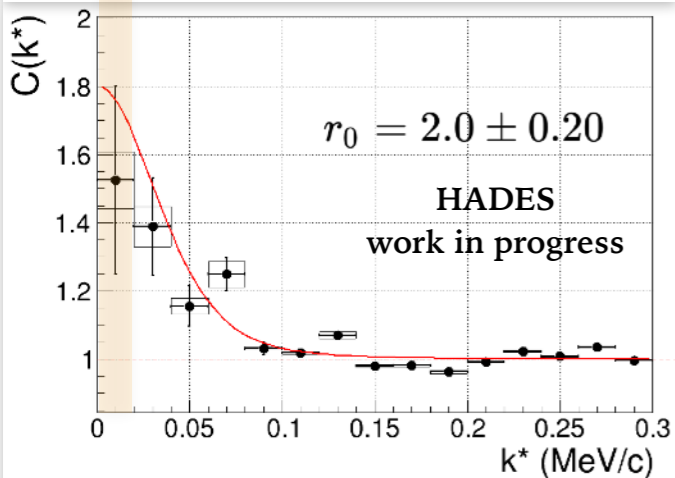
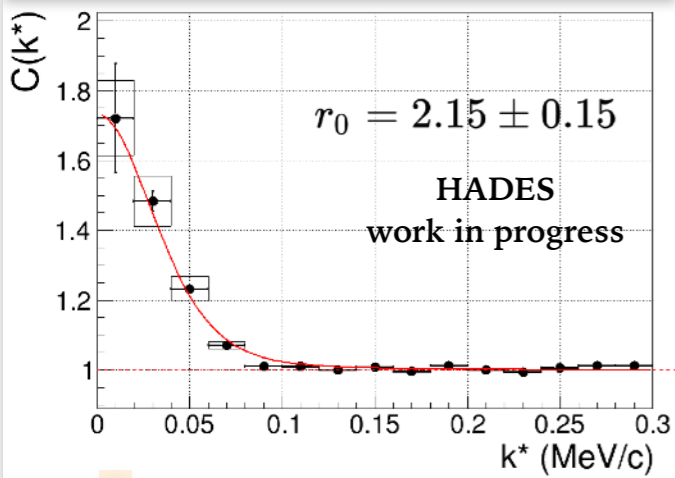
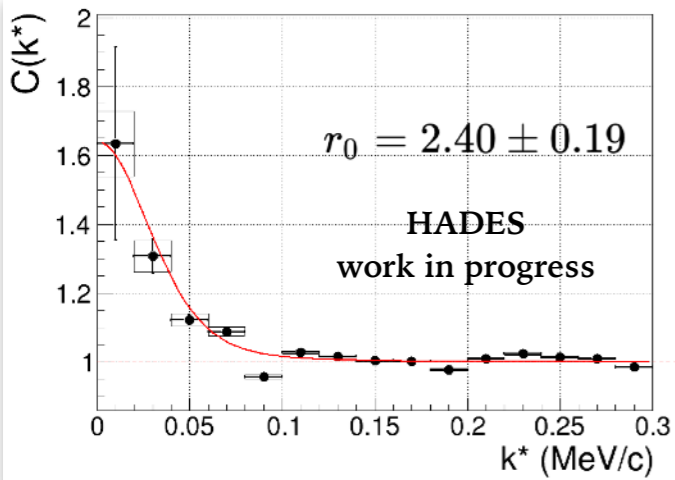
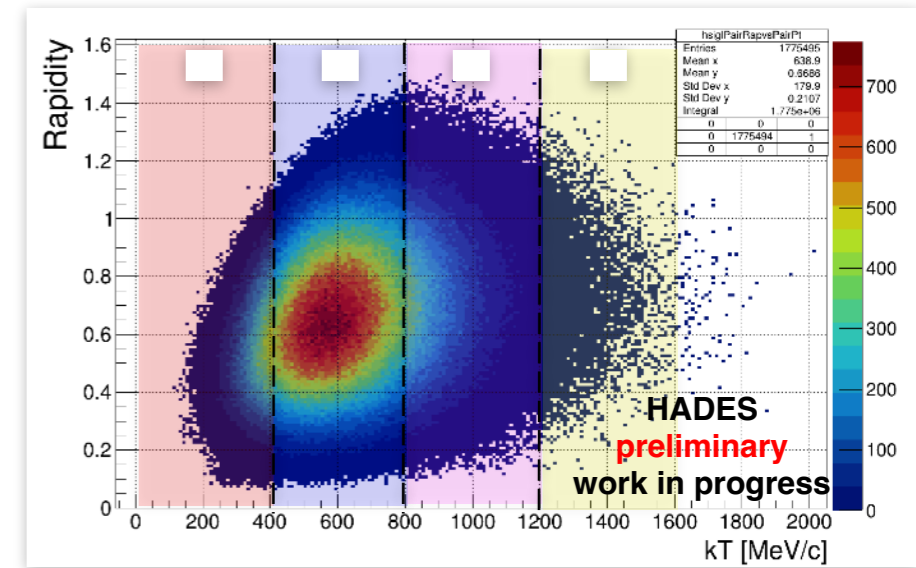
Centrality	Systematic Uncertainty
0 - 10 %	15.30 %
10 - 20 %	15.49 %
20 - 30 %	19.00 %



University

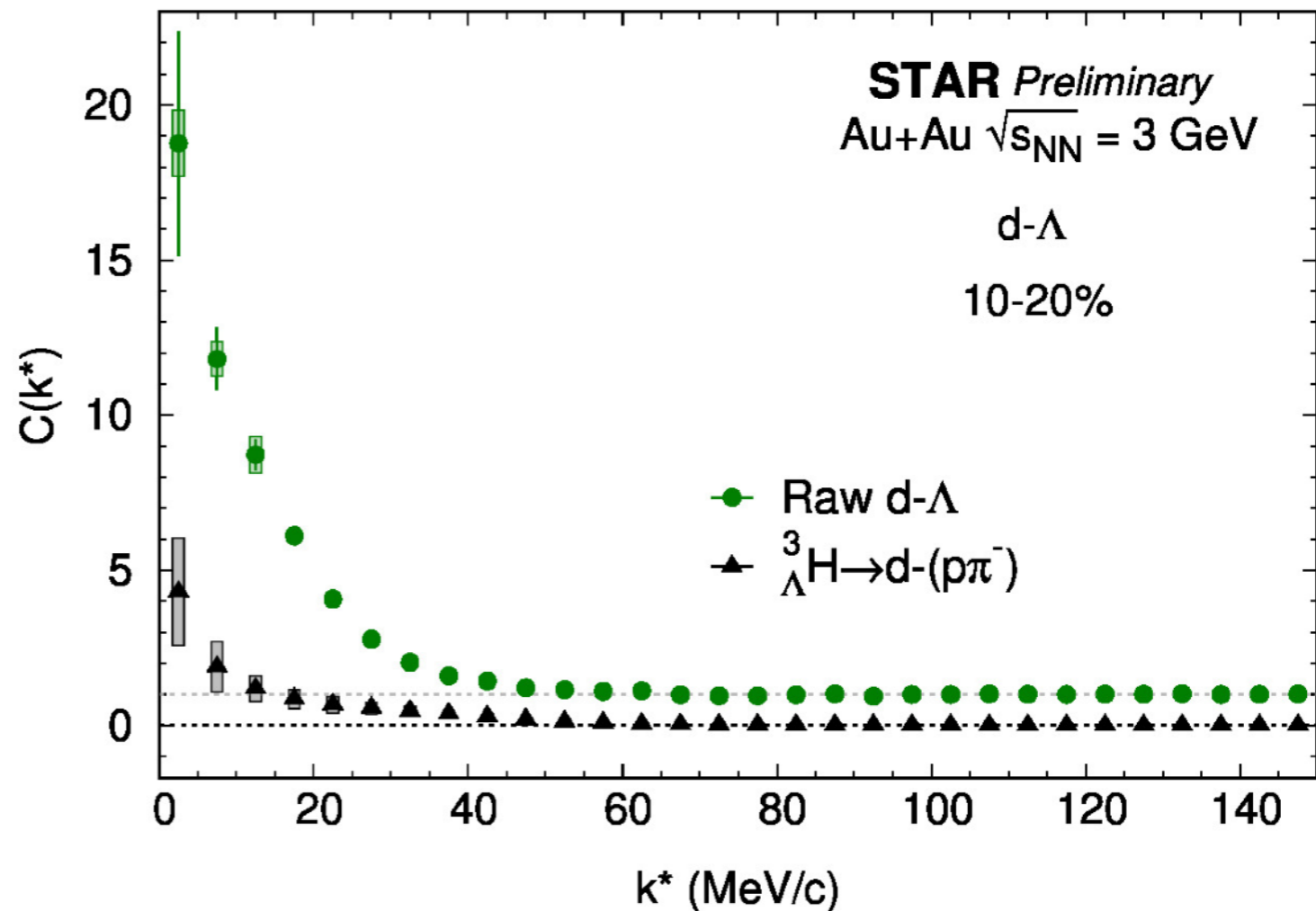
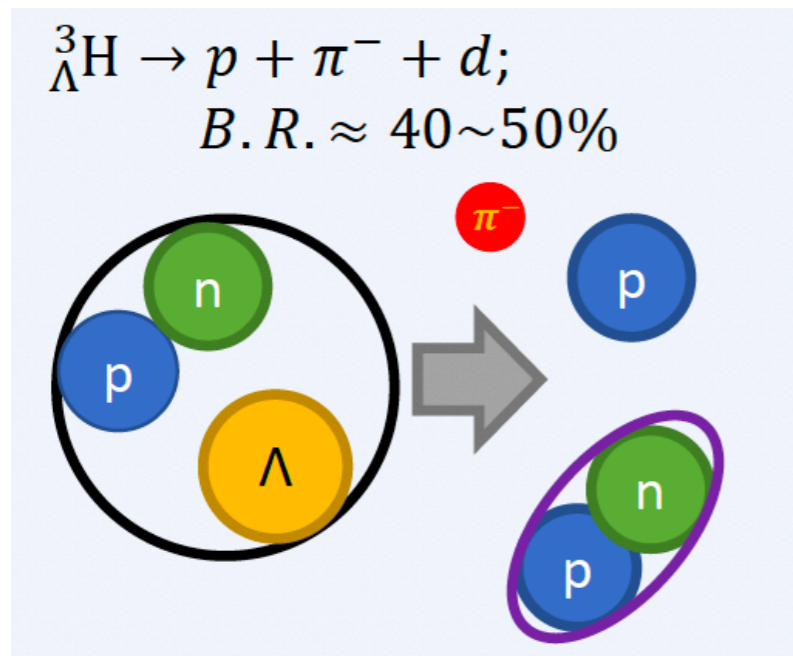
Results: k_T dependence

k_T [GeV/c]	Systematic Uncertainty
0 - 400	19 %
400 - 800	15 %
800 - 1200	22 %



$$k_T = \frac{p_{T1} + p_{T2}}{2}$$

YN ($d - \Lambda$) correlations at STAR



Simulation based on STAR ${}^3_{\Lambda}H$ yield measurement:

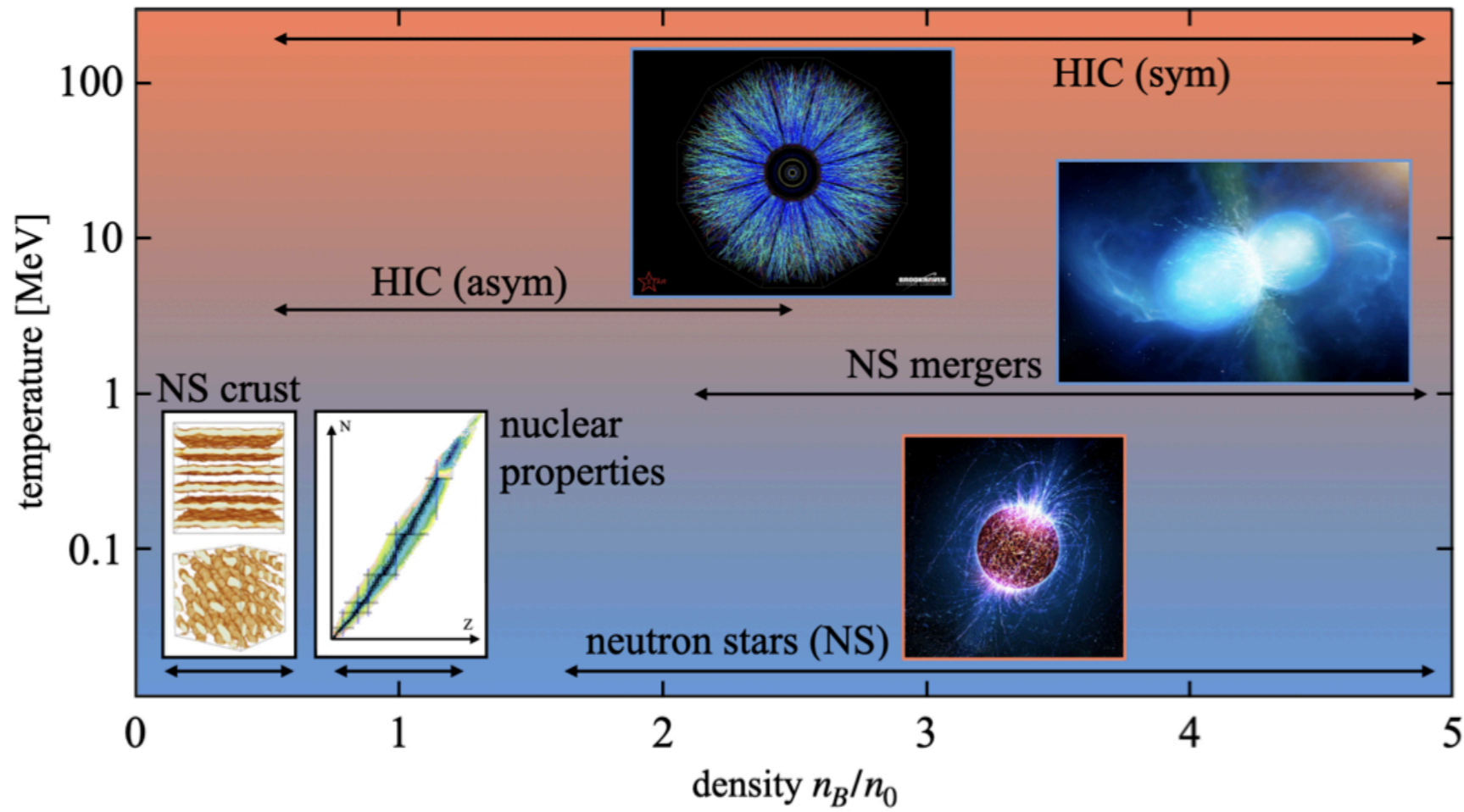
4 - 8% of $d - \Lambda$ entries come from ${}^3_{\Lambda}H$ decay for low k^* ;

Contamination subtracted from inclusive $d - \Lambda$ correlations;

Correlations of ${}^3_{\Lambda}H$ from $d - \Lambda$ and $d - (p - \pi^-)$
 are **not** experimentally distinguishable.

Various nuclear densities

A. Sorensen, HZ et al.
[arXiv:2301.13253 \[nucl-th\]](https://arxiv.org/abs/2301.13253)



$$\epsilon(n_n, n_p) = \epsilon_{SNM}(n) + S(n)\delta^2$$

$\epsilon_{SNM}(n)$ - energy per nucleon of symmetric nuclear matter

$\delta > 0.8$ - neutron stars

HIC (asym): < 600 AMeV

HIC (sym): broad range of energy, including high-energies