Mott Transition in the HRG and Onset of Deconfinement

David Blaschke (IFT UWr & HZDR/CASUS Görlitz)

Rubin Vase (2015)

„Various faces of QCD“, Wroclaw, 26.-28. April 2024

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CASUS CENTER FOR ADVANCED SYSTEMS UNDERSTANDING

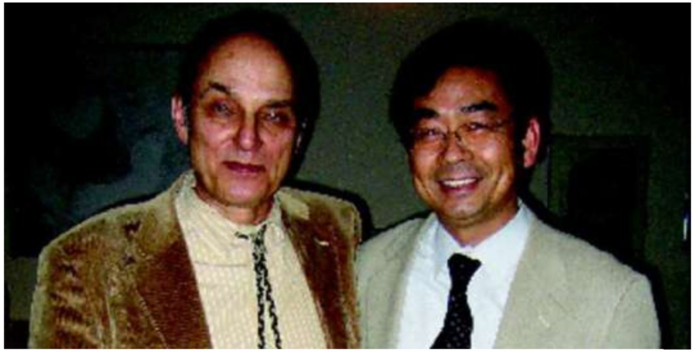
HZDR HELMHOLTZ ZENTRUM DRESDEN ROSSENDORF

Quark Deconfinement as a Mott Transition

Helmut Satz, Quark Matter 1984: A Summary, BI-TP 84/24 (1984)

QED	QCD
electric charge	colour charge
atom, positronium (electrically neutral)	hadron (colour neutral)
insulator (no electric conductivity)	hadronic matter (no colour conductivity)
metal (electric conductor)	chromoplasma (colour conductor)

In particular, we note that the transition from hadronic or nuclear matter to a chromoplasma corresponds to an insulator-metal transition in solid state physics. Deconfinement thus appears as the Mott transition[12] of QCD; it occurs because of the Debye screening of a given colour charge due to the presence of many other such charges[13].



J/ψ SUPPRESSION BY QUARK–GLUON PLASMA FORMATION ☆

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Received 17 July 1986

Mott Transition ?

Sir Neville Mott (1952)

In explaining the insulator-metal transition in solids under compression as bound state dissociation by screening, Sir Neville Mott considered the Debye potential. Analogously, Matsui and Satz considered the effect of J/psi suppression in HIC as charmonium bound state dissociation by screening of OGE in the quark plasma.

$$V_{\rm sc}(q) = V(q)/[1+F(0;\mathbf{q})/q^2]\ , \qquad V(q) = -\tfrac{4}{3}g^2/q^2,\ \ F(0;\mathbf{q}) = -\Pi_{00}(0;\mathbf{q})$$

$$\Pi_{00}(0;\mathbf{q}) = \frac{2N_{\rm dof}g^2}{\pi^2}\int_0^\infty dp\,p^2\frac{\partial f_\Phi}{\partial p} = -\frac{4N_{\rm dof}g^2}{\pi^2}\int_0^\infty dp\,p\,f_\Phi(p) = -\frac{N_{\rm dof}g^2T^2}{3}I(\Phi) = -m_D^2(T)$$

Debye potential: $V_{\rm sc}(q) = -4\pi\alpha/[q^2+m_D^2(T)]$

$$I(\Phi) = \frac{12}{\pi^2}\int_0^\infty dx\,x\frac{\Phi(1+2e^{-x})e^{-x}+e^{-3x}}{1+3\Phi(1+e^{-x})e^{-x}+e^{-3x}}$$

Schrödinger equation for quarkonia → Ritz Variational principle:

$$H = -\frac{\nabla^2}{m_Q} - \frac{\alpha}{r}e^{-m_D(T)r}\ ,$$

Trial wave function (H-atom): $\psi_{1S}(r;\gamma) = \sqrt{\frac{\gamma^3}{\pi}}\exp(-\gamma r)$

$$E_{1S}(\gamma,T) = \langle\psi_{1S}(\gamma)\,|\,H\,|\,\psi_{1S}(\gamma)\rangle = \frac{\gamma^2}{m_Q} - \frac{4\alpha\gamma^3}{(m_D(T)+2\gamma)^2}$$

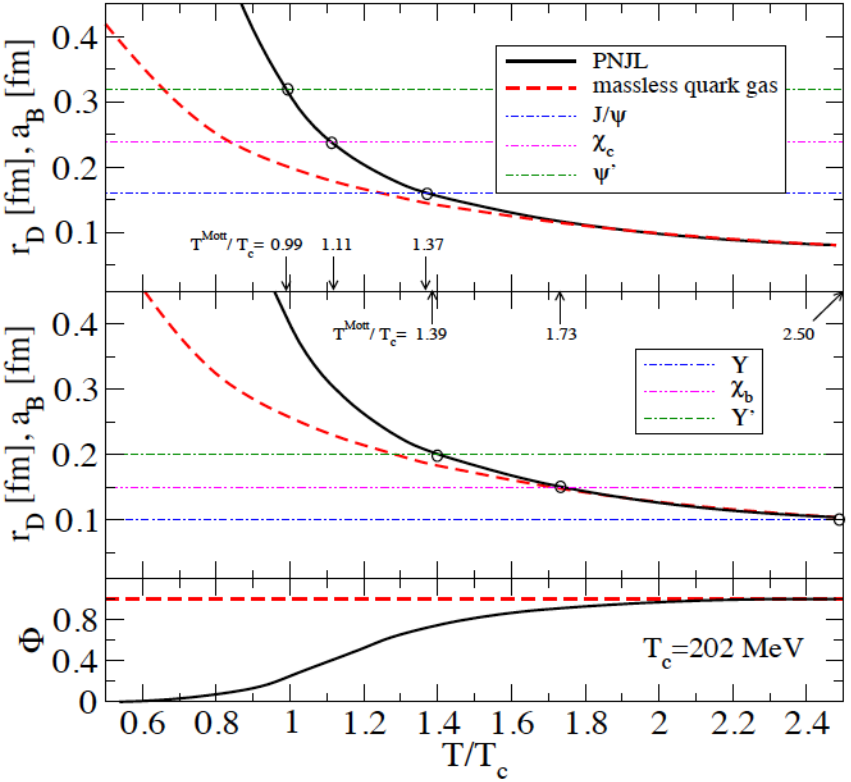
$$dE_{1S}(\gamma,T)/d\gamma = 0$$

Mott condition of vanishing binding energy: $E_{1S}(\gamma, T^{\rm Mott}) = 0$ → $r_D(T_{1S}^{\rm Mott}) = a_0\ ,$

$a_0 = 2/(\alpha m_Q) = 1/\sqrt{\varepsilon_0 m_Q}$ is the Bohr radius and $\varepsilon_0 = \alpha^2 m_Q/4$ the binding energy

Mott Transition !

$$T^{\text{Mott}} = \sqrt{3\varepsilon_0 m_Q / N_{\text{dof}} / g} = \sqrt{\sqrt{\varepsilon_0 m_Q^3} / (2\pi N_{\text{dof}})}$$



Temperature-dependent Debye radii $r_D(T) = 1/m_D(T)$ vs. Bohr radii

Jakub Jankowski in BLTP @ JINR Dubna (2008)

Effect of the Polyakov-loop on Mott temperatures for quarkonia

$$T^{\text{Mott},\Phi} = T^{\text{Mott}} / \sqrt{I(\Phi)}$$

	T^{Mott}/T_c	$T^{\text{Mott},\Phi}/T_c$
J/ψ	1.25	1.37
χ_c	0.83	1.11
ψ'	0.66	0.99
Υ	2.50	2.50
χ_b	1.72	1.73
Υ'	1.28	1.39

Mott temperatures T^{Mott} ($T^{\text{Mott},\Phi}$) for heavy quarkonia in a massless quark plasma without (with) coupling to Polyakov loop Φ

Jankowski, DB, DESY-PROC-2009-07 (2009) 271

Mott Transition in Nuclear Matter

Nuclear Physics **A399** (1983) 587–602

PARTICLE CLUSTERING AND MOTT TRANSITION IN NUCLEAR MATTER AT FINITE TEMPERATURE

(II). Self-consistent ladder Hartree-Fock approximation and model calculations for cluster abundances and the phase diagram

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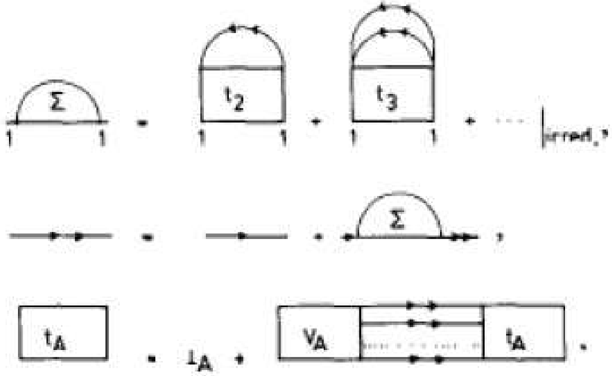
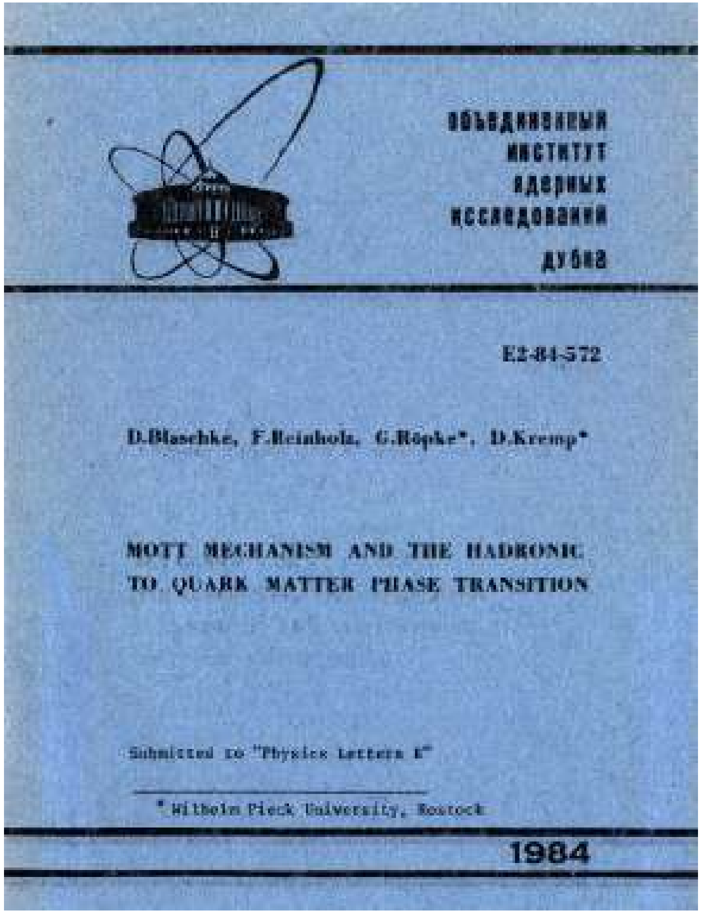
Received 11 June 1982
(Revised 24 September 1982)



With Gerd Röpke during a visit in Rostock for Erasmus lecturer exchange (~2010)

Quark Deconfinement as a Mott Transition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институт
ядерных
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Дубна

E2-84-572

D.Blaschke, F.Reinholz, G.Röpke*, D.Kremp*

MOTT MECHANISM AND THE HADRONIC TO QUARK MATTER PHASE TRANSITION

Submitted to "Physics Letters B"

* Wilhelm Pieck University, Rostock

1984

Potential energy density for confining interaction $V_{qq}(r)$

$$v_{\mathrm{pot}} = \frac{1}{2}\left(n_{\mathrm{q}} \int V_{\mathrm{q}\bar{\mathrm{q}}}(r)\, w_{\bar{\mathrm{q}}}(r)\, \mathrm{d}^3 r + n_{\bar{\mathrm{q}}} \int V_{\bar{\mathrm{q}}\mathrm{q}}(r)\, w_{\mathrm{q}}(r)\, \mathrm{d}^3 r\right)$$

Pair distribution function for

bound states $\quad w_{\mathrm{q}}^{\mathrm{b}}(r) = |\psi(r)|^2$

and

free states $\quad w_{\bar{\mathrm{q}}}^{\mathrm{HF}}(r) = \frac{1}{3} n_{\bar{\mathrm{q}}} \exp(-4\pi n_{\bar{\mathrm{q}}} r^3/9)$

Free energy density of free quark matter

$$f^{\mathrm{HF}}(T) = f_{\mathrm{q}}^{\mathrm{id}}(T) + V_{\mathrm{pot}}^{\mathrm{HF}}\,,$$

$$f_{\mathrm{q}}^{\mathrm{id}}(T) = 2 g_{\mathrm{q}} T \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \ln(\exp(\epsilon_{\mathrm{q}}(p)/T) + 1)\,,$$

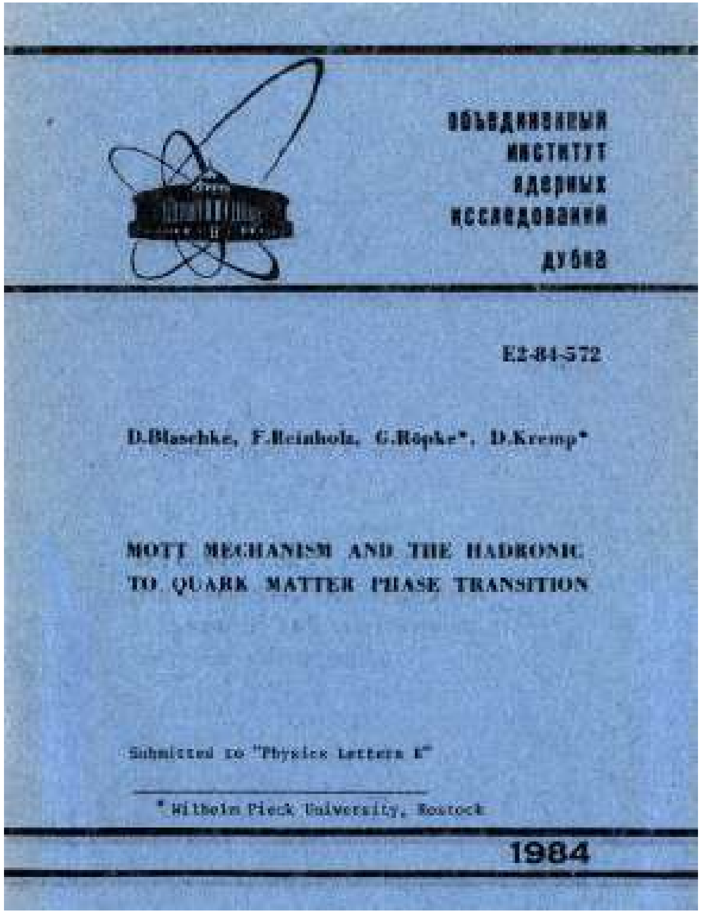
$$\epsilon_{\mathrm{q}}^2(p) = m_{\mathrm{q}}^2 + p^2\,.$$

$$V_{\mathrm{pot}}^{\mathrm{HF}} = \frac{1}{3} n_{\mathrm{q}} n_{\bar{\mathrm{q}}} \int V_{\mathrm{q}\bar{\mathrm{q}}}(r) \exp(-4\pi n_{\bar{\mathrm{q}}} r^3/9)\, \mathrm{d}^3 r\,.$$


The authors in Ahrenshoop (1983)

Published in Phys. Lett. 151B (1985) 439

Quark Deconfinement as a Mott Transition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ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E2-84-572

D.Blaschke, F.Reinholz, G.Röpke*, D.Kremp*

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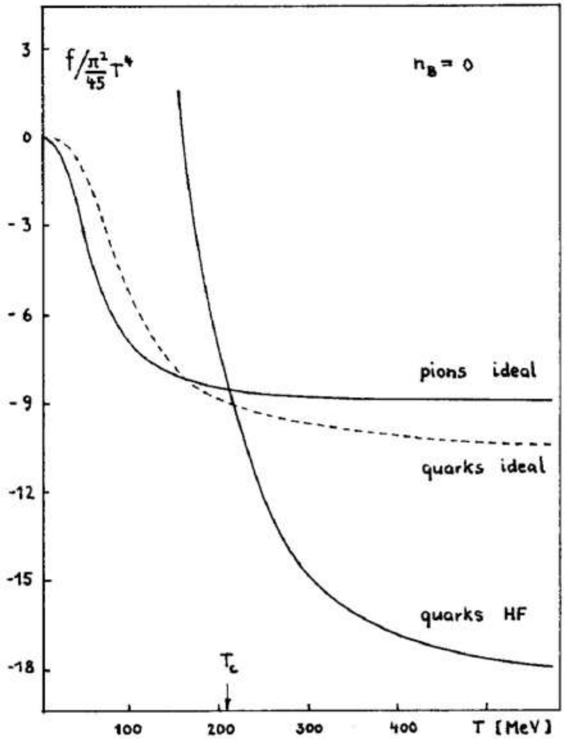
$$\epsilon_{\mathrm{q}}^2(p) = m_{\mathrm{q}}^2 + p^2 .$$

$$V_{\mathrm{pot}}^{\mathrm{HF}} = \frac{1}{3} n_{\mathrm{q}} n_{\bar{\mathrm{q}}} \int V_{\mathrm{q}\bar{\mathrm{q}}}(r) \exp(-4\pi n_{\bar{\mathrm{q}}} r^3/9)\, \mathrm{d}^3 r .$$



Published in Phys. Lett. 151B (1985) 439

String-flip model for quark matter

Röpke, Blaschke, Schulz, PRD 34 (1986) 3499

→ Relativistic reformulation: Kaltenborn, Bastian, Blaschke, PRD 96 (2017) 056024

$$\mathcal{Z} = \int \mathcal{D}\bar{q}\mathcal{D}q \exp\left\{\int_0^\beta d\tau \int_V d^3x \left[\mathcal{L}_{\rm eff} + \bar{q}\gamma_0\hat{\mu}q\right]\right\} , \quad q = \begin{pmatrix} q_u \\ q_d \end{pmatrix} , \quad \hat{\mu} = \mathrm{diag}(\mu_u, \mu_d)$$

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm free} - U(\bar{q}q, \bar{q}\gamma_0 q) , \quad \mathcal{L}_{\rm free} = \bar{q}\left(-\gamma_0\frac{\partial}{\partial\tau} + i\vec{\gamma}\cdot\vec{\nabla} - \hat{m}\right)q , \; \hat{m} = \mathrm{diag}(m_u, m_d)$$

General nonlinear functional of quark density bilinears: scalar, vector, isovector, diquark ...
Expansion around the expectation values:

$$U(\bar{q}q, \bar{q}\gamma_0 q) = U(n_{\rm s}, n_{\rm v}) + (\bar{q}q - n_{\rm s})\Sigma_{\rm s} + (\bar{q}\gamma_0 q - n_{\rm v})\Sigma_{\rm v} + \dots ,$$

$$\langle\bar{q}q\rangle = n_{\rm s} = \sum_{f=u,d} n_{{\rm s},f} = -\sum_{f=u,d}\frac{T}{V}\frac{\partial}{\partial m_f}\ln\mathcal{Z} , \quad \Sigma_{\rm s} = \left.\frac{\partial U(\bar{q}q, \bar{q}\gamma_0 q)}{\partial(\bar{q}q)}\right|_{\bar{q}q=n_{\rm s}} = \frac{\partial U(n_{\rm s}, n_{\rm v})}{\partial n_{\rm s}} ,$$

$$\langle\bar{q}\gamma_0 q\rangle = n_{\rm v} = \sum_{f=u,d} n_{{\rm v},f} = \sum_{f=u,d}\frac{T}{V}\frac{\partial}{\partial\mu_f}\ln\mathcal{Z} , \quad \Sigma_{\rm v} = \left.\frac{\partial U(\bar{q}q, \bar{q}\gamma_0 q)}{\partial(\bar{q}\gamma_0 q)}\right|_{\bar{q}\gamma_0 q=n_{\rm v}} = \frac{\partial U(n_{\rm s}, n_{\rm v})}{\partial n_{\rm v}}$$

$$\mathcal{Z} = \int \mathcal{D}\bar{q}\mathcal{D}q \exp\left\{\mathcal{S}_{\rm quasi}[\bar{q}, q] - \beta V\Theta[n_{\rm s}, n_{\rm v}]\right\} , \quad \Theta[n_{\rm s}, n_{\rm v}] = U(n_{\rm s}, n_{\rm v}) - \Sigma_{\rm s}n_{\rm s} - \Sigma_{\rm v}n_{\rm v}$$

$$\mathcal{S}_{\rm quasi}[\bar{q}, q] = \beta\sum_n\sum_{\vec{p}}\bar{q}\,G^{-1}(\omega_n, \vec{p})\,q , \quad G^{-1}(\omega_n, \vec{p}) = \gamma_0(-i\omega_n + \hat{\mu}^*) - \vec{\gamma}\cdot\vec{p} - \hat{m}^*$$

Relativistic density functional for quark matter

O. Ivanytskyi & D. Blaschke, Phys. Rev. D 105 (2022) 114042

Interaction model: $\mathcal{U} = D_0 \left[(1+\alpha)\langle \overline{q}q \rangle_0^2 - (\overline{q}q)^2 - (\overline{q}i\vec{\tau}\gamma_5 q)^2 \right]^{\varkappa}$

- **Parameters**

  D_0 - dimensionfull coupling, controls interaction strength

  α - dimensionless constant, controls vacuum quark mass

  $\langle \overline{q}q \rangle_0$ - χ-condensate in vacuum (introduced for the sake of convenience)

$\varkappa = 1/3$
$\Downarrow$
motivated by String Flip model

$\varkappa = 1$
$\Downarrow$
Nambu–Jona-Lasinio model

$$\mathcal{U}_{SFM} \propto \langle q^+ q \rangle^{2/3}$$

$$\Sigma_{SFM} = \frac{\partial \mathcal{U}_{SFM}}{\partial \langle q^+ q \rangle} \propto \langle q^+ q \rangle^{-1/3} \propto separation$$

- **Dimensionality**

$$[\mathcal{U}] = energy^4 \qquad [\overline{q}q] = energy^3 \quad \Rightarrow \quad [D_0]_{\varkappa=1/3} = energy^2 = [string\ \ tension]$$

self energy = string tension × separation $\Rightarrow$ **confinement**

J/ψ absorption in a multicomponent hadron gas

D. Prorok*, L. Turko* and D. Blaschke*,†

$$\mathcal{N}_{n.m.}(\epsilon_0) \cong \exp\left\{-\sigma_{J/\psi N}\,\rho_0 L\right\} \;,\quad \sigma_{J/\psi N} = 4.2 \pm 0.5 \text{ mb}$$

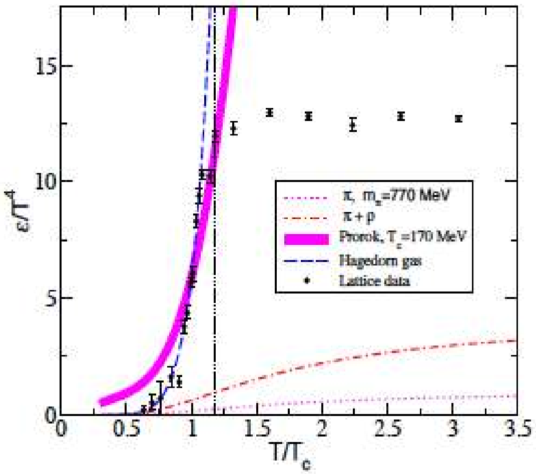
$$\rho_0 L(b) = \frac{1}{2T_{AB}} \int d^2\vec{s}\, T_A(\vec{s}) T_B(\vec{s}-\vec{b}) \left[\frac{A-1}{A} T_A(\vec{s}) + \frac{B-1}{B} T_B(\vec{s}-\vec{b})\right]$$

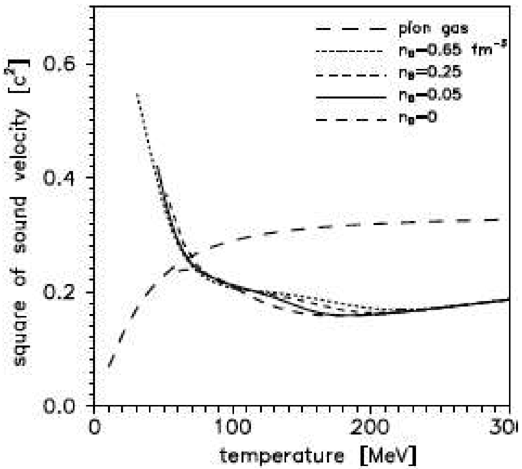
$$\mathcal{N}_{h.g.}(\epsilon_0) = \int dp_T\, g(p_T,\epsilon_0) \cdot \exp\left\{-\int_{t_0}^{t_{final}} dt \sum_{i=1}^{l} \int \frac{d^3\vec{q}}{(2\pi)^3} f_i(\vec{q},t)\sigma_i v_{rel,i} \frac{p_\nu q_i^\nu}{E E_i'}\right\} \;,$$

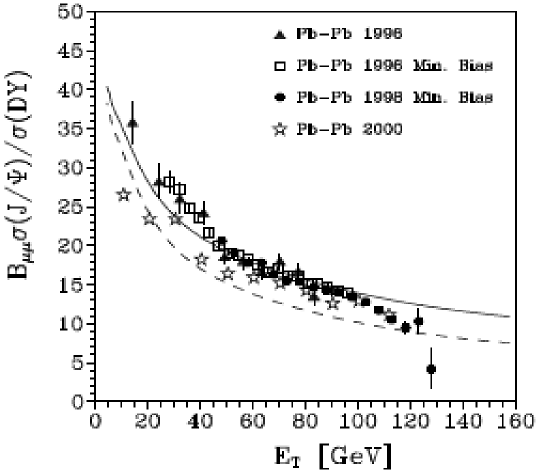
$$\mathcal{N}(\epsilon_0) = \mathcal{N}_{n.m.}(\epsilon_0) \cdot \mathcal{N}_{h.g.}(\epsilon_0) \;.$$

David Blaschke arrived in Wroclaw (2006)

AIP Conf. Proc. 1038 (2008) 73, „Three days of strong interactions ...“







Mott-Hagedorn Resonance Gas and Lattice QCD Results*

L. Turko[1]† D. Blaschke[1,2]‡ D. Prorok[1]§ and J. Berdermann[3]¶

Acta Physica Polonica B Proc. Suppl. 5 (2012) 485
„Three Days on Quarkyonic Island“

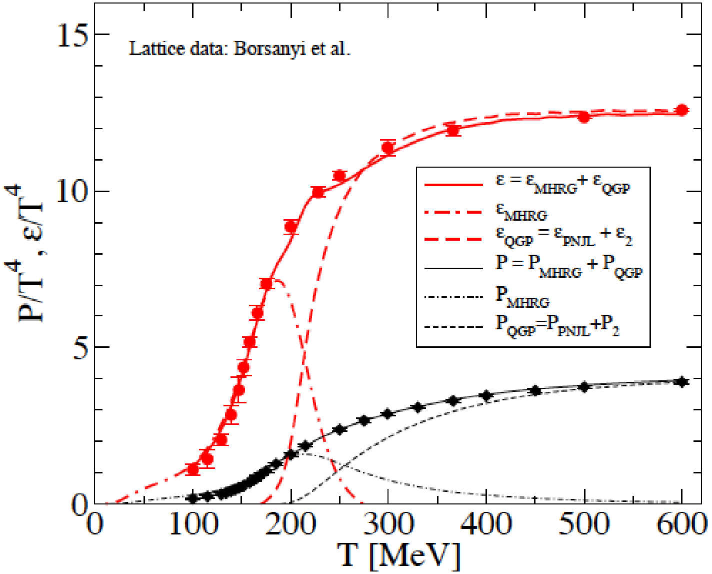
At Rynek in Wroclaw (2006)

$$\varepsilon(T) = \sum_{m_i<m_0} g_i\,\varepsilon_i(T;m_i) + \sum_{m_i\geq m_0} g_i \int_{m_0^2}^{\infty} d(M^2)\,A(M,m_i)\,\varepsilon_i(T;M),$$

$$\varepsilon_i(T;M) = \int \frac{d^3k}{(2\pi)^3} \frac{\sqrt{k^2+M^2}}{\exp\left(\frac{\sqrt{k^2+M^2}}{T}\right)+\delta_i},$$

$$A(M,m) = N_m \frac{\Gamma\cdot m}{(M^2-m^2)^2+\Gamma^2\cdot m^2},$$

$$\Gamma(T) = C_\Gamma \left(\frac{m}{T_H}\right)^{N_m} \left(\frac{T}{T_H}\right)^{N_T} \exp\left(\frac{m}{T_H}\right),$$



D. B. Blaschke, K. A. Bugaev, Fizika B **13**, 491 (2004)
Phys. Part. Nucl. Lett. **2**, 305 (2005).

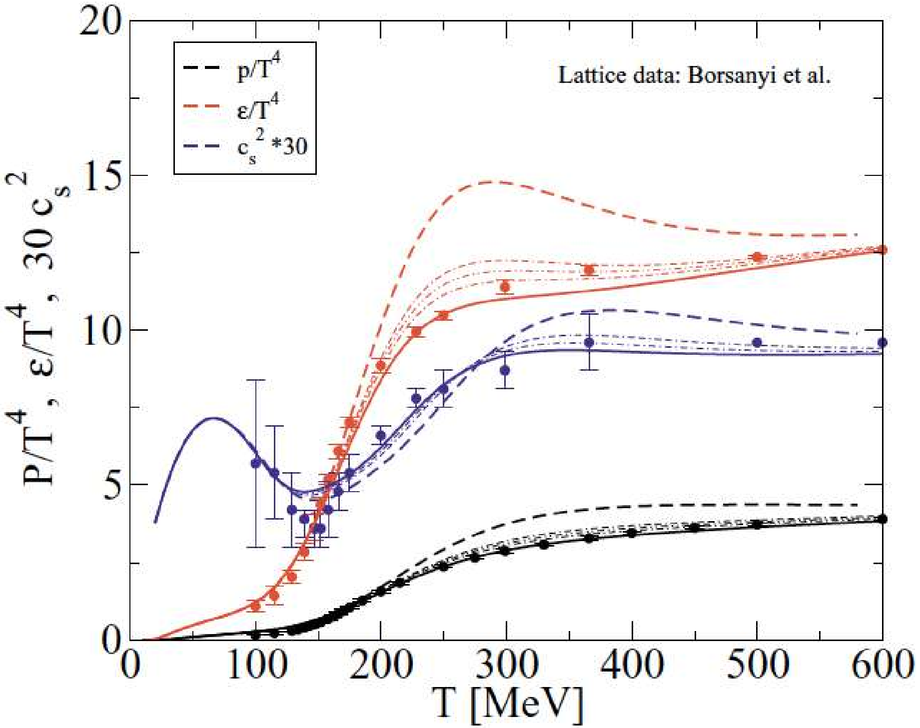
Effective degrees of freedom in QCD thermodynamics

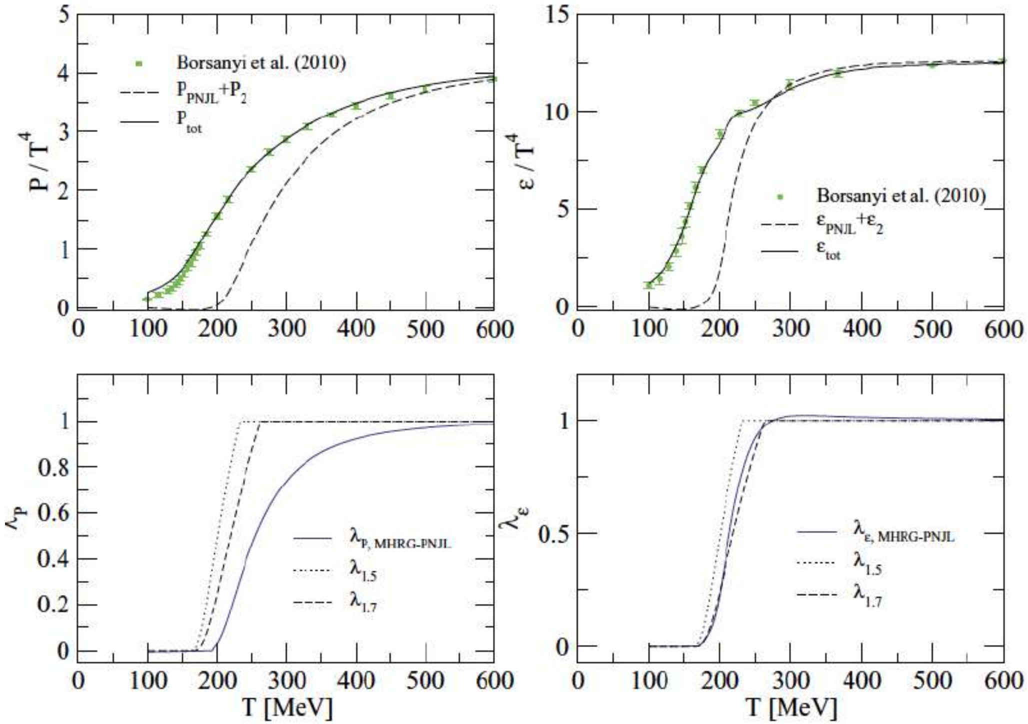
L. Turko[1,a], D. Blaschke[1,2,b], D. Prorok[1,c], and J. Berdermann[3,d]

EPJ Web of Conferences **71**, 00134 (2014)

$$\Gamma_i(T) = \tau^{-1}_{\mathrm{coll,i}}(T) = \sum_j \lambda \langle r_i^2 \rangle_T \langle r_j^2 \rangle_T \, n_j(T) \, , \quad r_\pi^2(T) = \frac{3}{4\pi^2} f_\pi^{-2}(T) = \frac{3M_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_T|^{-1} \, .$$

Wroclaw group in Knoxville (2009)





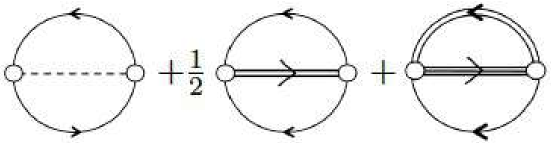
Generalized Beth-Uhlenbeck approach to the equation of state for quark-hadron matter*

D. Blaschke[a,b,c], A. Dubinin[a], L. Turko[a]

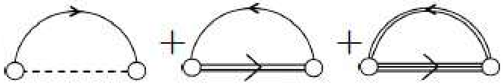
APPB Proc. Suppl. 10 (2017) 473; Proc. CPOD 2016

$$\Omega = \sum_{i=q,d,M,B} \frac{c_i}{2}\left[\mathrm{Tr}\ln(G_i^{-1}) - \mathrm{Tr}(\Sigma_i G_i)\right] + \Phi\left[\{G_i\}\right] + \mathscr{U}[\phi;T] + \Omega_{\mathrm{pert}}.$$

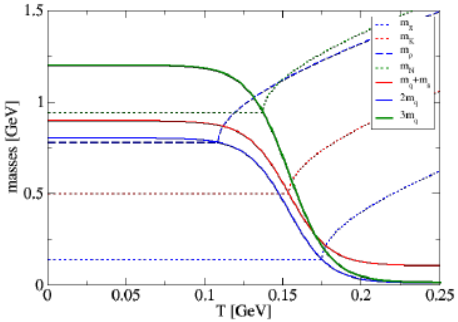
$\Phi\left[\{G_i\}\right] =$ $\frac{1}{2}$ + $\frac{1}{2}$ +

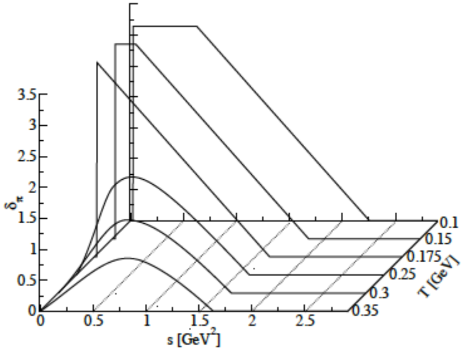


$\Sigma_q = \frac{\delta\Phi}{\delta G_q} =$ + +



$$n_i(T) = \int_0^\infty \frac{d\hat{M}}{\pi} n_{s,i}(\hat{M})[\delta_i(\hat{M}^2) - \frac{1}{2}\sin 2\delta_i(\hat{M}^2)]. \quad n_{s,i}(\hat{M}) = d_i T^3 \int_0^\infty \frac{d\hat{p}\,\hat{p}^2}{2\pi^2} \frac{\hat{M}}{\sqrt{\hat{p}^2+\hat{M}^2}} f_i(\sqrt{\hat{p}^2+\hat{M}^2})$$



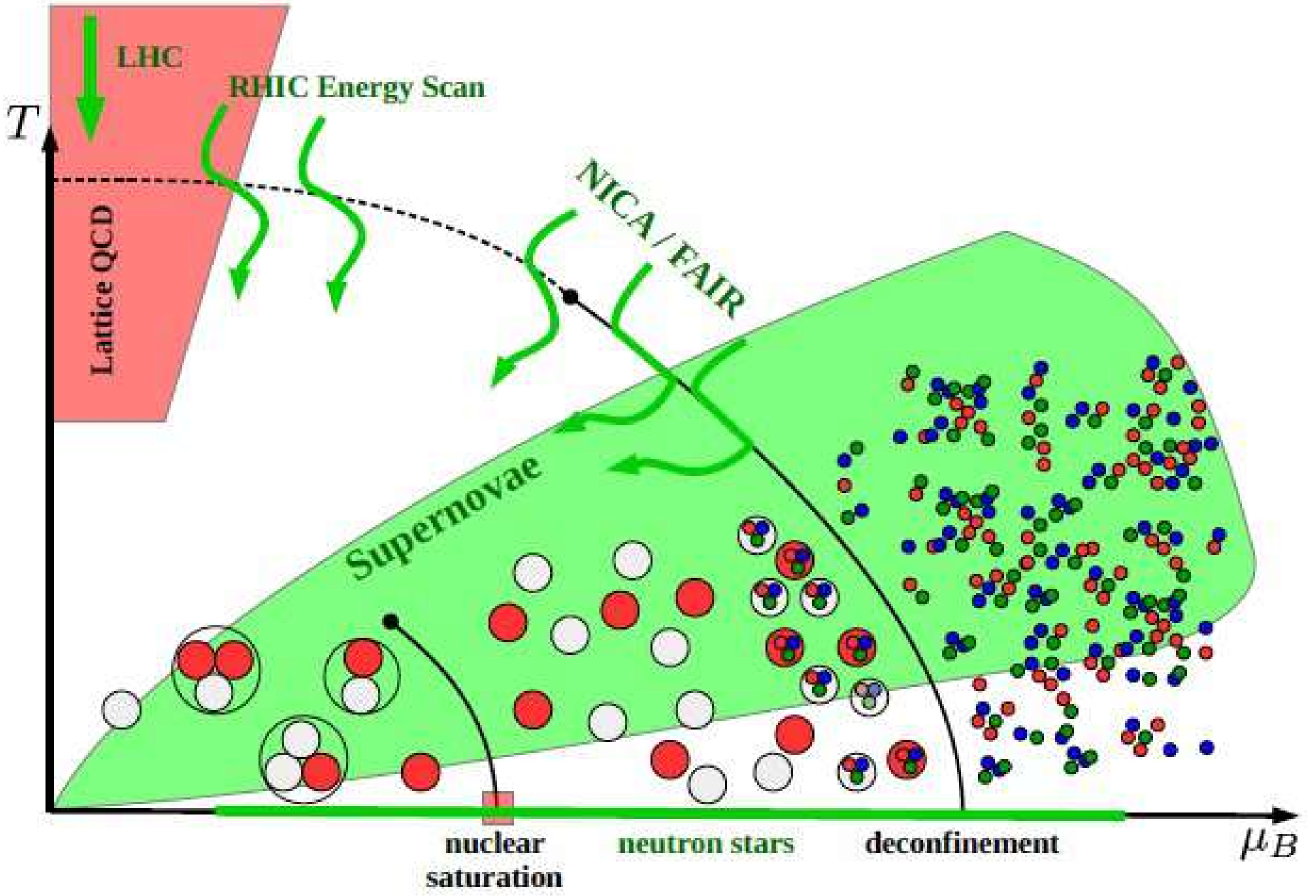


Admiring stars of the southern Sky, Fishhoek (2009)

CPOD 2016

Wroclaw

Unified approach: Clustering aspects in the QCD phase diagram

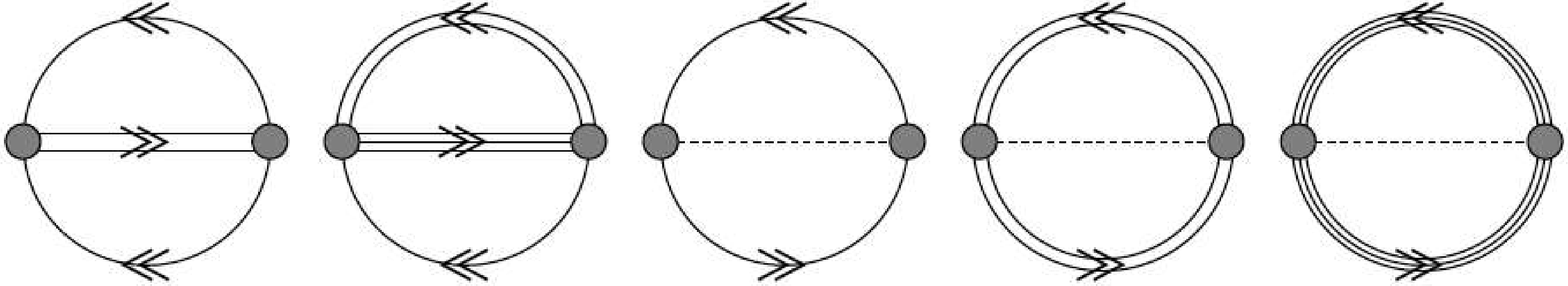


At David's 50. birthday in Prerow (2009)

From: N.-U. Bastian, D.B., et al., Universe 4 (2018) 67; arxiv:1804.10178

Unified approach: Cluster virial expansion for quark-hadron matter

$$\Omega = \sum_{i=Q,M,D,B} c_i \left[\mathrm{Tr}\ln\left(-G_i^{-1}\right) + \mathrm{Tr}\left(\Sigma_i\ G_i\right)\right] + \Phi\left[G_Q, G_M, G_D, G_B\right] ,$$



When Φ functional for the system is given by 2-loop diagrams holds

$$n = -\frac{\partial\Omega}{\partial\mu} = \sum_a a\, n_a(T,\mu)$$

$$= \sum_a a\, d_a \int \frac{d\omega}{\pi} \int \frac{d^3q}{(2\pi)^3} \left\{ f_\phi^{(a),+} - \left[f_\phi^{(a),-}\right]^* \right\} 2\sin^2\delta_a(\omega,q) \frac{\partial\delta_a(\omega,q)}{\partial\omega} ,$$

Analogous for the entropy density $s = -\partial\Omega/\partial T$.

Cluster virial expansion for quark-hadron matter

The cluster decomposition of the thermodynamic potential is given as

$$\Omega_{\rm total}(T,\mu,\phi,\bar\phi) = \Omega_{PNJL}(T,\mu,\phi,\bar\phi) + \Omega_{\rm pert}(T,\mu,\phi,\bar\phi) + \Omega_{MHRG}(T,\mu,\phi,\bar\phi),$$

where the first two terms describe the quark and gluon degrees of freedom via the mean–field thermodynamic potential for quark matter in a gluon background field $\mathcal{U}$

$$\Omega_{PNJL}(T,\mu,\phi,\bar\phi) = \Omega_Q(T,\mu,\phi,\bar\phi) + \mathcal{U}(T,\phi,\bar\phi)$$

with a perturbative correction $\Omega_{\rm pert}(T,\mu,\phi,\bar\phi)$.
The Mott-Hadron-Resonance-Gas (MHRG) part for the multi-quark clusters is

$$\Omega_{MHRG}(T,\mu,\phi,\bar\phi) = \sum_{i=M,B,\dots} \Omega_i(T,\mu,\phi,\bar\phi),$$

where the multi–quark states are described by the GBU formula:

$$\begin{aligned} n &= -\frac{\partial\Omega}{\partial\mu} = \sum_a a\, n_a(T,\mu) \\ &= \sum_a a\, d_a \int \frac{d\omega}{\pi} \int \frac{d^3q}{(2\pi)^3} \left\{ f_\phi^{(a),+} - \left[f_\phi^{(a),-} \right]^* \right\} 2\sin^2\delta_a(\omega,q) \frac{\partial\delta_a(\omega,q)}{\partial\omega} , \end{aligned}$$

where d_i is the degeneracy factor, a is the number of valence quarks in the cluster an $f_\phi^{(a),+}$, $\left[f_\phi^{(a),-}\right]^*$ are the Polyakov-loop modified distribution functions.
Analogous for the entropy density $s = -\partial\Omega/\partial T$.

A big steak for Ludwik at his 65 birthday party in Fishhoek (South Africa)

D.B., M. Cierniak, O. Ivanytskyi and G. Röpke, Eur. Phys. J. A60 (2024) 14; arXiv:22308.07950

Polyakov-loop modified distribution functions

For multiquark clusters with net number a of valence quarks holds

$$f_\phi^{(a),\pm} \overset{(a\ \text{even})}{=} \frac{(\phi - 2\bar\phi y_a^\pm) y_a^\pm + y_a^{\pm 3}}{1 - 3(\phi - \bar\phi y_a^\pm) y_a^\pm - y_a^{\pm 3}} ,$$

$$f_\phi^{(a),\pm} \overset{(a\ \text{odd})}{=} \frac{(\bar\phi + 2\phi y_a^\pm) y_a^\pm + y_a^{\pm 3}}{1 + 3(\bar\phi + \phi y_a^\pm) y_a^\pm + y_a^{\pm 3}} ,$$

where $y_a^\pm = e^{-(E_p \mp a\mu)/T}$ and $E_p = \sqrt{\vec{p}^2 + M^2}$.
It is instructive to consider the two limits $\phi = \bar\phi = 1$ (deconfinement)

$$f_{\phi=1}^{(a=0,2,4,\dots),\pm} = \frac{y_a^\pm}{1 - y_a^\pm} , \quad f_{\phi=1}^{(a=1,3,5,\dots),\pm} = \frac{y_a^\pm}{1 + y_a^\pm} ,$$

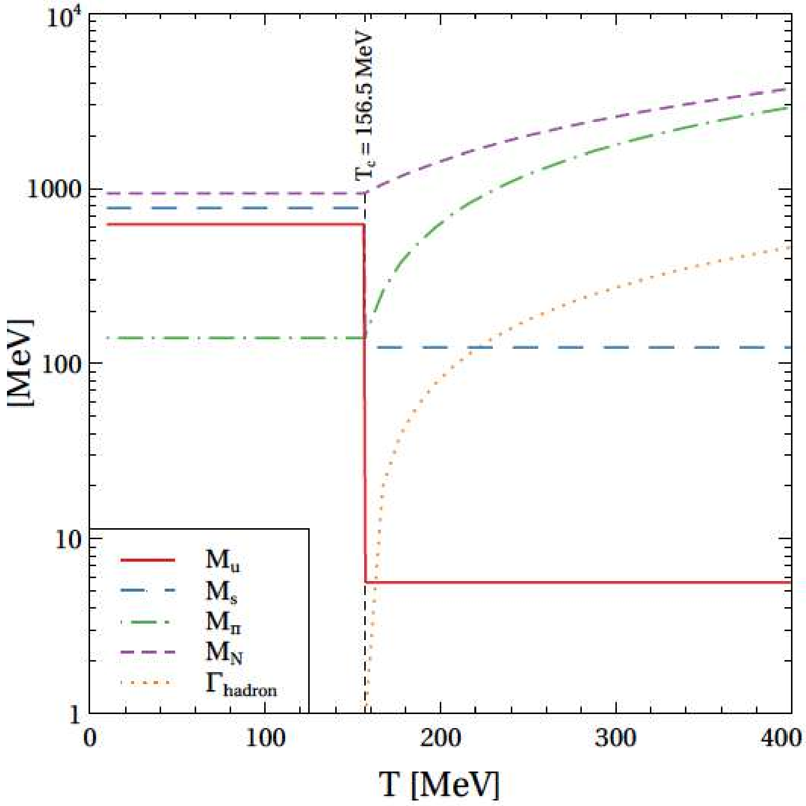
and $\phi = \bar\phi = 0$ (confinement),

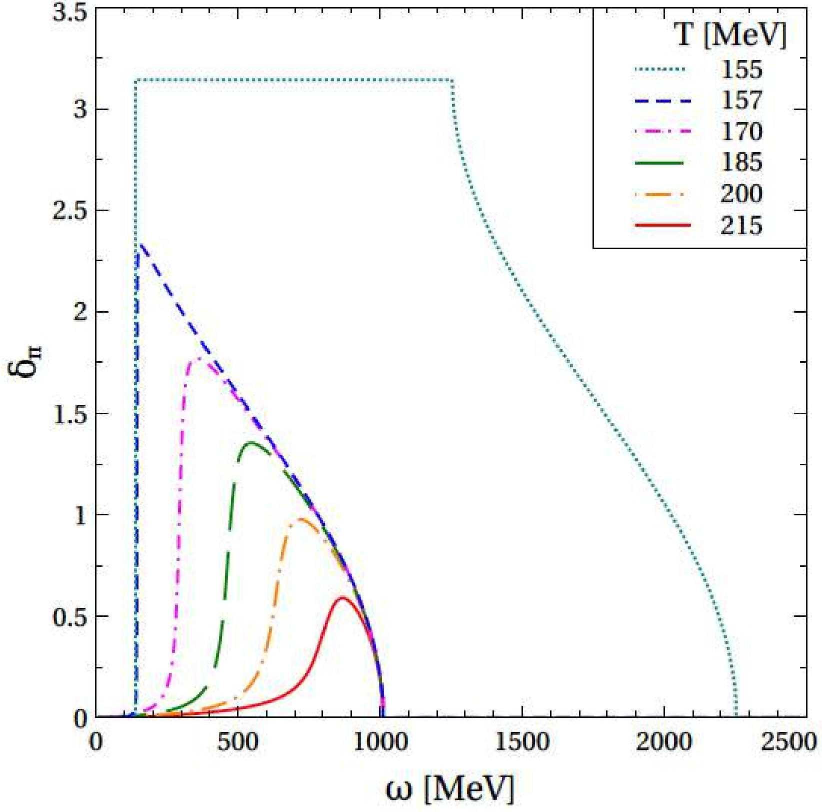
$$f_{\phi=0}^{(a=0,2,4,\dots),\pm} = \frac{y_a^{\pm 3}}{1 - y_a^{\pm 3}} , \quad f_{\phi=0}^{(a=1,3,5,\dots),\pm} = \frac{y_a^{\pm 3}}{1 + y_a^{\pm 3}} .$$


... it is almost ready ...

D.B., M. Cierniak, O. Ivanytskyi and G. Röpke, Eur. Phys. J. A60 (2024) 14; arXiv:22308.07950

Inputs: mass spectrum & phase shifts (models)





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Inputs: mass spectrum (Particle Data Tables)

Mesons

PDG mesons	d_i	M_{PDG} [MeV]	M_i [MeV]	$M^<_{\mathrm{th},i}$ [MeV]	$M^>_{\mathrm{th},i}$ [MeV]
π^+/π^0	3	140	140	1254	11.2
K^+/K^0	4	494	494	1397	129.6
η	1	548	878	1349	90.1
ρ^+/ρ^0	9	775	783	1254	11.2
ω	9	783	783	1254	11.2
K^{*+}/K^{*0}	12	895	$806^{*)}$	2651	140.8
η'	1	960	878	1349	90.1
a_0	3	980	$1095^{*)}$	2508	22.4
f_0	1	980	$1095^{*)}$	2508	22.4
ϕ	3	1020	1069	1540	248
..					
$\pi_2(1880)$	15	1895	$1095^{*)}$	2508	22.4
$f_2(1950)$	5	1944	$1095^{*)}$	2508	22.4
$a_4(2040)$	27	1996	$1095^{*)}$	2508	22.4
$f_2(2010)$	5	2011	$1095^{*)}$	2508	22.4
$f_4(2050)$	9	2018	$1095^{*)}$	2508	22.4
$K_4^*(2045)$	36	2045	$1238^{*)}$	2651	140.8
$\phi(2170)$	3	2175	$1381^{*)}$	2794	259.2
$f_2(2300)$	5	2297	$1095^{*)}$	2508	22.4
$f_2(2340)$	5	2339	$1095^{*)}$	2508	22.4

Baryons

PDG baryons	d_i	M_{PDG} [MeV]	M_i [MeV]	$M^<_{\mathrm{th},i}$ [MeV]	$M^>_{\mathrm{th},i}$ [MeV]
n/p	4	939	939	1881	16.8
Λ	2	1116	1082	2024	135.2
Σ	6	1193	1082	2024	135.2
Δ	16	1232	$1251^{**)}$	3135	28
Ξ^0	2	1315	1225	2167	253.6
Ξ^-	2	1322	1225	2167	253.6
$\Sigma(1385)$	6	1385	$1394^{**)}$	3278	146.4
$\Lambda(1405)$	2	1405	$1394^{**)}$	3278	146.4
$N(1440)$	4	1440	$1251^{**)}$	3135	28
..					
$N(2195)$	36	2220	$1251^{**)}$	3135	28
$\Sigma(2250)$	6	2250	$1394^{**)}$	3278	146.4
$\Omega^-(2250)$	2	2252	$1680^{**)}$	3564	383.2
$N(2250)$	20	2275	$1251^{**)}$	3135	28
$\Lambda(2350)$	10	2350	$1394^{**)}$	3278	146.4
$\Delta(2420)$	48	2420	$1251^{**)}$	3135	28
$N(2600)$	24	2600	$1251^{**)}$	3135	28

… and colored clusters (model) !

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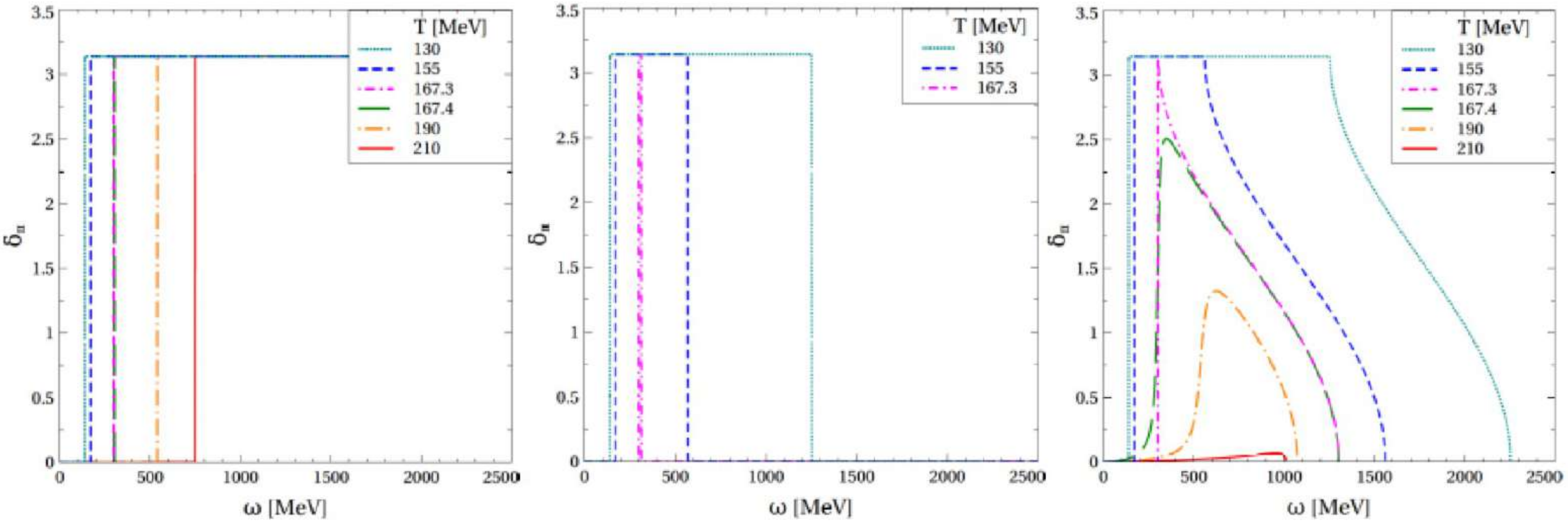
Inputs for the phase shifts (models)



Step-up (SU) model → Hadron Resonance Gas

Step-up-step-down model Step-up-continuum model → Mott Hadron Resonance Gas (MHRG)

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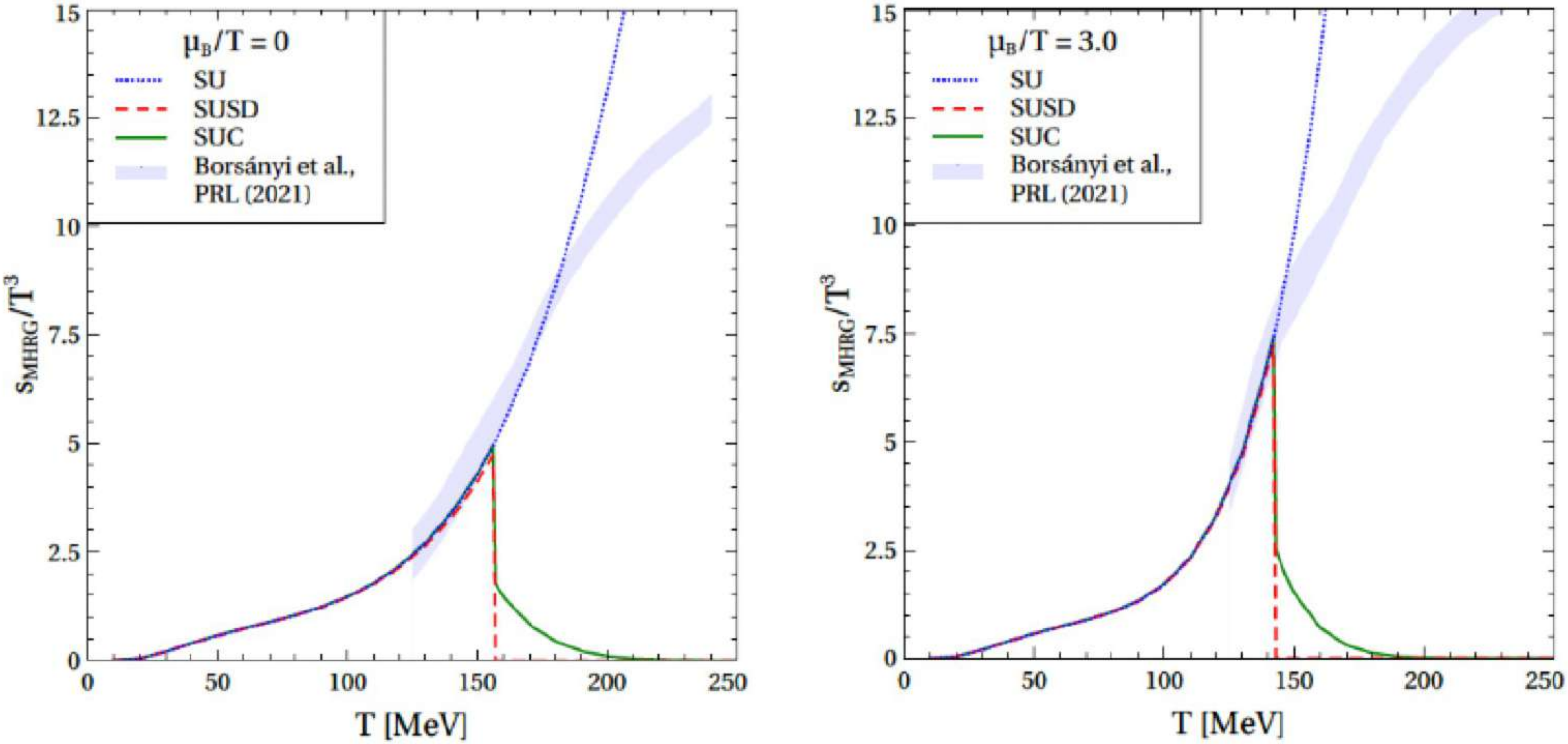
Results for Mott-Hadron Resonance Gas (MHRG)



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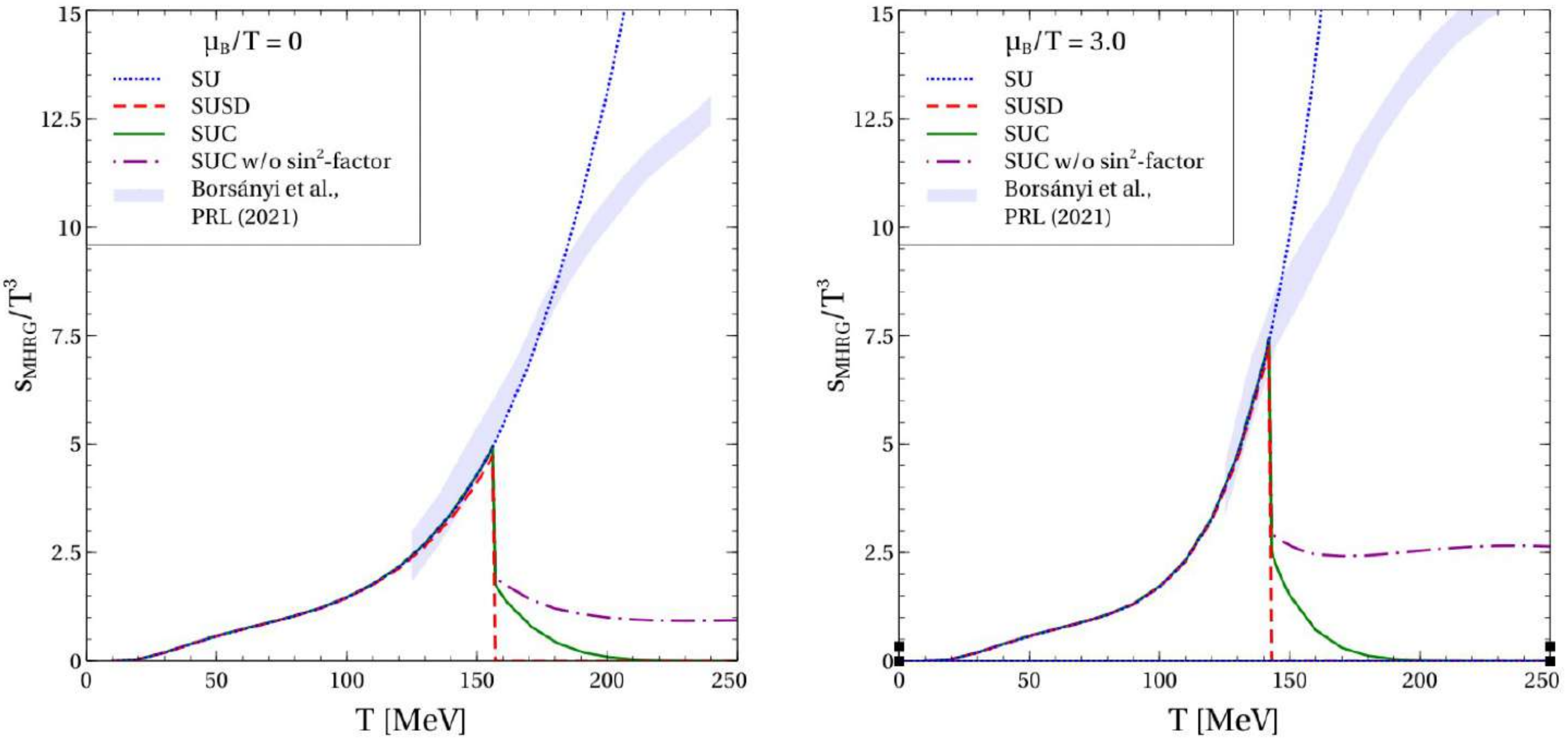
Entropy for MHRG: role of the $\sin^2$-term



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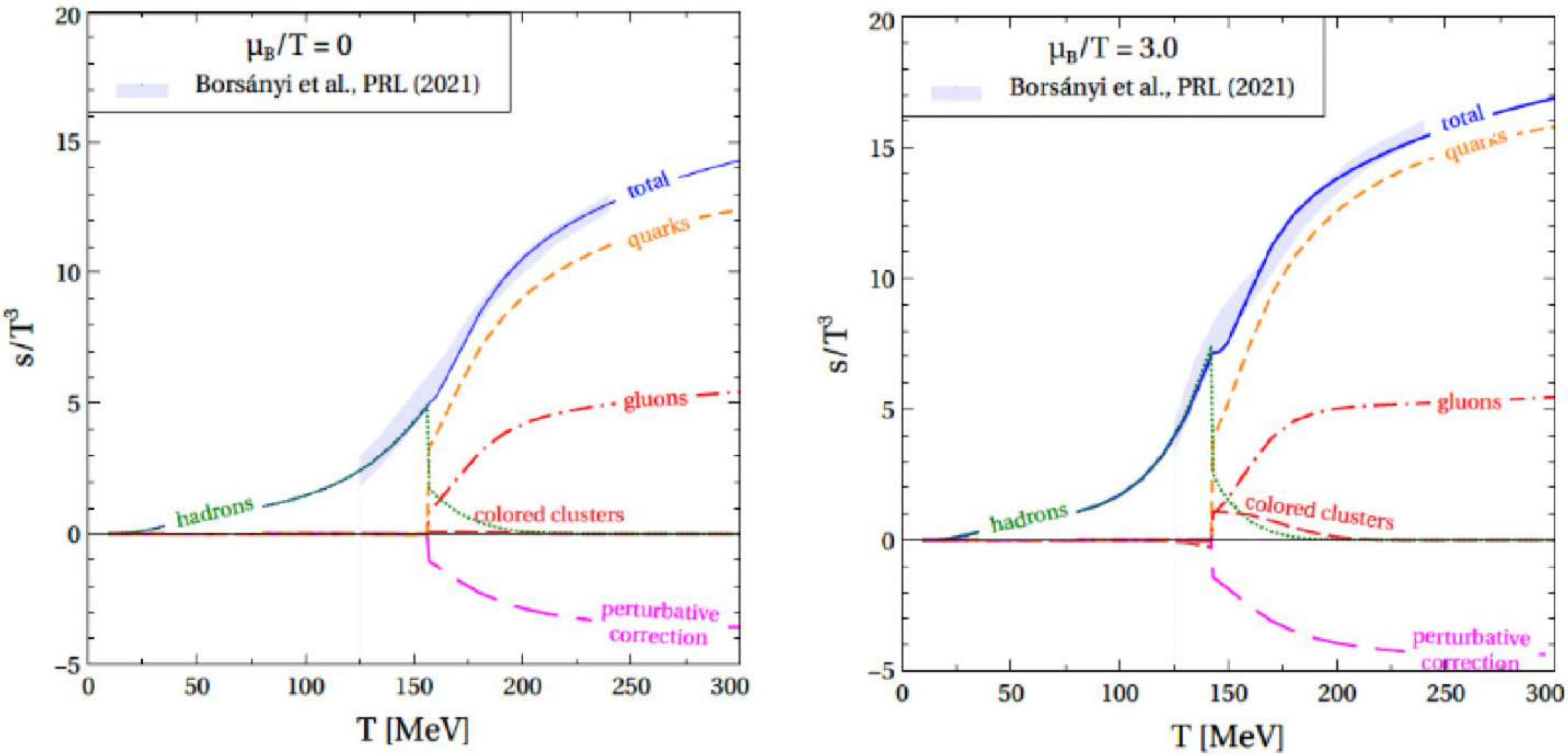
Results for the entropy density of the unified model QGP+clusters



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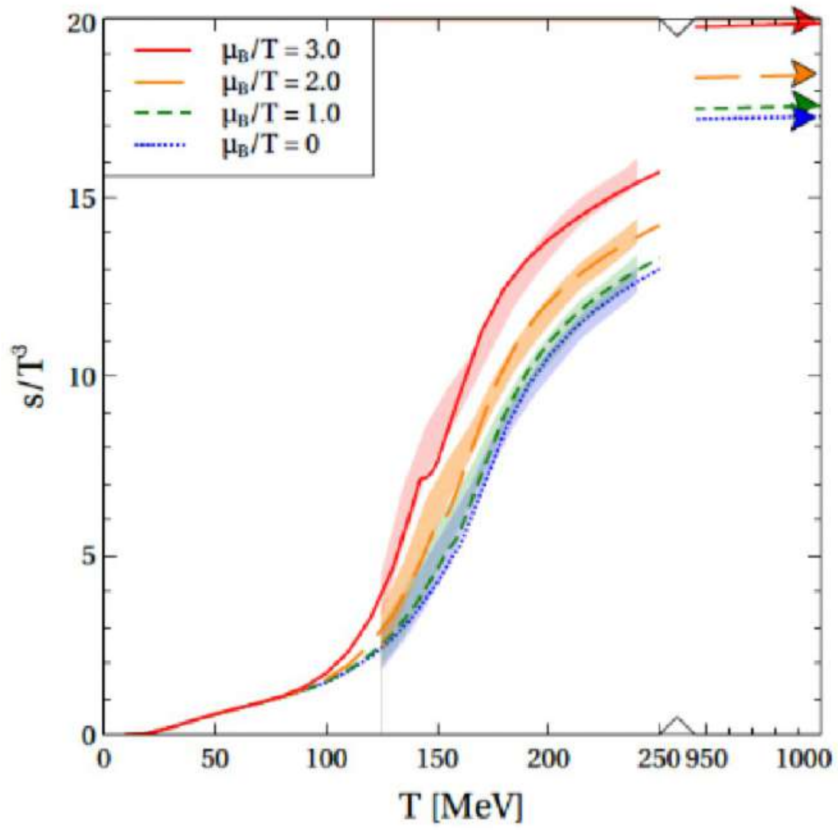
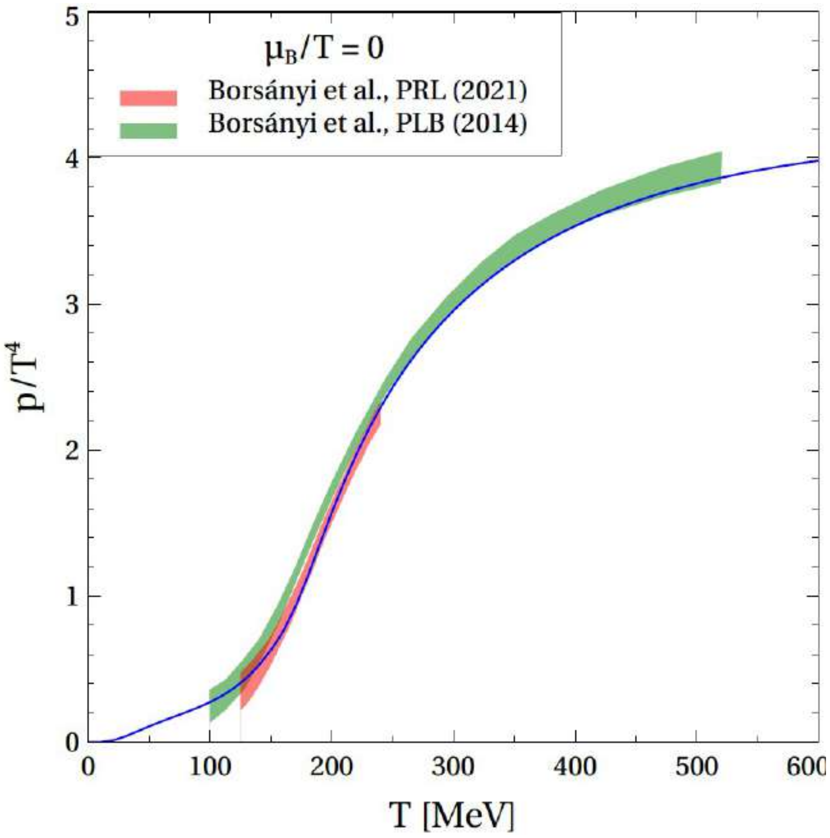
Results for the entropy density & pressure





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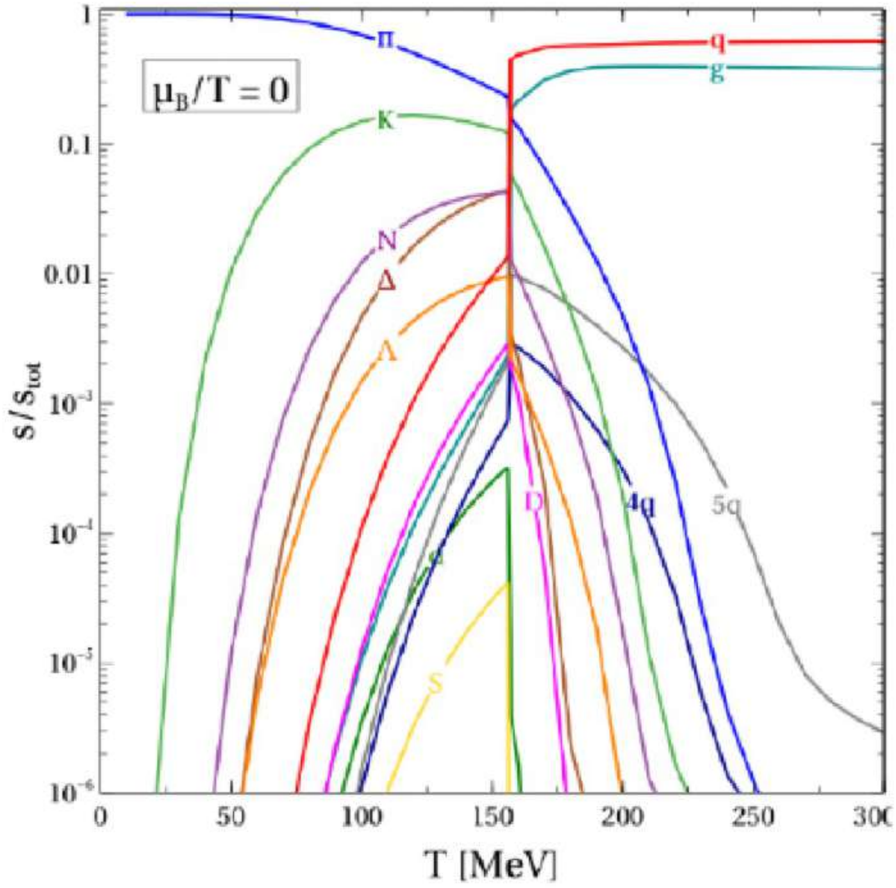
Results for the composition



Gerd Röpke in discussion with Simon Liebing and Oleksii Ivanytskyi in Darea Sushi Wroclaw (2024)

Despite smooth crossover behaviour of the thermodynamic functions (entropy, density), the composition changes abruptly from HRG to QGP at the pseudocritical (Mott) temperature Tc ~ 156.5 MeV

This is due to the effective confinement model!

Can this be an explanation for the success of the thermal statistical model in explaining hadron production in heavy-ion collisions within a sudden chemical freezeout model?

D.B., M. Cierniak, O. Ivanytskyi and G. Röpke, Eur. Phys. J. A60 (2024) 14; arXiv:22308.07950

… dear **Ludwik**, and best wishes from my family and my parents … **Sto Lat** !!!