Glasma properties from a proper time expansion

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Introduction

goal: describe early time ($\tau \leq 1$ fm) dynamics of heavy-ion collisions
- evolution of system during this early stage not well understood
- importance: initial conditions for subsequent hydro evolution

more generally:
want to understand transition btwn the early-time dynamics and hydro phase
1. microscropic theory of non-abelian gauge fields (far from equilibrium)
$\rightarrow$
2. macroscopic effective theory based on universal conservation laws
- valid close to equilibrium

for more details on our work:
MEC, Czajka, Mrówczyński:
2001.05074, 2012.03042, 2105.05327, 2112.06812, 2202.00357
MEC, Cowie, Friesen, Mrówczyński, Pickering: 2304.03241.

method - Colour Glass Condensate (CGC) effective theory

CGC = high energy density largely gluonic matter
- associated with wavefunction of a high energy hadron
- initial state in high energy hadronic collisions

after collision CGC fields are transformed into glasma fields
- initially longitudinal colour electric and magnetic fields

method is based on a separation of scales between
1. valence partons with large nucleon momentum fraction (x)
2. gluon fields with small x and large occupation numbers

<u>basic picture</u>

dynamics of gluon fields determined from classical YM equation
$\rightarrow$ source provided by the valence partons

L. D. McLerran and R. Venugopalan, Phys. Rev. D, **49**, 2233 (1994); Phys. Rev. D, **49**, 3352 (1994); Phys. Rev. D, **50**, 2225 (1994).

method - notation

light-cone coordinates $x^{\pm} = (t \pm z)/\sqrt{2}$
Milne coordinates $\tau = \sqrt{2x^+x^-} = \sqrt{t^2 - z^2}$ and $\eta = \ln(x^+/x^-)/2 = \ln((t+z)/(t-z))$.

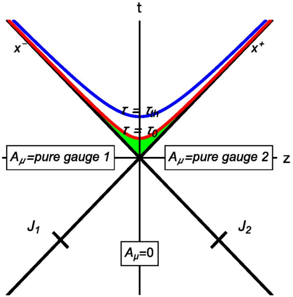
gauge: $A^\mu_{\rm milne} = \theta(\tau)(0, \alpha(\tau, \vec{x}_\perp), \vec{\alpha}_\perp(\tau, \vec{x}_\perp))$

sources: $J^\mu(x) = J_1^\mu(x) + J_2^\mu(x)$

$J_1^\mu(x) = \delta^{\mu +} g\rho_1(x^-, \vec{x}_\perp)$ and $J_2^\mu(x) = \delta^{\mu -} g\rho_2(x^+, \vec{x}_\perp)$

ansatz:

$$A^+(x) = \Theta(x^+)\Theta(x^-)x^+\alpha(\tau, \vec{x}_\perp)$$
$$A^-(x) = -\Theta(x^+)\Theta(x^-)x^-\alpha(\tau, \vec{x}_\perp)$$
$$A^i(x) = \Theta(x^+)\Theta(x^-)\alpha^i_\perp(\tau, \vec{x}_\perp) + \Theta(-x^+)\Theta(x^-)\beta^i_1(x^-, \vec{x}_\perp) + \Theta(x^+)\Theta(-x^-)\beta^i_2(x^+, \vec{x}_\perp)$$



description of method:

$$\underbrace{\rho^n(x^\pm,\vec{x}_\perp)}_{\text{static valence parton sources}} \rightarrow \underbrace{\beta^n(x^\pm,\vec{x}_\perp)}_{\text{CGC pre-collision fields}} \rightarrow \underbrace{\alpha(0,\vec{x}_\perp)}_{\text{initial glasma fields (boost invariant)}} \rightarrow \underbrace{\alpha(\tau,\vec{x}_\perp)}_{\text{glasma fields}}$$

parton sources $\rightarrow$ solve YM equation in the pre-collision region

apply boundary conditions $\rightarrow$ initial glasma potentials
use a proper time expansion *(dimensionless small parameter is $\tilde{\tau} = \tau Q_s$)*

$$\alpha(\tau,\vec{x}_\perp) \;=\; \alpha(0,\vec{x}_\perp) + \tau\alpha^{(1)}(\vec{x}_\perp) + \tau^2\alpha^{(2)}(\vec{x}_\perp) + \cdots$$

$\rightarrow$ find gluon potentials at finite τ

$\rightarrow$ colour electric and magnetic fields

$\Rightarrow$ calculate observables

next: colour charge distributions are not known
CGC: use Gaussian distributed random variables and average
$\rightarrow$ use Wick's theorem and the glasma graph approximation $\cdots$

result for correlator of 2 potentials: $(\vec{R} = \frac{1}{2}(\vec{x}_\perp + \vec{y}_\perp),\ \vec{r} = \vec{x}_\perp - \vec{y}_\perp)$

$$\delta_{ab} B^{ij}(\vec{x}_\perp, \vec{y}_\perp) \equiv \lim_{\mathrm{w}\to 0} \langle \beta^i_a(x^-, \vec{x}_\perp) \beta^j_b(y^-, \vec{y}_\perp) \rangle$$

$$\lim_{r\to 0} B^{ij}(\vec{x}_\perp, \vec{y}_\perp) = \delta^{ij} g^2 \frac{\mu(\vec{R})}{8\pi} \left(\ln\left(\frac{Q_s^2}{m^2} + 1 \right) - \frac{Q_s^2}{Q_s^2 + m^2} \right) + \cdots$$

infra-red regulator $m \sim \Lambda_{\mathrm{QCD}} \sim 0.2$ GeV
ultra-violet regulator = saturation scale = $Q_s = 2$ GeV

$\mu(\vec{R})$ is a surface colour charge density

dots indicate kept terms to 2nd order in grad expansion of $\mu(\vec{R})$

collisions with non-zero impact parameter (non-central collisions)
$\Rightarrow$ expand $\mu_{1/2}(\vec{z}_\perp)$ around ave coord $\vec{R} \mp \vec{b}/2$

surface charge density μ

must specify the form of the surface colour charge density $\mu(\vec{x}_\perp)$
- 2-dimensional projection of a Woods-Saxon potential

$$\mu(\vec{x}_\perp) = \left(\frac{A}{207}\right)^{1/3} \frac{\bar{\mu}}{2a\log(1+e^{R_A/a})} \int_{-\infty}^{\infty} dz \frac{1}{1+\exp\left[(\sqrt{(\vec{x}_\perp)^2+z^2}-R_A)/a\right]}$$

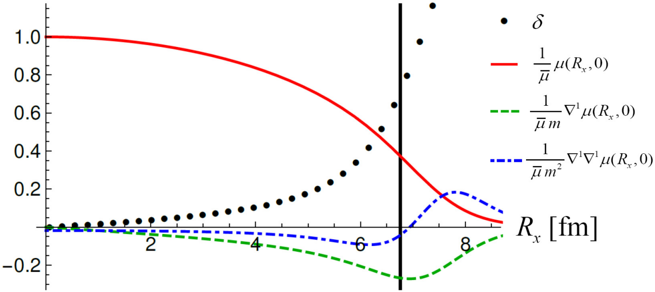
R_A and a = radius and skin thickness of nucleus mass number A
$r_0 = 1.25$ fm, $a = 0.5$ fm $\rightarrow$ when $A = 207$ gives
$R_A = r_0 A^{1/3} = 7.4$ fm

normalization: $\mu(\vec{0}) = \bar{\mu} = Q_s^2/g^4$ lead nucleus
$g^2\sqrt{\bar{\mu}}$ = McLerran-Venugopalan (MV) scale
proportional to Q_s - exact value not determined with CGC approach

** numerical results for $\mathcal{E}$ ... are order of magnitude estimates
ratios of different elements of the energy momentum tensor
$\rightarrow$ will have much weaker dependence on the MV scale.

gradient expansion

the parameter that we assume small is $\delta = \frac{|\nabla^i \mu(\vec{R})|}{m\mu(\vec{R})}$



derivatives are appreciable only in a very small region at the edges

summary of method:

YM eqn with average over gaussian distributed valence sources
$\rightarrow$ correlators of pre-collision fields
$\rightarrow$ glasma field correlators (b. conds, sourceless YM eqn, τ exp)
$\rightarrow$ correlators of glasma chromodynamic $\vec{E}$ and $\vec{B}$ fields
$\Rightarrow$ observables

we work to order τ^8 and study

1. isotropization of transverse/longitudinal pressures
2. azimuthal momentum distribution and spatial eccentricity
3. angular momentum
4. momentum broadening of hard probes - talk by Stanisław Mrówczyński

comment: many numerical approaches to study initial dynamics
our method is fully analytic
- allows control over different approximations and sources of errors
- can be systematically extended
- it has limitations (classical / no fluctuations of positions of nucleons)

isotropization

at $\tau = 0^+$ the energy-momentum tensor has the diagonal form

$$T(\tau = 0) = \begin{pmatrix} \mathcal{E}_0 & 0 & 0 & 0 \\ 0 & -\mathcal{E}_0 & 0 & 0 \\ 0 & 0 & \mathcal{E}_0 & 0 \\ 0 & 0 & 0 & \mathcal{E}_0 \end{pmatrix}$$

$\rightarrow$ the longitudinal pressure is large and negative
- system is far from equilibrium

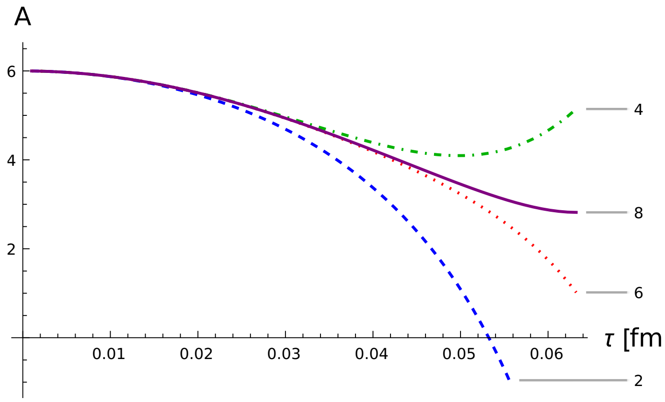
if the system approaches equilibrium as it evolves
- the longitudinal pressure must grow
- transverse pressure must decrease ($T_{\mu\nu}$ is traceless)

to study pressure isotropization

$$A_{TL} \equiv \frac{3(p_T - p_L)}{2p_T + p_L}$$

J. Jankowski, S. Kamata, M. Martinez and M. Spaliński, Phys. Rev. D **104**, 074012 (2021).

in equilibrium $(p_L = p_T = \mathcal{E}/3) \longrightarrow A_{TL} = 0$



$R = 5$ fm, $\eta = 0$ and $b = 0$

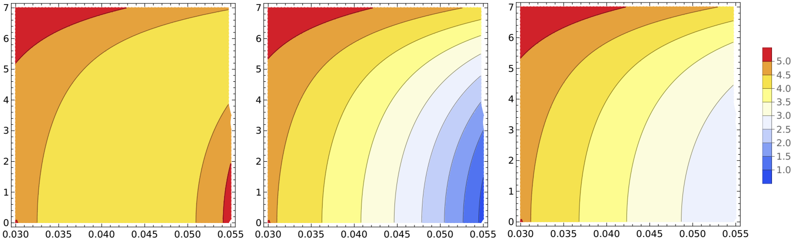


Figure: A_{TL} at fourth, sixth and eighth order. The vertical/horizontal axes are R and τ in fm.

dependence on confinement and saturation scales

the correlator $\langle\beta^i_a(x^-,\vec{x}_\perp)\beta^j_b(y^-,\vec{y}_\perp)\rangle$

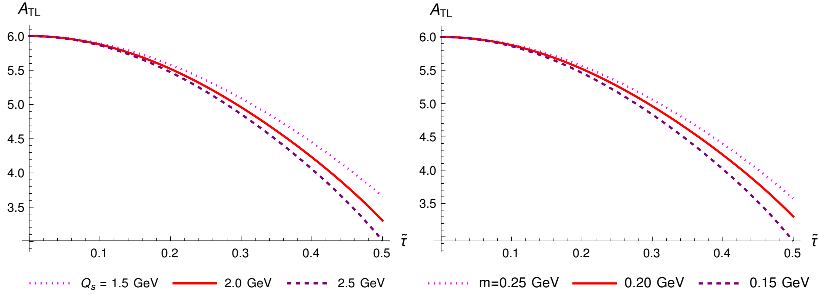
depends on two regulators: m (infra-red) and Q_s (ultra-violet)
- physically related to confinement / saturation scales
$\rightarrow$ constraints on how to choose them

we used: $m = 0.2$ GeV and $Q_s = 2.0$ Gev - standard choices

- want results $\approx$ independent of these numbers

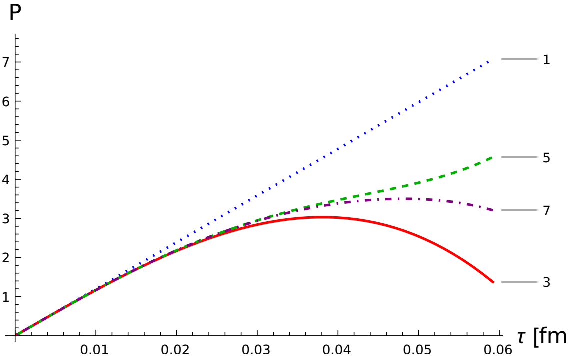
- especially since the two scales are pretty close together

A_{TL} at order τ^6 as a function of time
3 different values of Q_s with $m = 0.2$ GeV (left)
3 different values of m with $Q_s = 2.0$ GeV (right)
$R = 5$ fm, $b = 0$ and $\eta = 0$ at order τ^6
$\Rightarrow$ dependence on these scales is weak



Radial Flow

transverse momentum flow vector $= T_{i0}$ (trans. Poynting vector)
radial flow of the expanding glasma = radial projection $P \equiv \hat{R}_i T_{i0}$
$R = 3$ fm, $b = 1$ fm, $\phi = \pi/2$ (perpendicular to the reaction plane)



- P slows as system expands
- limit of validity of the expansion $\tau \sim 0.06$ fm

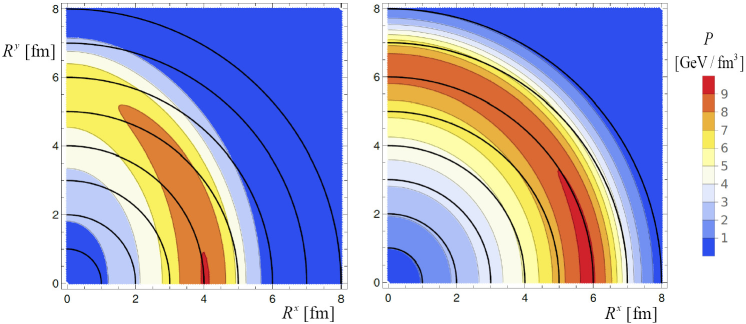
reaction plane defined by collision axis and impact parameter

$\vec{b} = b\hat{i} \rightarrow$ reaction plane is x-z

$\phi = 0$ is in reaction plane

$\phi = \pi/2$ is perpendicular to reaction plane

expect radial flow greater at $\phi = 0$



left panel $b = 6$ fm $\rightarrow R \lesssim 5$ fm

right panel $b = 2$ fm $\rightarrow R \lesssim 7$ fm

azimuthal asymmetry

in a non-central collision

- initial spatial asymmetry - moments of the energy density

$\rightarrow$ final state momentum asymmetry

physically: momentum anisotropy is generated by pressure gradients

asymmetry in momentum from Fourier coefficients of the flow

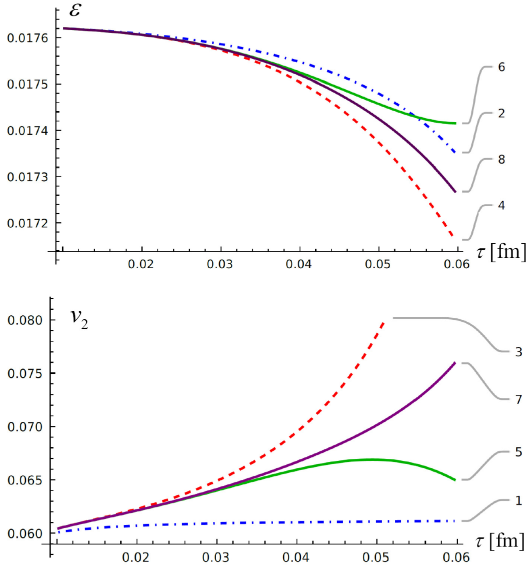
$\cdots$ write in terms of components T^{00}, T^{0x} and T^{0y}

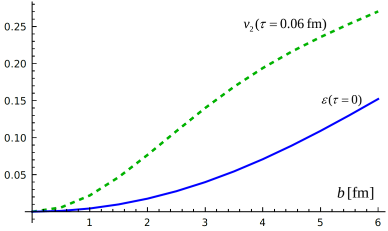
$$\varepsilon = -\frac{\int d^2R \, \frac{R_x^2 - R_y^2}{\sqrt{R_x^2 + R_y^2}} \, T^{00}}{\int d^2R \, \sqrt{R_x^2 + R_y^2} \, T^{00}} \qquad \text{and} \qquad v_2 = \frac{\int d^2R \, \frac{T_{0x}^2 - T_{0y}^2}{\sqrt{T_{0x}^2 + T_{0y}^2}}}{\int d^2R \, \sqrt{T_{0x}^2 + T_{0y}^2}}$$

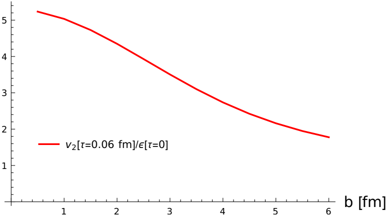
in a relativistic collision spatial asymmetries rapidly decrease

$\rightarrow$ anisotropic momentum flow can develop only in the first fm$/c$

- sensitive to system properties very early in its evolution
- provides direct information about the early stages of the system


ε
0.0176
0.0175
0.0174
0.0173
0.0172
0.02
0.03
0.04
0.05
0.06
τ [fm]
6
2
8
4
v2
0.080
0.075
0.070
0.065
0.060
0.02
0.03
0.04
0.05
0.06
τ [fm]
3
7
5
1





relative change in v_2 as b: $1 \to 6$ fm $\gg$ relative change in $v_2/\varepsilon(0)$
$\to$ correlation btwn spatial asymmetry from initial geometry and anisotropy of azimuthal momentum distribution

- mimics behaviour of hydrodynamics

angular momentum

define tensor $M^{\mu\nu\lambda} = T^{\mu\nu} R^{\lambda} - T^{\mu\lambda} R^{\nu}$

$\nabla_\mu M^{\mu\nu\lambda} = 0 \to$ conserved charges $J^{\nu\lambda} = \int_\Sigma d^3y \sqrt{|\gamma|} n_\mu M^{\mu\nu\lambda}$
- n^μ is a unit vector perpendicular to the hypersurface Σ
- γ is the induced metric on this hypersurface
- d^3y is the corresponding volume element

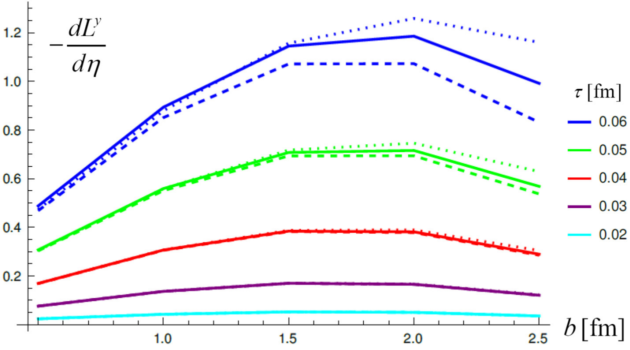
$n^\mu = (1,0,0,0)$ in Milne coordinates
$\to$ $J^{\nu\lambda}$ defined on a hypersurface of constant τ

Pauli-Lubanski vector: $L_\mu = -\frac{1}{2}\epsilon_{\mu\alpha\beta\gamma} J^{\alpha\beta} u^\gamma$

result: angular momentum per unit rapidity (symmetric collision)

$$\frac{dL^y}{d\eta} = -\tau^2 \int d^2\vec{R}\, R^x T^{0z}$$

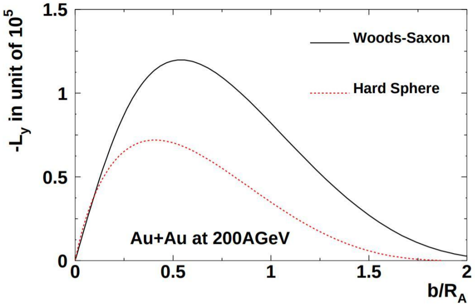
result:
dotted/dashed/solid lines show orders 4/6/8



ions moving in $+$/-z dirns displaced in $+$/- x dirns $\rightarrow$ L_y is negative comparison:

$L_y \sim 10^5$ at RHIC energies for initial system of colliding ions

J. H. Gao, S. W. Chen, W. t. Deng, Z. T. Liang, Q. Wang and X. N. Wang, Phys. Rev. C **77**, *024906 (2008).*



- even larger at LHC energies

F. Becattini, F. Piccinini and J. Rizzo, Phys. Rev. C **77**, *024906 (2008).*

idea: initial rapid rotation of glasma
$\rightarrow$ could be observed via polarization of final state hadrons
- large $\vec{L}$ & spin-orbit coupling $\rightarrow$ alignment of spins with $\vec{L}$

many experimental searches for this polarization
- effect of a few percent observed at RHIC
- at LHC result consistent with zero
- *difficult to measure ...*

F. Becattini, M.A. Lisa, Ann. Rev. Nucl. Part. Sci. 70, 395 (2020).

these results support our calculation:
glasma carries only tiny imprint of the $\vec{L}$ of the intial state
$\rightarrow$ majority of the angular momentum is carried by valence quarks

conclusions

1. 8th order τ expansion can be trusted to $\tau \approx 0.07$ fm
2. glasma moves towards equilibrium
3. correlation btwn elliptic flow coef v_2 / spatial eccentricity
   - spatial asymmetry introduced by initial geometry is effectively transmitted to azimuthal distribution of gluon momentum field

   $\rightsquigarrow$ this behaviour mimics hydrodynamics
4. most of the angular momentum of the intial system not transmitted to glasma
   - contradicts picture of a rapidly rotating initial glasma state