Modelling choices in multi-fluid-dynamical descriptions of low energy collisions

3 fluid code: MUFFIN

Cimerman, Karpenko, Tomàšik, Huovinen PRC 107 (2023) 044902

Clemens Werthmann

University of Wrocław

Uniwersytet Wrocławski

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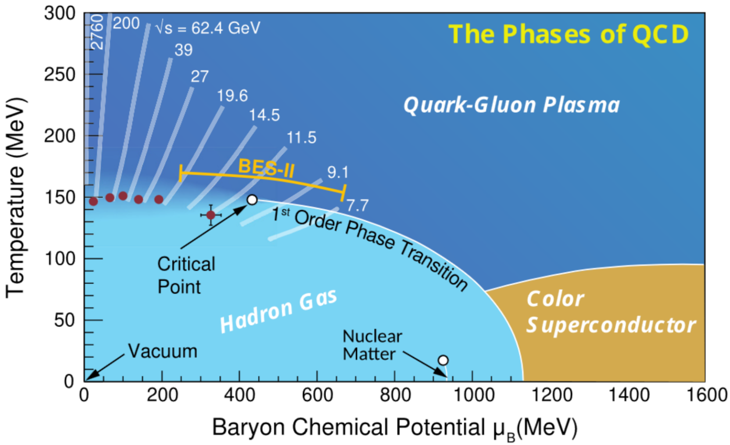
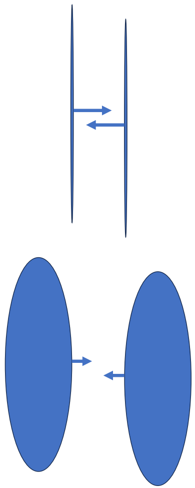
QCD Phase Diagram



- goal: probe QCD phase diagram via heavy ion collisions
- expect 1st Order transition line and critical end point
- to probe this region, need higher $\mu_B \Leftrightarrow$ lower $\sqrt{s_{NN}}$

Fluiddynamical simulations for low energy collisions

Hydrodynamics works well at high $\sqrt{s_{NN}}$.
What are the challenges at low $\sqrt{s_{NN}}$?



Lorentz contraction:

- at high energies, nuclei are flat
  $\Rightarrow$ timescale separation of:
  1. energy deposition
  2. pre-equilibrium
  3. hydrodynamic stage

  many early time descriptions rely on this!
- low energies: extended interpenetration; all 3 processes happen simultaneously!

Fluiddynamical simulations for low energy collisions

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What are the challenges at low $\sqrt{s_{NN}}$?

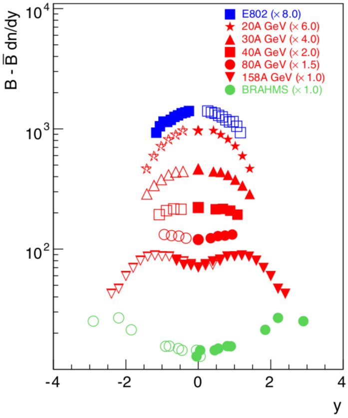
Baryon stopping vs baryon transparency:

- high beam energies: baryons from nuclei escape collision region
  $\Rightarrow$ this is why $\mu_B \approx 0$!
- very low energies: nuclei completely stopped
- intermediate energies: partial baryon transparency causes double peak!

State of the art:
Start hydro at late time with a very complicated intial condition

Shen, Schenke PRC 105 (2022) 064905

net baryon rapidity distribution for low $\sqrt{s_{NN}}$'s:



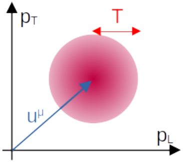
Multi-fluid Hydrodynamics

idea: Hydrodynamics can also describe cold nuclei ("liquid drop model")

⇒ can we use it throughout entire collision evolution?

problem: Hydrodynamics describes a system close to equilibrium, i.e. isotropic in momentum space

not true for colliding nuclei!



hydro

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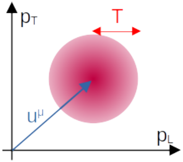
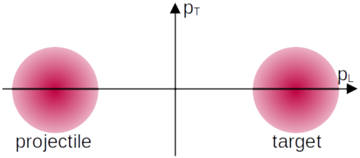


hydro



2 fluid hydro

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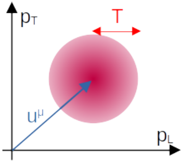

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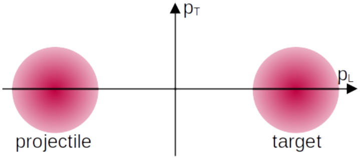

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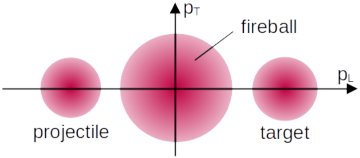

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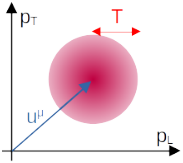

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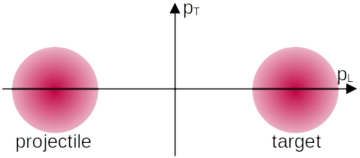

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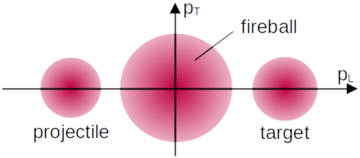

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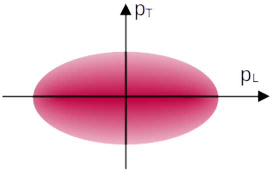

anisotropic hydro

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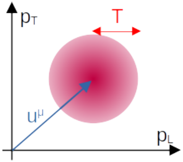

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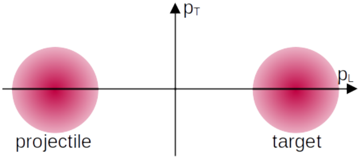

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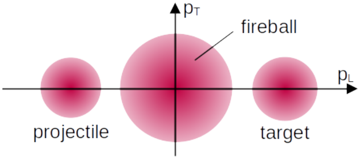

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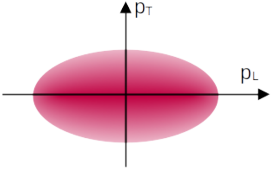

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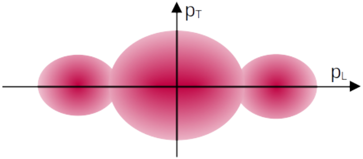

somewhat realistic distribution

Equations governing multi-fluid hydro

3 versions of energy-momentum tensor

$$T_p^{\mu\nu}\,,\ T_t^{\mu\nu}\,,\ T_f^{\mu\nu}$$

and 3 versions of baryon current

$$J^\mu_{B,p}\,,\ J^\mu_{B,t}\,,\ J^\mu_{B,f}$$

conservation equations hold only globally:

$$\partial_\mu(T_p^{\mu\nu} + T_t^{\mu\nu} + T_f^{\mu\nu}) = 0\,, \quad \partial_\mu(J^\mu_{B,p} + J^\mu_{B,t} + J^\mu_{B,f}) = 0$$

$\Rightarrow$ fluids can exchange energy, momentum and baryon charge via "friction"

$$\partial_\mu T_\alpha^{\mu\nu} = F_\alpha^\nu\,, \quad \partial_\mu J^\mu_{B,\alpha} = R_{B,\alpha} \qquad \text{where} \qquad \sum_\alpha F_\alpha^\nu = 0\,,\ \sum_\alpha R_{B,\alpha} = 0$$

a priori, any friction is allowed! <u>but:</u>

- system should not deviate too much from modelling assumptions
- needs to reproduce observed behaviour

Example: Csernai's model

"next-to-trivial" model: all that scatters is dumped into the fireball

Csernai, Lovas, Maruhn, Rosenhauer, Zimányi, Greiner PRC 26 (1982) 149

$\Rightarrow$ nuclei stay cold!

projectile/target:

$$\partial_\mu T^{\mu\nu}_{p/t} = F^\nu_{p/t} = u^\nu_{p/t} m_N R_{B,p/t}\,, \quad \partial_\mu J^\mu_{B,p/t} = R_{B,p/t}$$

fireball:

$$\partial_\mu T^{\mu\nu}_f = -F^\nu_p - F^\nu_t\,, \quad \partial_\mu J^\mu_{B,p/t} = -R_{B,p} - R_{B,t}$$

Only need to model $R_{B,p/t}$: how nuclei loose baryon charge

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Only need to model $R_{B,p/t}$: how nuclei loose baryon charge

<u>problem:</u> no double peak!

Deriving friction from kinetic theory

consider fluids as phase space distributions: $f_\alpha = \frac{\mathrm{d}N_\alpha}{\mathrm{d}^3x d^3p}$,
can obtain hydro d.o.f. as:

$$T^{\mu\nu}_\alpha = \int \frac{\mathrm{d}^3p}{p^0} p^\mu p^\nu f_\alpha \,, \quad J^\mu_{B,\alpha} = B_\alpha \int \frac{\mathrm{d}^3p}{p^0} p^\mu f_\alpha$$

time evolution is described via Boltzmann equation

$$p^\mu \partial_\mu f_\alpha = C_\alpha[f_p, f_t, f_f] = \sum_{\beta,\gamma} C^{\beta\gamma}_\alpha [f_\beta, f_\gamma]$$

for given $C^{\beta\gamma}_\alpha$, hydrodynamic equations obtained as

$$\partial_\mu T^{\mu\nu}_\alpha = \int \frac{\mathrm{d}^3p}{p^0} p^\nu C_\alpha = F^\nu_\alpha \,, \quad \partial_\mu J^\mu_{B,\alpha} = B_\alpha \int \frac{\mathrm{d}^3p}{p^0} C_\alpha = R_{B,\alpha}$$

Wait, how many different terms do we need to model now?

$$C_\alpha[f_p, f_t, f_f] = \sum_{\beta,\gamma} C_\alpha^{\beta\gamma}[f_\beta, f_\gamma] \longrightarrow F_\alpha^{\beta\gamma,\nu}$$

How many unique $F_\alpha^{\beta\gamma}$ are there?

a priori $\qquad 3 \cdot 3 \cdot 3 = 27$

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How many unique $F_\alpha^{\beta\gamma}$ are there?

a priori	$3 \cdot 3 \cdot 3 = 27$
$\beta = \gamma$ is hydro	$3 \cdot 3 \cdot 2 = 18$
symmetry $\beta \Leftrightarrow \gamma$	$3 \cdot 3 = 9$
symmetry $\alpha = t \leftrightarrow \alpha = p$	$2 \cdot 3 = 6$
symmetry $\beta = f$: $\gamma = t \leftrightarrow \gamma = p$	$2 \cdot 2 = 4$
conservation	$4 - 2 = 2$

have to model

- how p-t interaction affects p/f
- how p-f interaction affects p/f

Microscopic input

collision integrals are given in terms of scattering crosssections

$\alpha\beta \to \underline{\underline{\alpha}} X$: Modelling goes <u>here</u>!

$$C_\alpha^{\alpha\beta}[f_\alpha, f_\beta](p_\alpha) = \int \mathrm{d}^3 p_\beta \, p_\alpha^0 \Bigg[\underbrace{-f_\alpha(p_\alpha) f_\beta(p_\beta) v_{\mathrm{rel}} \sigma_{\alpha\beta\to X}}_{\text{loss}} + \underbrace{\int d^3 q_\alpha f_\alpha(q_\alpha) f_\beta(p_\beta) v_{\mathrm{rel}} \frac{\mathrm{d}\sigma_{\alpha\beta\to\alpha X}}{\mathrm{d}^3 p_\alpha}}_{\text{gain}} \Bigg]$$

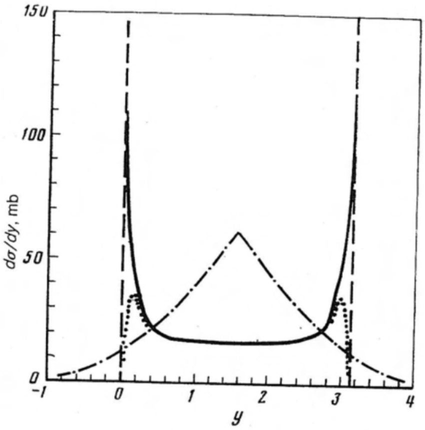
from these, approximative friction formulae are derived

<u>problems:</u>

- crosssections may not be fully measured in experiment
- d.o.f. change in deconfinement transition

Ivanov's model

- N+N scattering: N strongly peaked at ingoing rapidities, π at midrapitidy
  $\Rightarrow$ in p-t friction: N stay in p/t, π go to f
- $\pi + N$ mostly resonance formation
  $\Rightarrow$ all outgoing particles from p-f friction go to p
- uncertainty in deconfined phase: densities multiplied with $\sqrt{s}$-dependent prefactor



can describe the double peak!

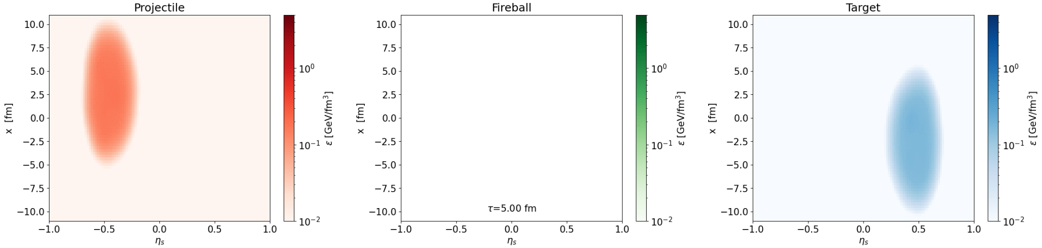
Ivanov, Russkikh, Toneev PRC 73 (2006) 044904

MUFFIN

- implementation of 3-fluid (ideal) hydrodynamics in Ivanov's formulation
    - hybrid with SMASH

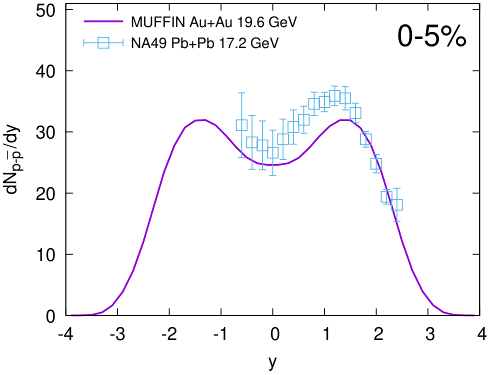
Cimerman, Karpenko, Tomàšik, Huovinen PRC 107 (2023) 044902

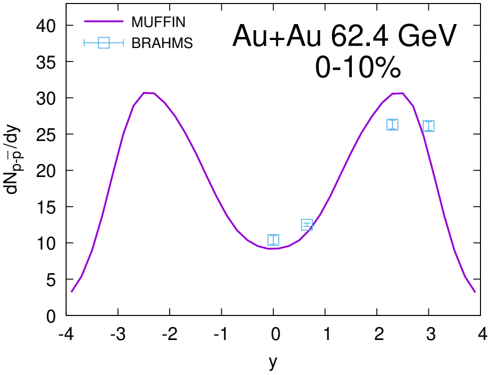
- allows easy modification of e.o.s.
- all 3 fluids heat up, p and t slow down a lot



Au+Au 7.7 GeV $b = 5$ fm

Baryon transparency/stopping



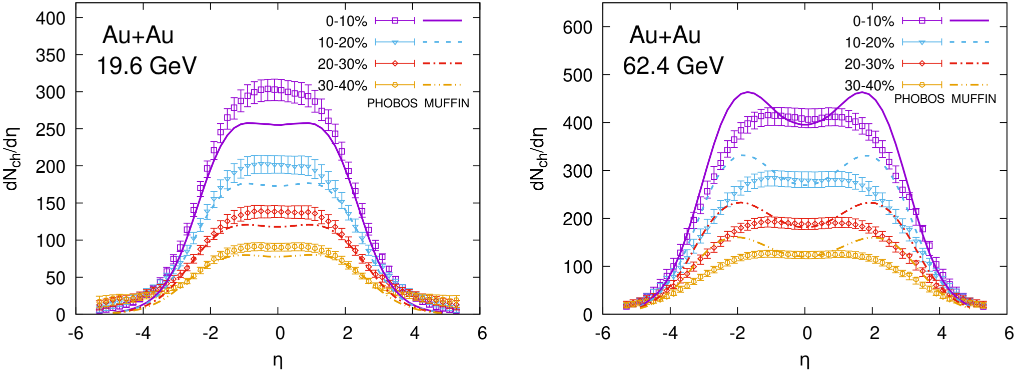


net proton rapidity distributions:

simulated results agree well with experiment

$\Rightarrow$ correct baryon stopping!

Charged hadron yield



reasonable agreement also in $\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}\eta$

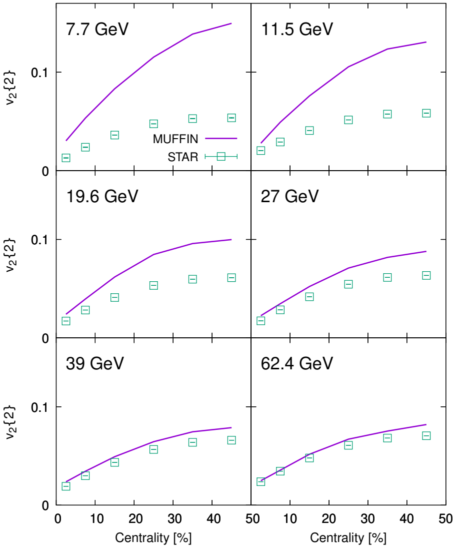
$\Rightarrow$ correct entropy production

Collective Flow

v_2 is strongly overestimated at small $\sqrt{s_{NN}}$

$\Rightarrow$ probably need viscosity

$\Rightarrow$ introduce transfer of dissipative quantities



Summary

- low $\sqrt{s_{NN}}$ collisions allow to examine QCD phase diagram
- 3 fluid hydrodynamics models 3 momentum space components of the collision:
  projectile, target, fireball
- fluid friction obtained from kinetic theory; requires crosssection input
- can describe full time evolution, correct results for baryon stopping and entropy production