Effect of color superconductivity on QCD Phase Diagram

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VARIOUS FACES OF QCD
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Table of Contents

(non-local) Nambu-Jona-Lasinio Model

$$m^*(p) = m - \gamma(p) G_s n_s$$

$$\mu^*(p) = \mu - \gamma(p) G_v n_v$$

Quasi-Particle Model

Non-local interaction

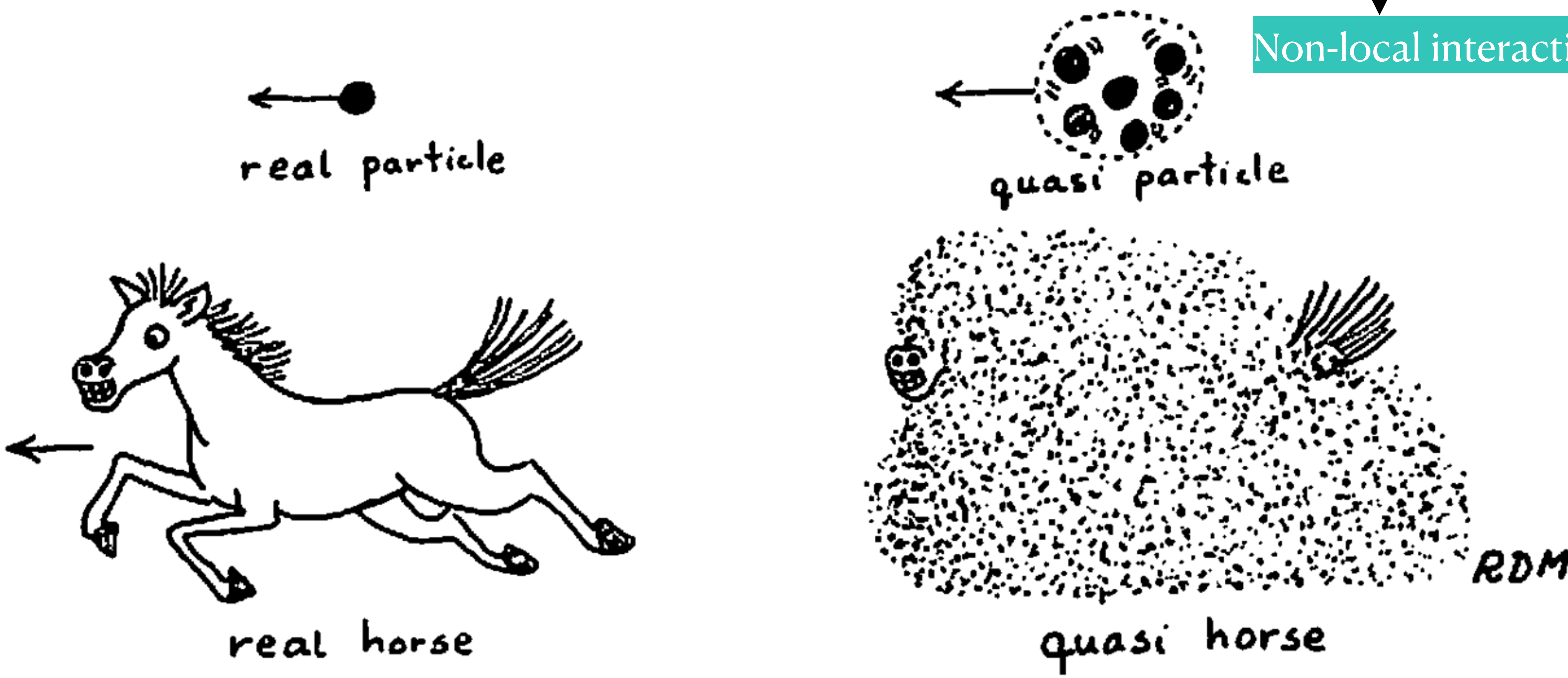


Fig. 0.4 ***Quasi Particle Concept***

Adapted from Richard D. Mattuck, “A Guide to Feynman Diagrams in the Many-Body problem”

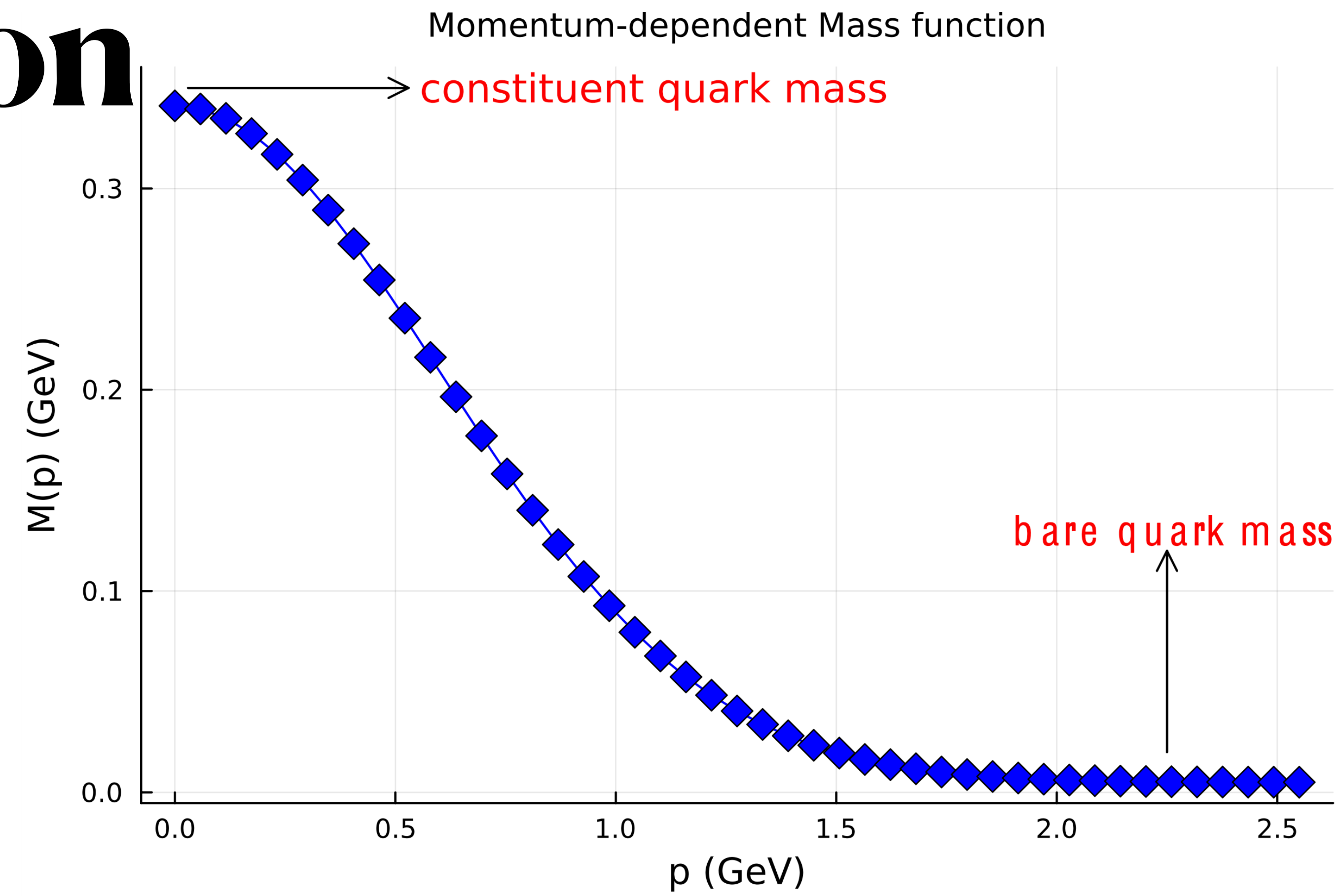
Non-local interaction

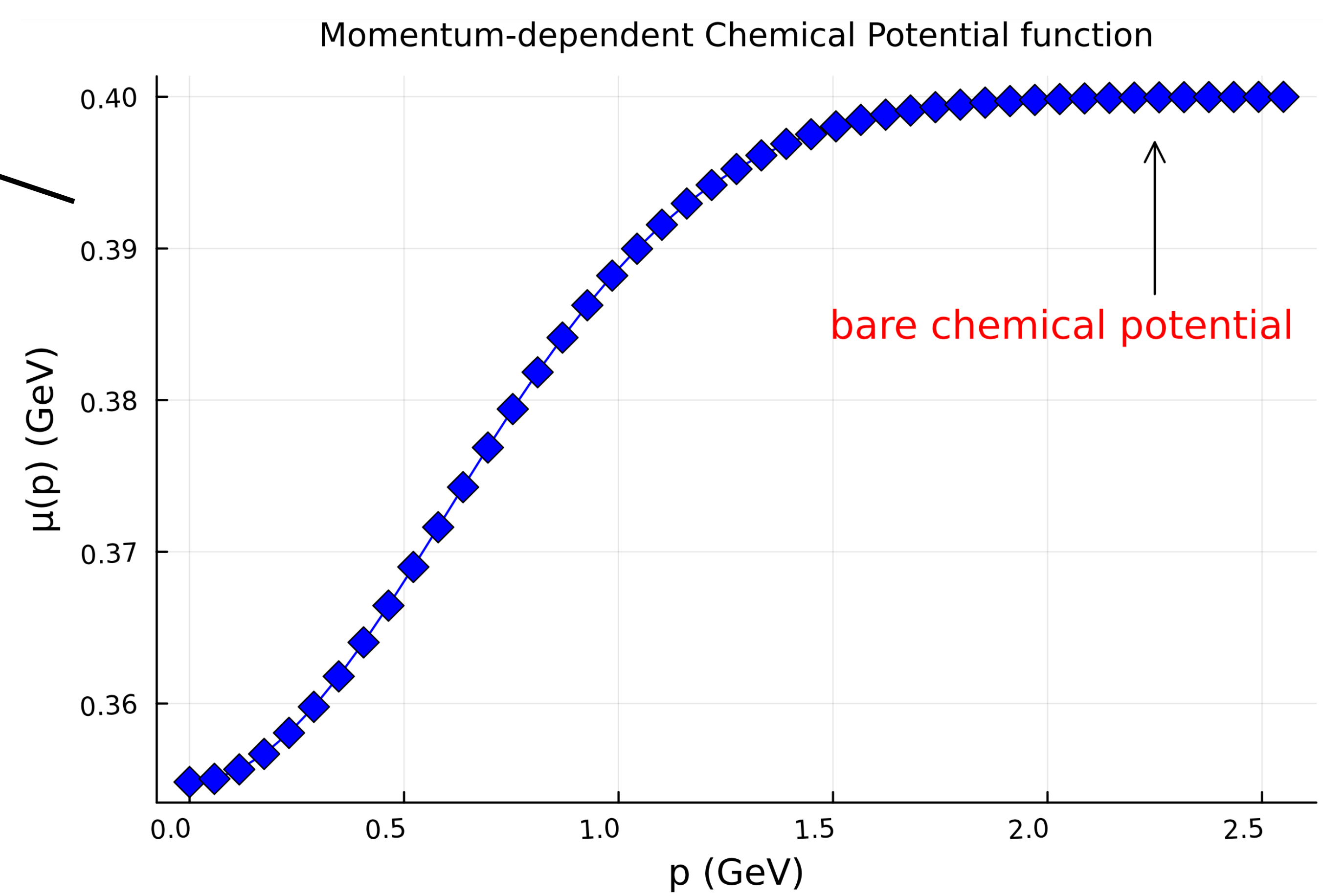
- M(p) saturates asympto--tically to current quark mass
- μ(p) saturates asympto--tically to bare μ

?

- Change in dispersion relation -

$$E(p) = \sqrt{p^2 + M(p)^2}$$





Local v/s Non-local
$T \to 0$

Local Interaction

$$n_v = N_c N_f \frac{1}{\pi^2} \int_0^\infty dq q^2 \Theta(\mu^* - q)$$

$$= N_c N_f \frac{1}{\pi^2} \int_0^{\mu^*} dq q^2$$

$$= N_c N_f \frac{1}{3\pi^2} \mu^*(\mu)^3$$

$$n_v = n_v(\mu^*(\mu))$$

$$\mu^* = \mu - G_v n_v$$

↓

Trivial Fermi surface

Non-local Interaction

$$n_v = N_c N_f \frac{1}{\pi^2} \int_0^\infty dq q^2 \Theta(\mu^*_q - q)$$

$$= N_c N_f \frac{1}{\pi^2} \int_0^{\mu^*_{p_f(\mu)}} dq q^2$$

$$n_v = n_v\left[\mu^*_{p_f(\mu)}\right]$$

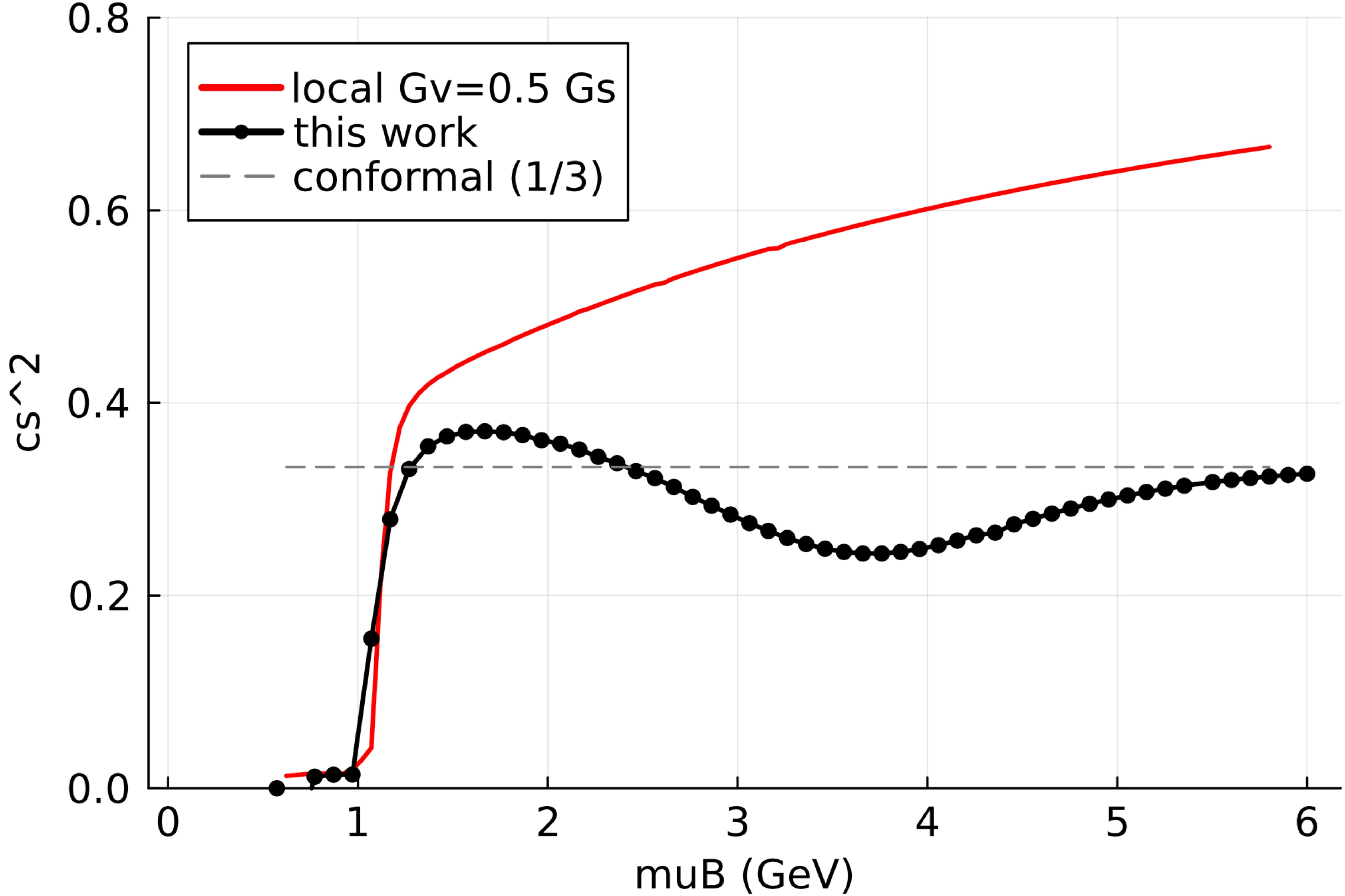
$$\mu^*(p) = \mu - \gamma(p) G_v n_v$$

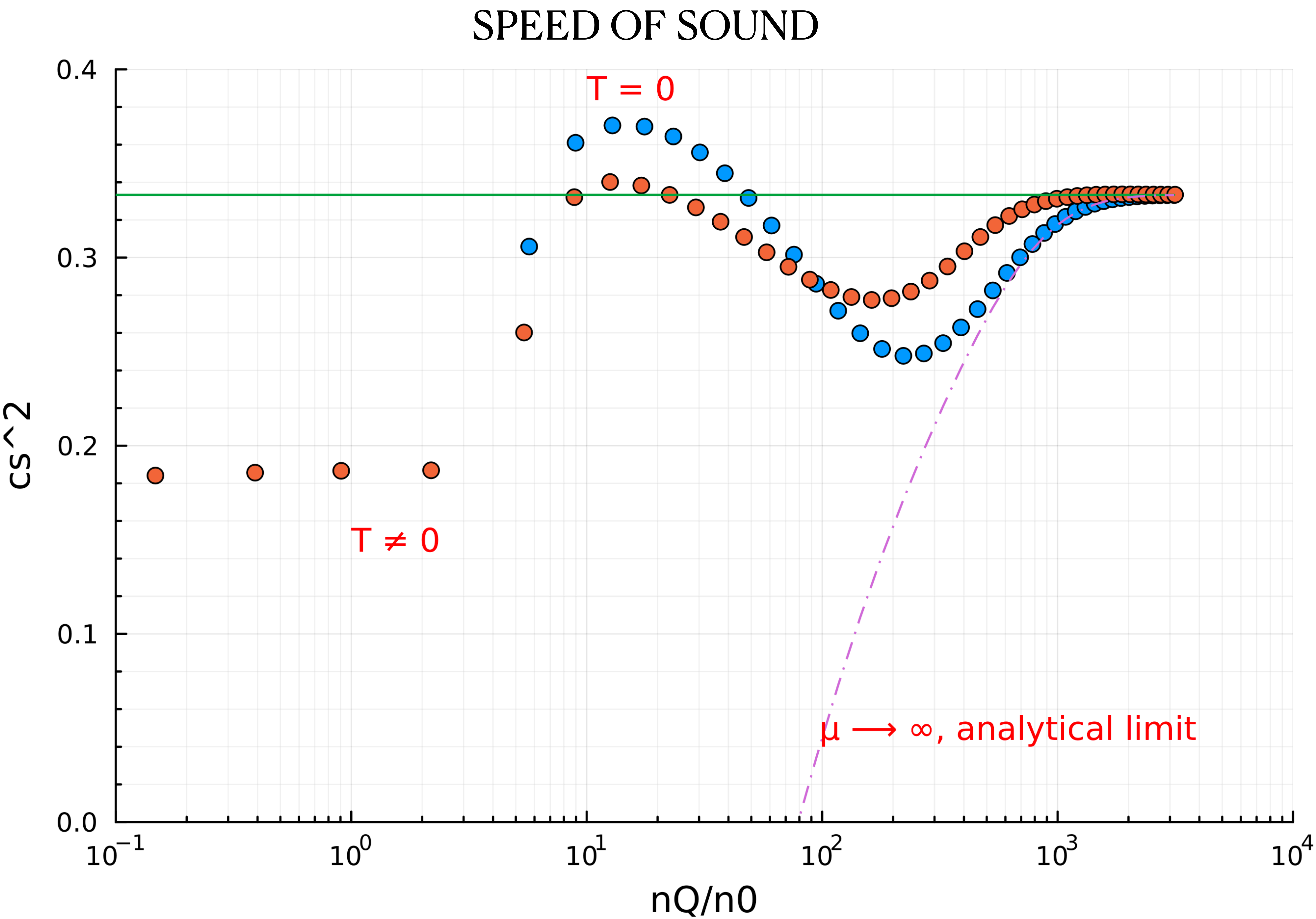
NATURAL DENSITY-DEPENDENCE

↓

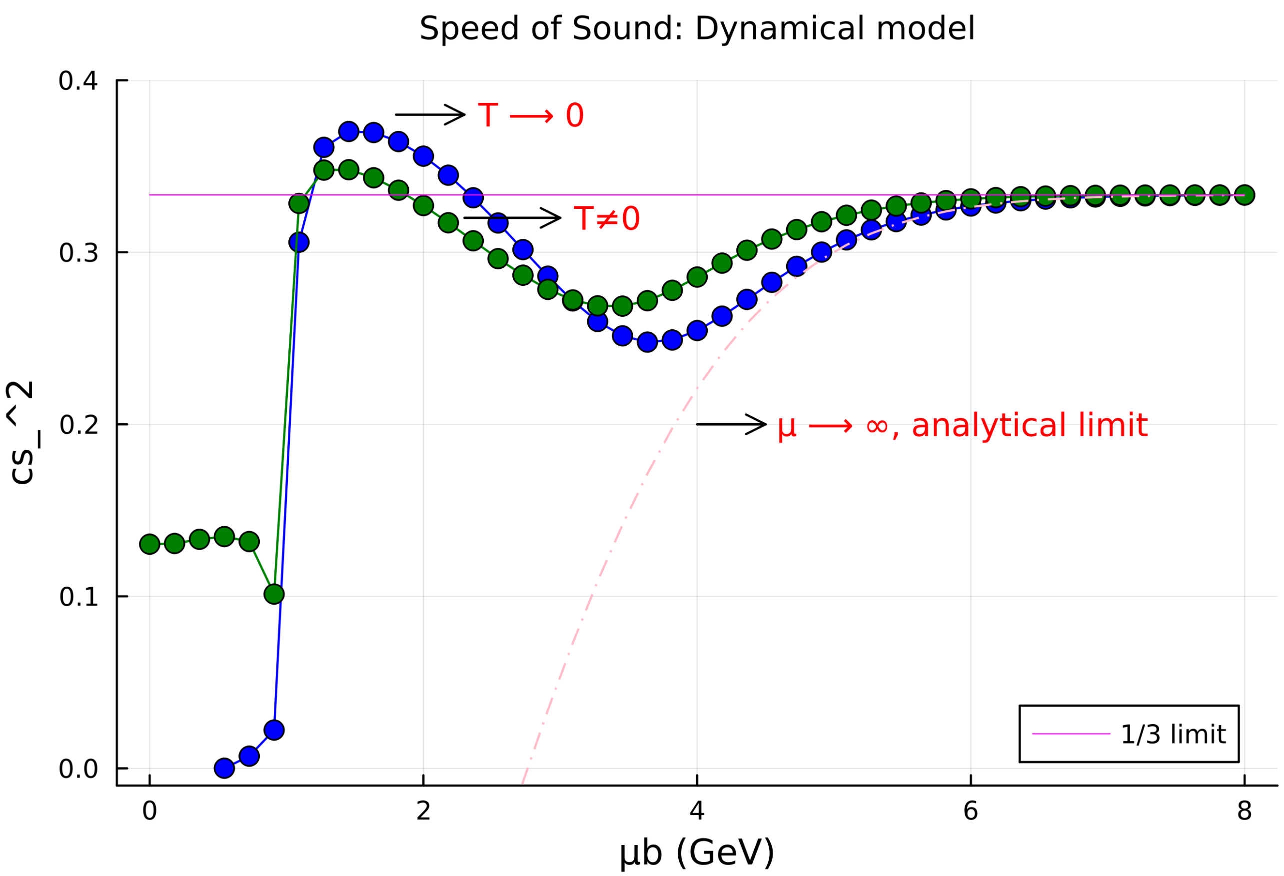
Non-trivial Interacting Fermi surface !

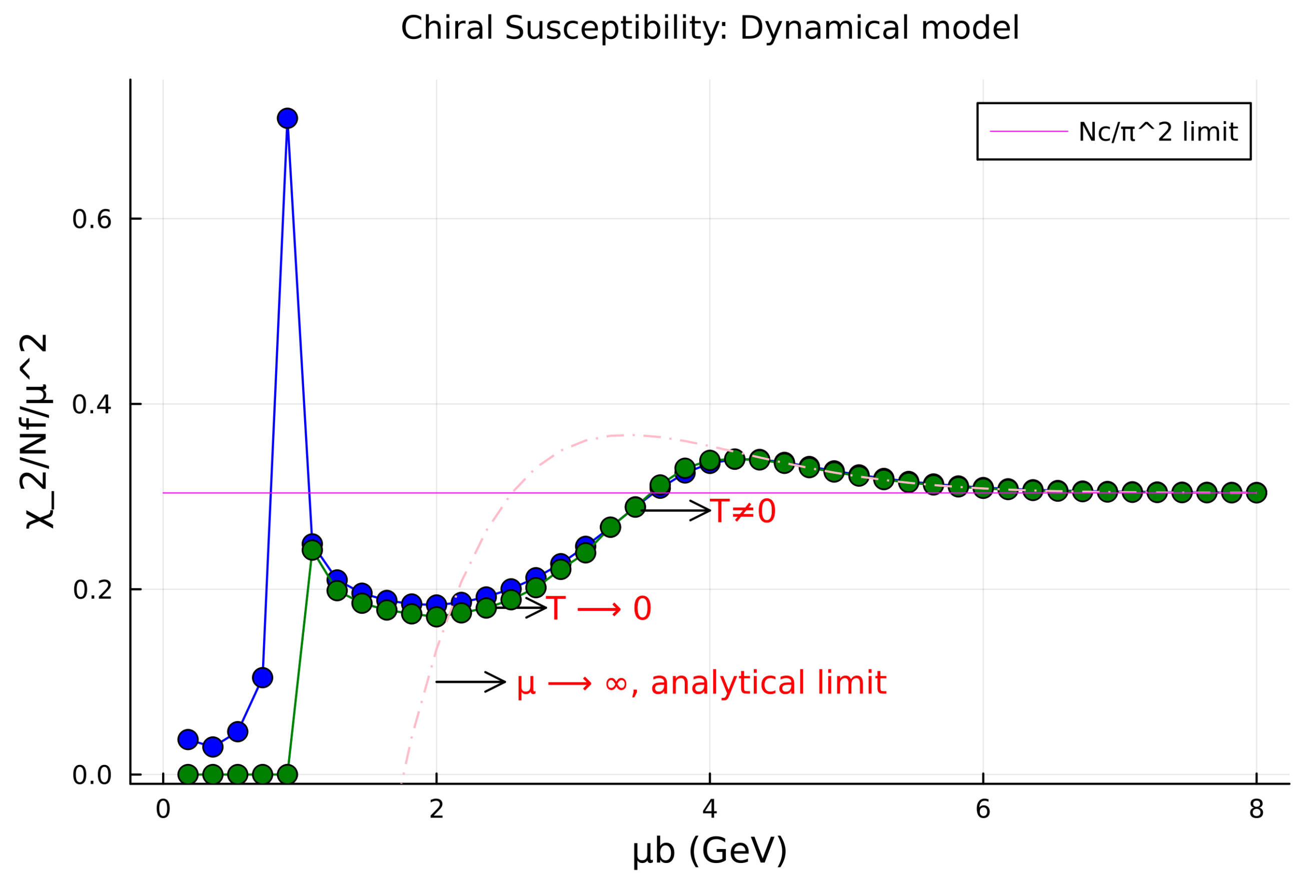
Speed of Sound




SPEED OF SOUND
T = 0
T ≠ 0
μ ⟶ ∞, analytical limit
cs^2
nQ/n0
0.4
0.3
0.2
0.1
0.0
10^-1
10^0
10^1
10^2
10^3
10^4

SPEED OF SOUND V/S CHIRAL SUSCEPTIBILITY



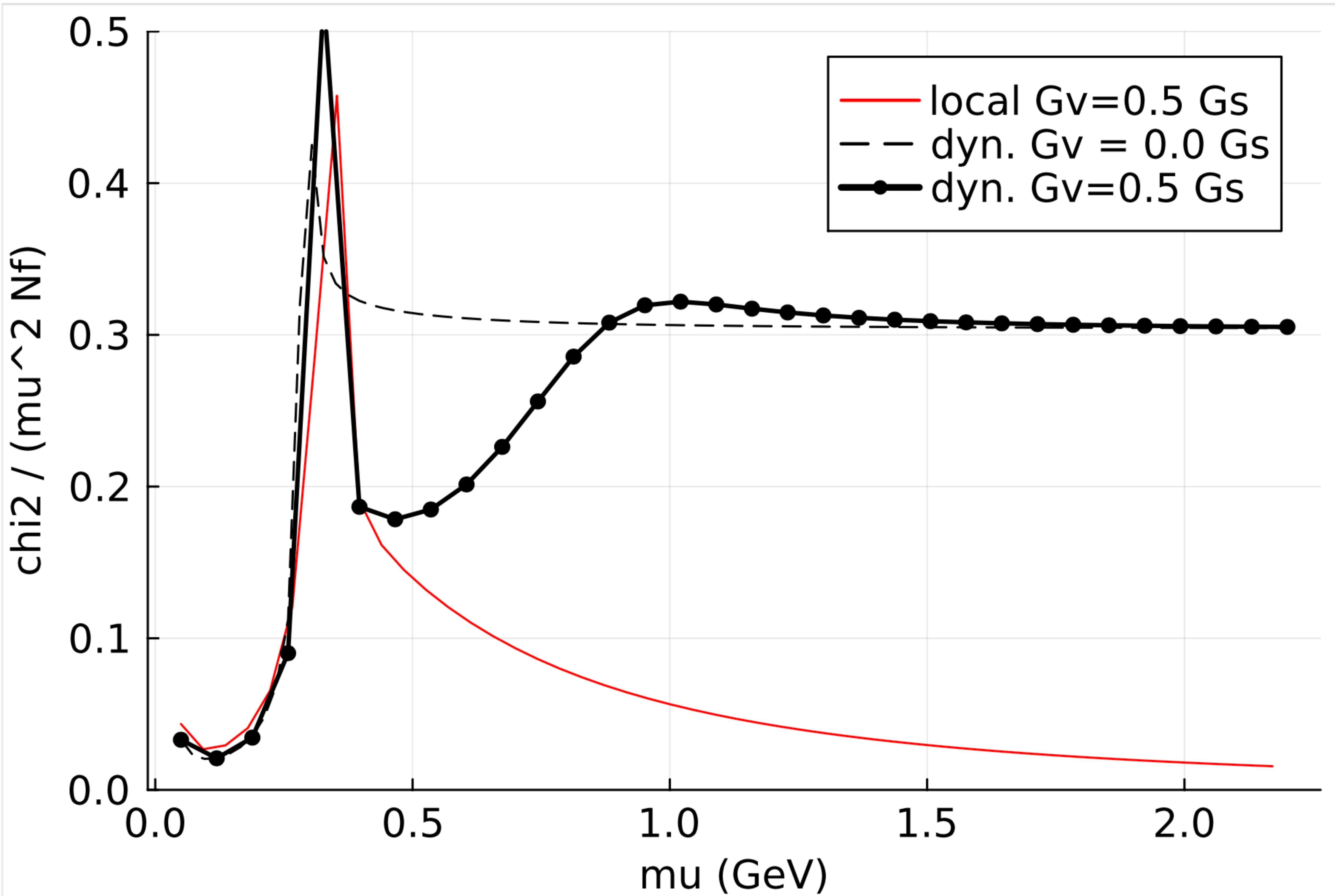


$$cs^2_{\mu\to\infty} \approx \frac{1}{3}\left[1 - \frac{\omega_\infty}{\mu}\left(1+\frac{2\mu^2}{UV^2}\right)e^{-\frac{\mu^2}{UV^2}}\right]$$

$$\chi 2_{\mu\to\infty} \approx \frac{N_c N_f}{\pi^2}\mu^2\left|1 - \frac{2\omega_\infty}{\mu}\left(1-\frac{\mu^2}{UV^2}\right)e^{-\frac{\mu^2}{UV^2}}\right|$$

Non-local cut-off is inherent gluon interaction scale

CHIRAL SUSCEPTIBILITY



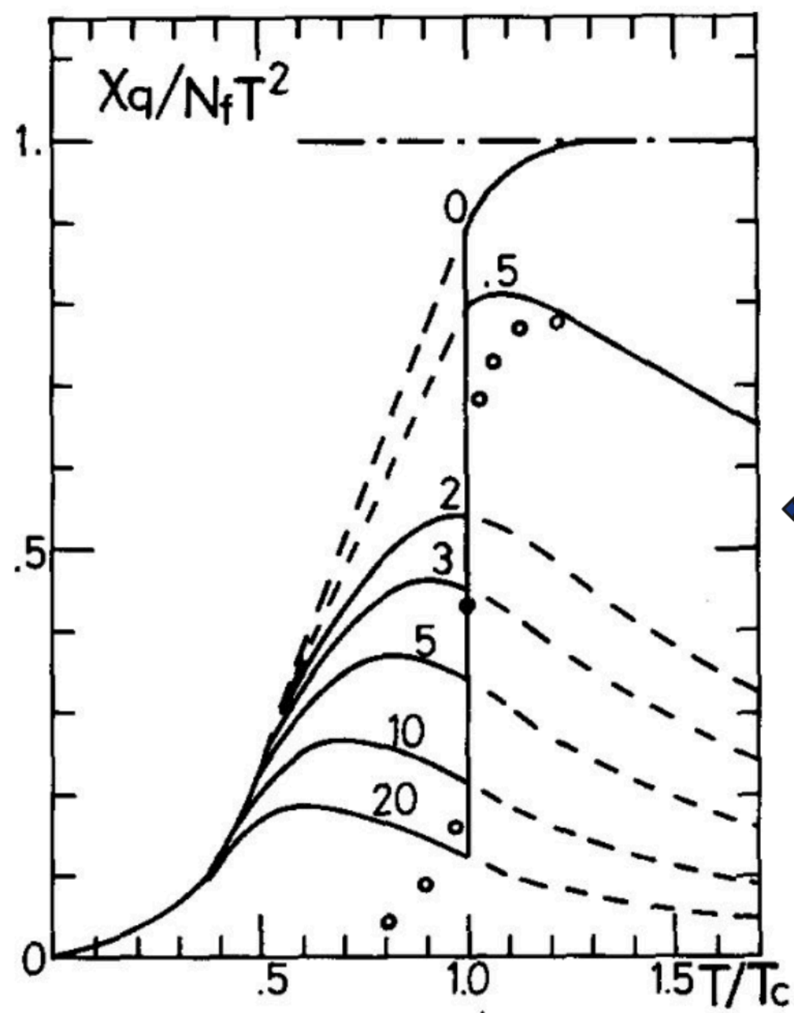


Fig. 1. The temperature dependence of the quark-number susceptibility χ_q in the unit of $N_f T^2$ with some of the vector coupling $g_V\Lambda^2$: $g_V\Lambda^2 = 0$, 0.5, 2, 3, 5, 10, 20, which are indicated with the numbers attached to the respective curves. The dash-dotted line shows the free massless case. The small circles are the lattice result on an $8^3 \times 4$ lattice with the quark mass $m/T = 0.2$ [7] compiled in ref. [9].

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Quark-number susceptibility and fluctuations in the vector channel at high temperatures ☆

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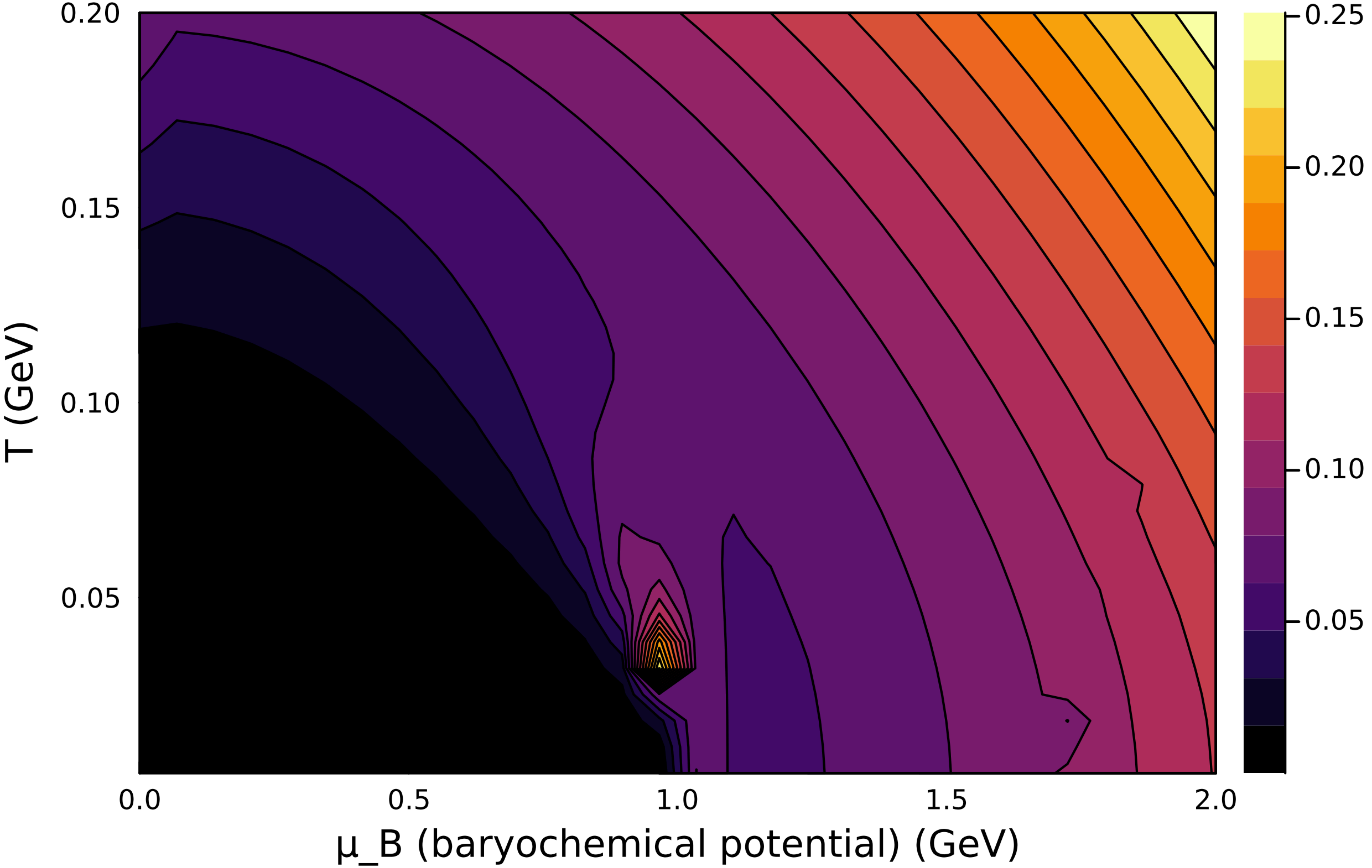
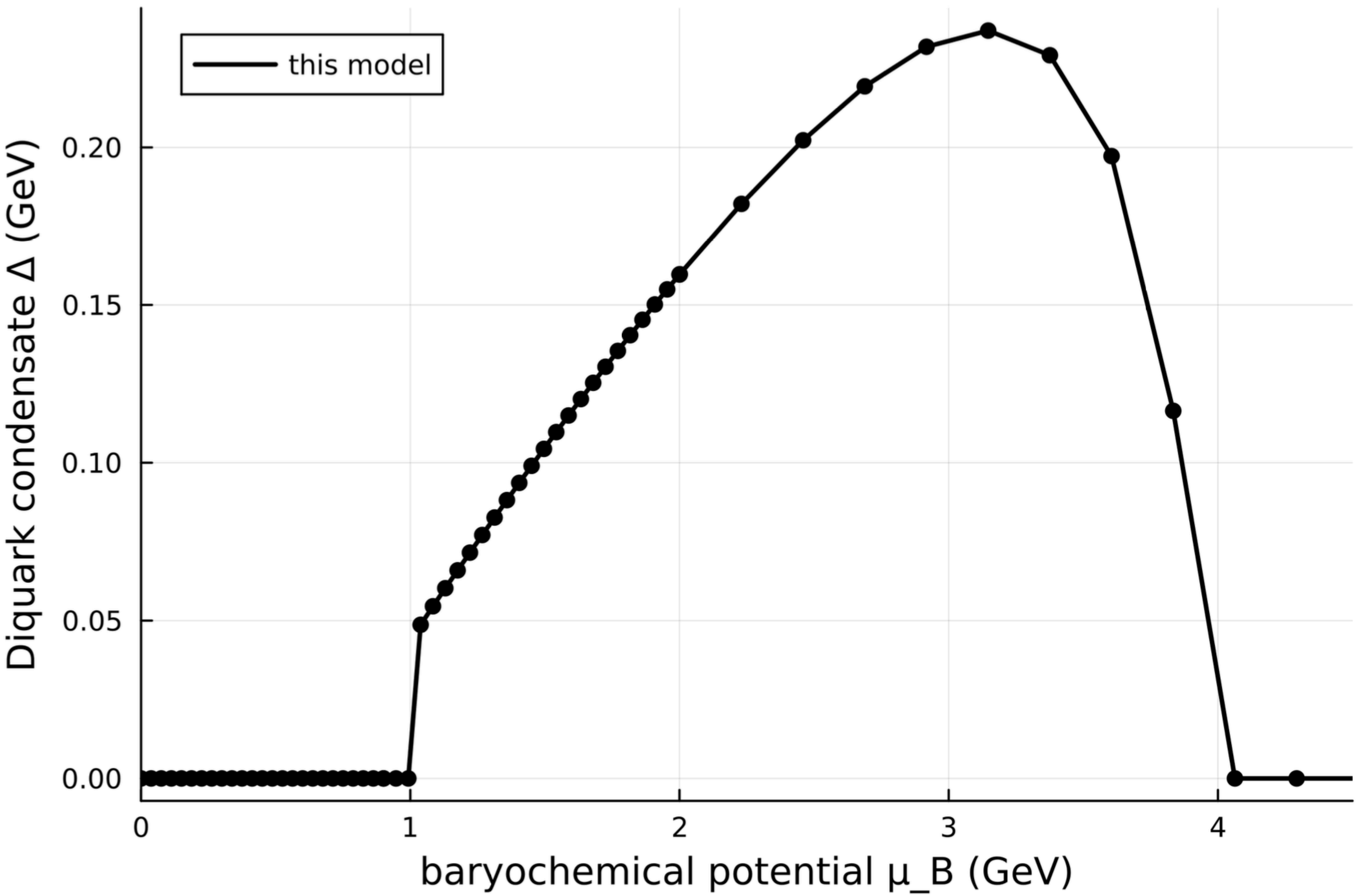
The quark-number susceptibility χ_q is examined as an observable which may help to reveal the physical picture of the high-temperature phase of QCD. It is emphasized that χ_q is intimately related with the fluctuations in the vector channel of the system. It is shown that the results of the recent lattice simulations of χ_q can be understood in terms of a possible change of the interactions between quark and anti-quarks in the vector channel, and imply that the fluctuations in the vector channel is greatly suppressed in the high-temperature phase in contrast with those in the scalar and pseudo-scalar ones.

CHIRAL SUSCEPTIBILITY

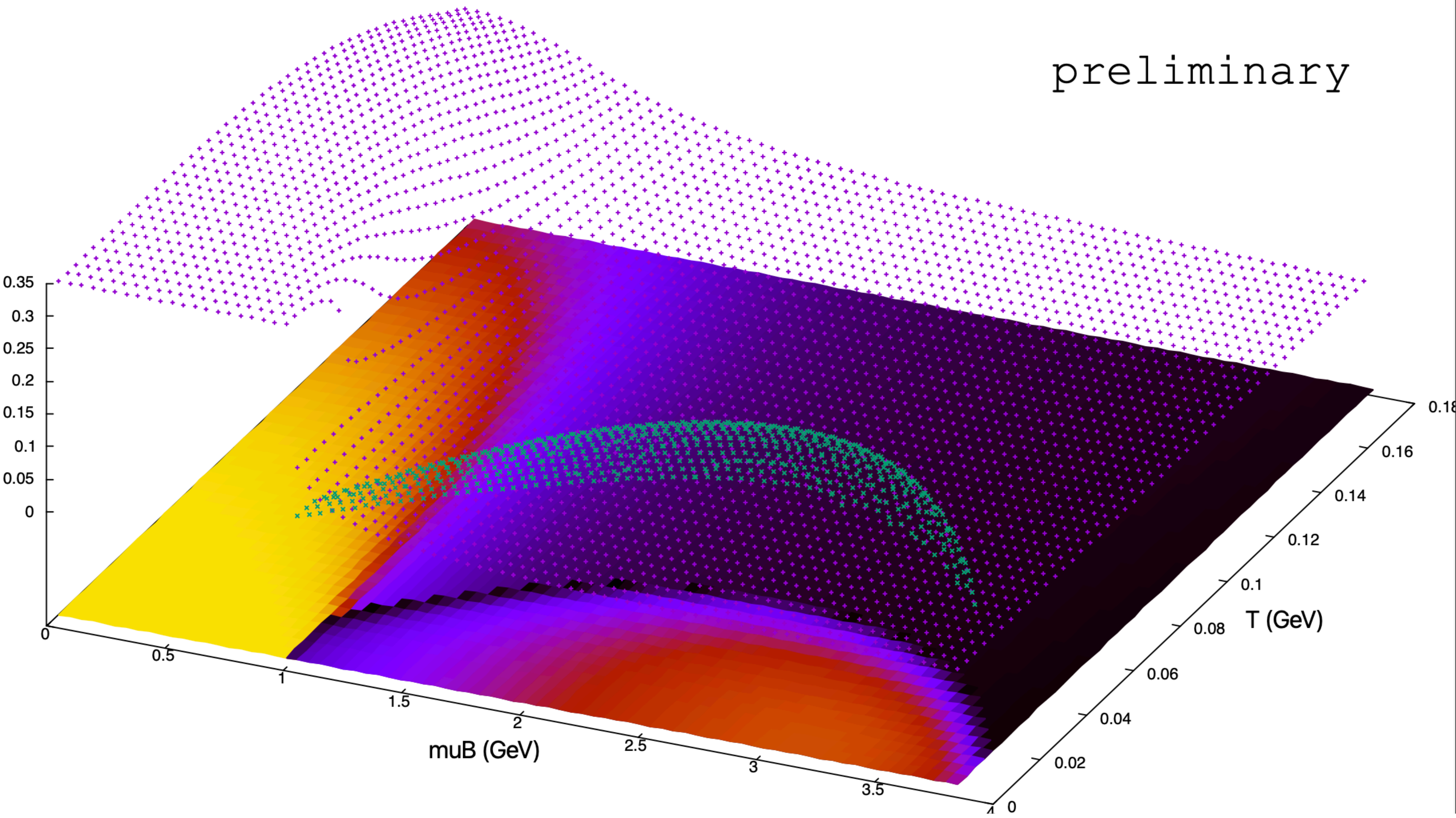


Diquark Gap



this model
Diquark condensate Δ (GeV)
0.00
0.05
0.10
0.15
0.20
0
1
2
3
4
baryochemical potential μ_B (GeV)

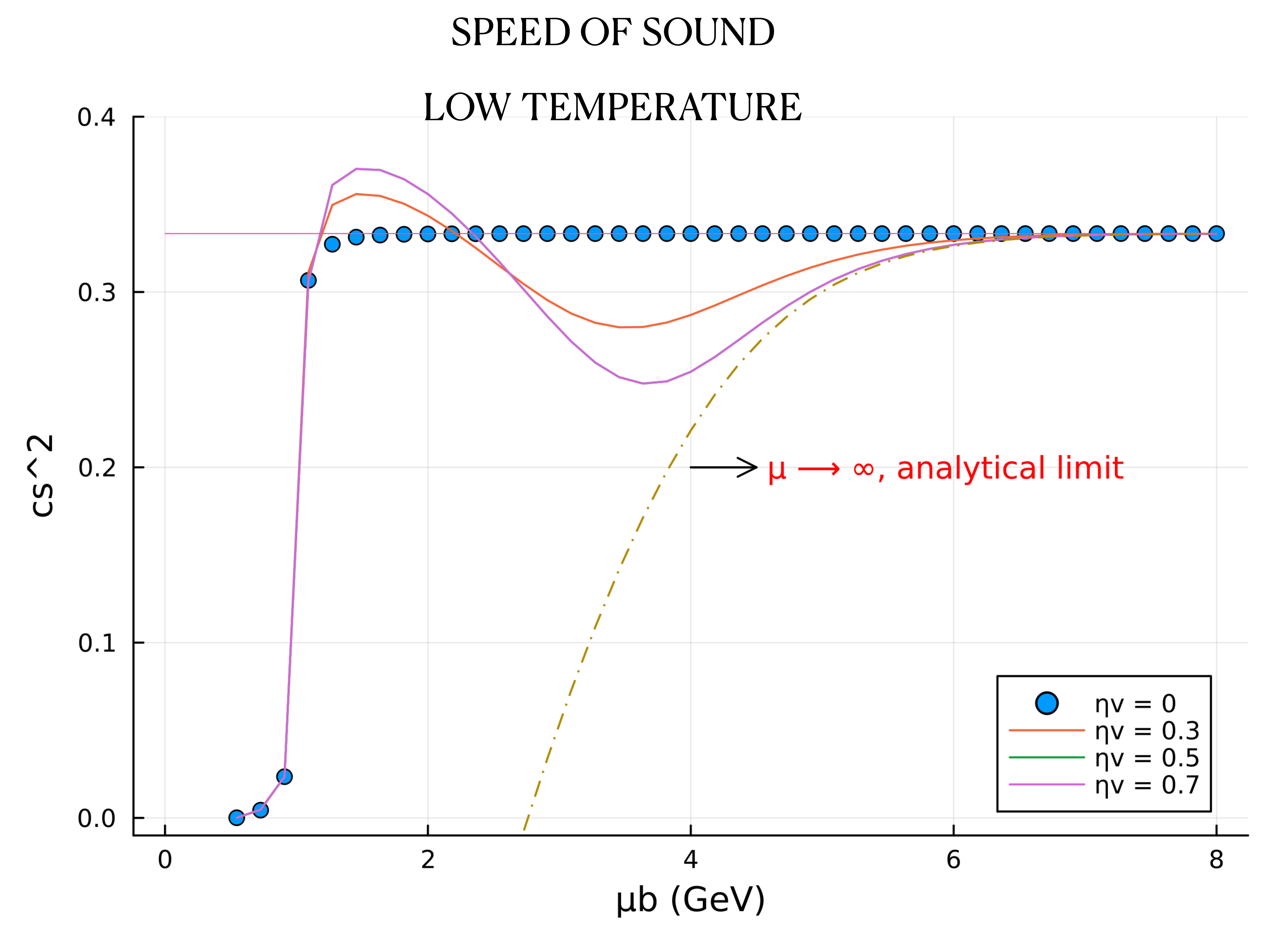
QCD Phase diagram



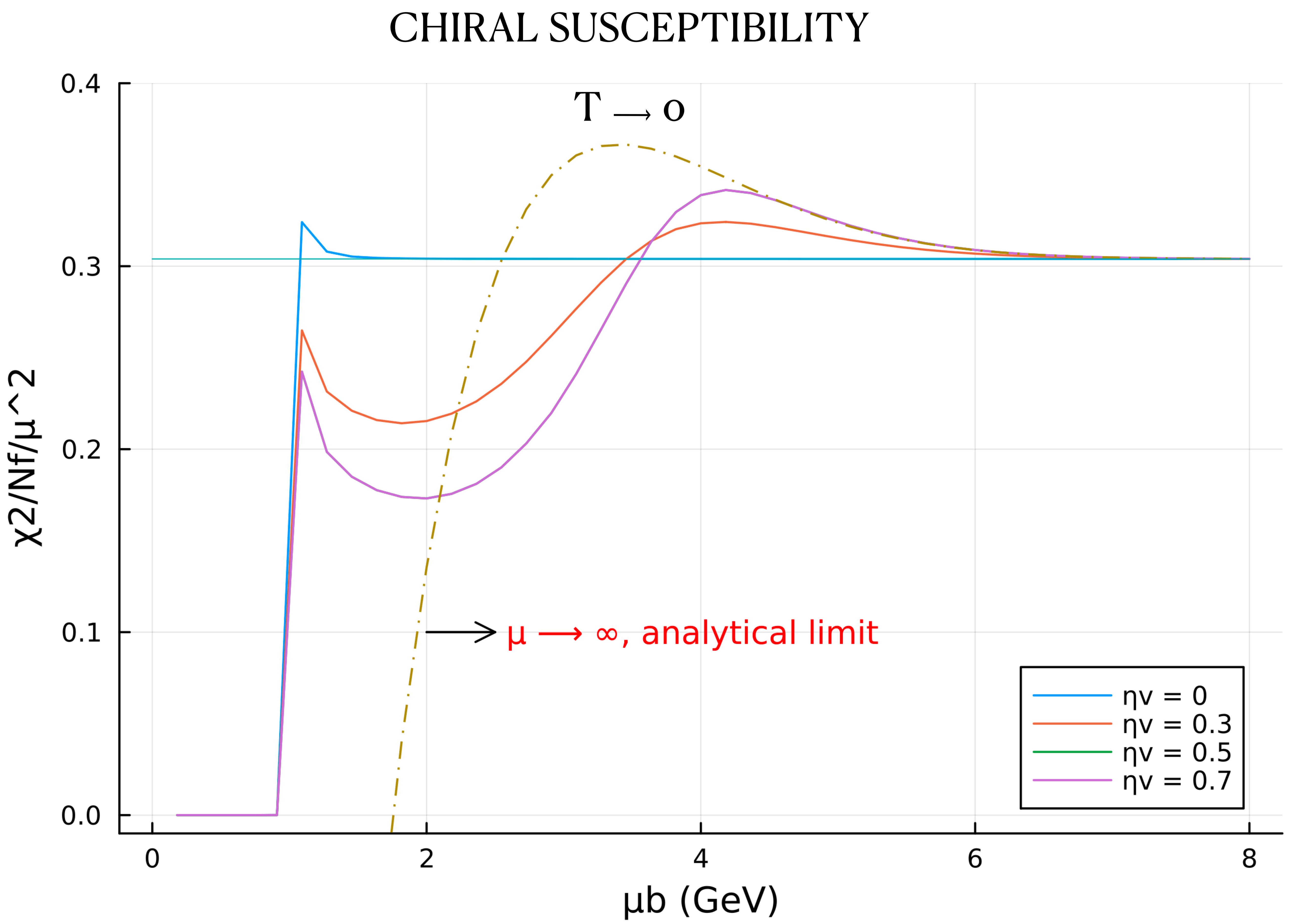
SUMMARY & CONCLUSIONS

- Inherent gluon scale essential for modelling interacting Fermi surface.
- Interacting fermi surface leads to natural density-dependence.
- Speed of sound and chiral susceptibility reach conformal limit in a non-local model.
- Non-local cut-off gives essential, quantifiable contributions to speed of sound and chiral susceptibility & not a scale to be removed.

BACK UP SLIDES



SPEED OF SOUND
LOW TEMPERATURE
cs^2
μb (GeV)
0.0
0.1
0.2
0.3
0.4
0
2
4
6
8
μ ⟶ ∞, analytical limit
ηv = 0
ηv = 0.3
ηv = 0.5
ηv = 0.7



CHIRAL SUSCEPTIBILITY
T ⟶ o
μ ⟶ ∞, analytical limit
ηv = 0
ηv = 0.3
ηv = 0.5
ηv = 0.7
χ2/Nf/μ^2
μb (GeV)
0.0
0.1
0.2
0.3
0.4
0
2
4
6
8

$$n(x) = \frac{1}{1 + \exp{(x/T)}}$$

$$\sigma = f(\sigma, \omega, \Delta)$$

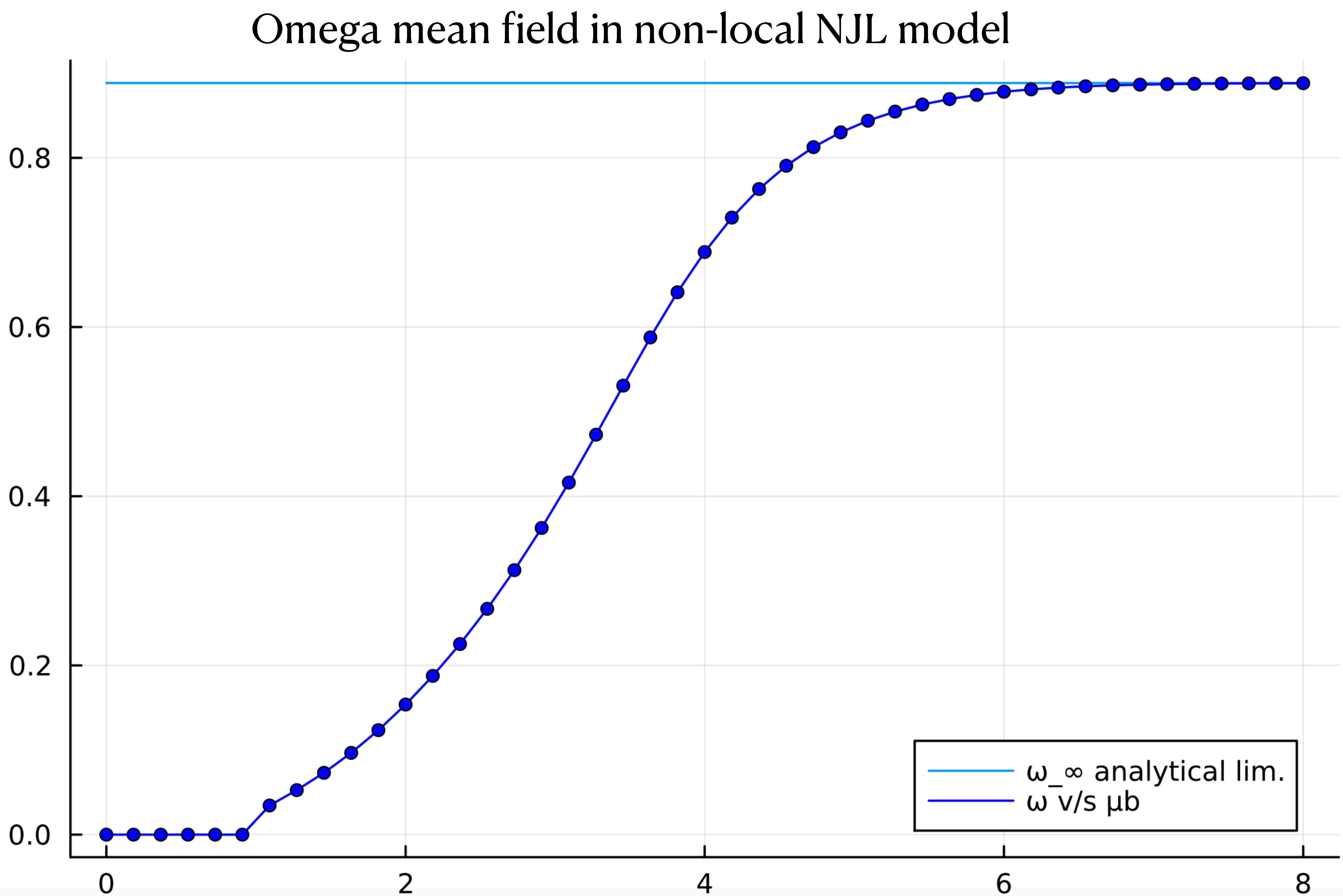
$$\sigma = \frac{4 * 2\, G_s}{2\pi^2} \int dq\, q^2\, \gamma(q)\, \frac{M_q}{2E_q} \left[1 - n(E_q^+) - n(E_q^-) + \frac{E_q^+}{\epsilon_q^+} \left(1 - 2n(\epsilon_q^+)\right) + \frac{E_q^-}{\epsilon_q^-} \left(1 - 2n(\epsilon_q^-)\right)\right]$$

$$\omega = f(\sigma, \omega, \Delta)$$

$$\omega = \frac{4 * 2\, G_v}{2 * 2\pi^2} \int dq\, q^2\, \gamma(q) \left[n(E_q^-) - n(E_q^+) + \frac{E_q^+}{\epsilon_q^+} \left(1 - 2n(\epsilon_q^+)\right) - \frac{E_q^-}{\epsilon_q^-} \left(1 - 2n(\epsilon_q^-)\right)\right]$$

$$\Delta = f(\sigma, \omega, \Delta)$$

$$\Delta = \frac{4 * 2\, G_d}{2 * 2\pi^2} \int dq\, q^2\, \gamma(q)\, \Delta \left[\frac{1}{\epsilon_q^+} \left(1 - 2n(\epsilon_q^+)\right) + \frac{1}{\epsilon_q^-} \left(1 - 2n(\epsilon_q^-)\right)\right]$$


Omega mean field in non-local NJL model
0.0
0.2
0.4
0.6
0.8
0
2
4
6
8
ω_∞ analytical lim.
ω v/s μb