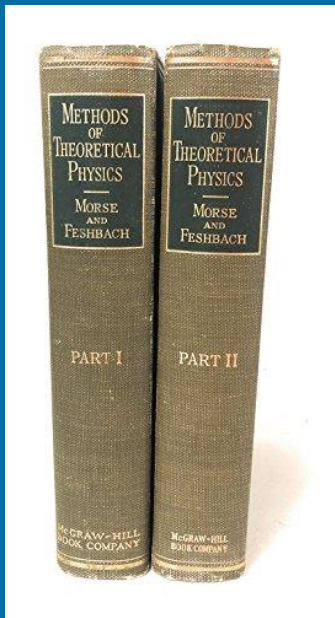# DIFFERENT ~~PHASES~~ FACES OF PHYSICS

A few personal remarks
and observations

Ludwik Turko
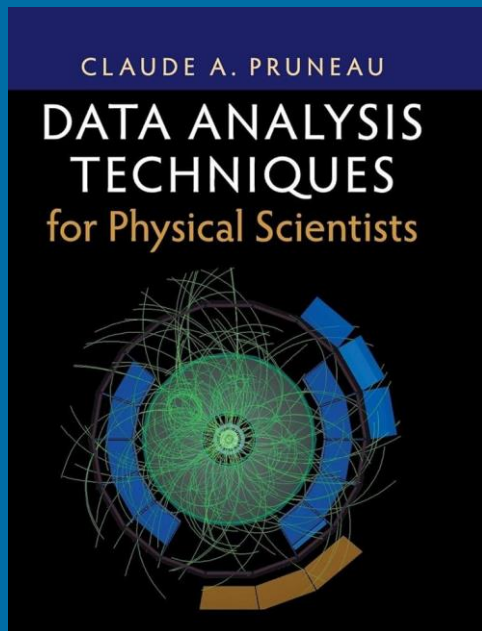
1953

2017

DIFFERENT
~~PHASES~~ FACES OF PHYSICS

A few personal remarks
and observations

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PROCRUSTEAN BED OF PHYSICS

METHODS OF THEORETICAL PHYSICS

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PART I: CHAPTERS 1 TO 8

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This Procrustean tendency to force the physical situation to fit the requirements of a partial differential equation results in a field which is both more regular and more irregular than the "actual" conditions. A solution of a differential equation is more smoothly continuous over most of space and time than is the corresponding physical situation, but it usually is also provided with a finite number of mathematical discontinuities which are considerably more "sharp" than the "actual" conditions exhibit.

To begin with ...

$$S = \int d^{2D}x\, e^{-2d}\left(L^{(0)} + L^{(2)} + L^{(4)} + L^{(6)}\right),$$

$$L^{(0)} = \frac{1}{2}\mathcal{M}^{M}{}_{M} - \frac{1}{6}\mathcal{M}^{MN}\mathcal{M}_{M}{}^{P}\mathcal{M}_{NP},$$

Just a simple formula only

And some explanations about the coefficients

$$
\begin{aligned}
L^{(2)} = & -\frac{3}{2}\partial^{MN}\mathcal{M}_{MN} + 6\,\mathcal{M}^{MN}\partial_{MN}d + 6\,\partial^{M}\mathcal{M}_{M}{}^{N}\partial_{N}d - 6\,\mathcal{M}^{MN}\partial_{M}d\,\partial_{N}d \\
& -\frac{1}{12}\,\mathcal{M}^{MN}\mathcal{M}^{PQ}\partial_{MN}\mathcal{M}_{PQ} + \frac{1}{2}\,\mathcal{M}^{MN}\mathcal{M}^{PQ}\partial_{MP}\mathcal{M}_{NQ} + \mathcal{M}^{MN}\mathcal{M}_{M}{}^{P}\partial_{N}{}^{Q}\mathcal{M}_{PQ} \\
& +\frac{1}{2}\,\mathcal{M}^{MN}\partial^{P}\mathcal{M}_{MP}\,\partial^{Q}\mathcal{M}_{NQ} + \frac{1}{2}\,\mathcal{M}^{MN}\partial^{P}\mathcal{M}_{M}{}^{Q}\,\partial_{Q}\mathcal{M}_{NP} - \frac{1}{12}\,\mathcal{M}^{MN}\partial^{P}\mathcal{M}_{MN}\,\partial^{Q}\mathcal{M}_{PQ} \\
& +\frac{1}{2}\,\mathcal{M}^{MN}\partial_{M}\mathcal{M}^{PQ}\,\partial_{P}\mathcal{M}_{NQ} + \frac{1}{24}\,\mathcal{M}^{MN}\partial_{M}\mathcal{M}^{PQ}\,\partial_{N}\mathcal{M}_{PQ} + \mathcal{M}^{MN}\partial_{M}\mathcal{M}_{N}{}^{P}\,\partial^{Q}\mathcal{M}_{PQ} \\
& -2\,\mathcal{M}^{MN}\mathcal{M}_{M}{}^{P}\mathcal{M}_{N}{}^{Q}\partial_{PQ}d - 2\,\mathcal{M}^{MN}\mathcal{M}^{PQ}\partial_{M}\mathcal{M}_{NP}\,\partial_{Q}d + \frac{1}{6}\,\mathcal{M}^{MN}\mathcal{M}^{PQ}\partial_{M}\mathcal{M}_{PQ}\,\partial_{N}d \\
& -2\,\mathcal{M}^{MN}\mathcal{M}_{M}{}^{P}\partial_{N}\mathcal{M}_{P}{}^{Q}\,\partial_{Q}d - 2\,\mathcal{M}^{MN}\mathcal{M}_{M}{}^{P}\partial^{Q}\mathcal{M}_{NQ}\,\partial_{P}d + 2\,\mathcal{M}^{MN}\mathcal{M}_{M}{}^{P}\mathcal{M}_{N}{}^{Q}\partial_{P}d\,\partial_{Q}d\,,
\end{aligned}
$$

$$L^{(4)} = \mathcal{M}_{MN}\mathcal{M}_{PQ}\partial_{IJKL}\mathcal{M}_{RS}\left(-\frac{1}{4}\eta^{MI}\eta^{NJ}\eta^{PK}\eta^{QR}\eta^{LS} - \frac{1}{2}\eta^{MP}\eta^{NI}\eta^{QJ}\eta^{KR}\eta^{LS}\right)$$

$$+\mathcal{M}_{MN}\partial_P\mathcal{M}_{QI}\,\partial_{JKL}\mathcal{M}_{RS}\left(-\frac{1}{4}\eta^{MJ}\eta^{NR}\eta^{PQ}\eta^{IK}\eta^{LS} - \frac{1}{4}\eta^{MJ}\eta^{NK}\eta^{PQ}\eta^{IR}\eta^{LS}\right.$$

$$-\frac{1}{2}\eta^{MJ}\eta^{NR}\eta^{PS}\eta^{QK}\eta^{IL} + \frac{1}{6}\eta^{MJ}\eta^{NK}\eta^{PR}\eta^{QL}\eta^{IS} - \frac{1}{2}\eta^{MQ}\eta^{NJ}\eta^{PI}\eta^{KR}\eta^{LS}$$

$$\left.-\eta^{MQ}\eta^{NJ}\eta^{PR}\eta^{IK}\eta^{LS} - \frac{1}{2}\eta^{MP}\eta^{NJ}\eta^{QK}\eta^{IR}\eta^{LS} - \frac{1}{2}\eta^{MP}\eta^{NQ}\eta^{IJ}\eta^{KR}\eta^{LS}\right)$$

$$+\mathcal{M}_{MN}\partial_{PQ}\mathcal{M}_{IJ}\,\partial_{KL}\mathcal{M}_{RS}\left(-\frac{1}{2}\eta^{MP}\eta^{NR}\eta^{QI}\eta^{JK}\eta^{LS} - \frac{1}{2}\eta^{MP}\eta^{NK}\eta^{QI}\eta^{JR}\eta^{LS}\right.$$

$$\left.-\frac{1}{4}\eta^{MP}\eta^{NR}\eta^{QS}\eta^{IK}\eta^{JL} - \frac{1}{12}\eta^{MP}\eta^{NK}\eta^{QR}\eta^{IL}\eta^{JS} - \eta^{MP}\eta^{NI}\eta^{QR}\eta^{JK}\eta^{LS} + \frac{1}{4}\eta^{MP}\eta^{NQ}\eta^{IK}\eta^{JR}\eta^{LS}\right)$$

$$+\partial_M\mathcal{M}_{NP}\,\partial_Q\mathcal{M}_{IJ}\,\partial_{KL}\mathcal{M}_{RS}\left(-\frac{1}{4}\eta^{MN}\eta^{PK}\eta^{QI}\eta^{JR}\eta^{LS} - \frac{1}{2}\eta^{MN}\eta^{PR}\eta^{QS}\eta^{IK}\eta^{JL}\right.$$

$$+\frac{1}{6}\eta^{MN}\eta^{PK}\eta^{QR}\eta^{IL}\eta^{JS} - \eta^{MN}\eta^{PI}\eta^{QR}\eta^{JK}\eta^{LS} + \frac{1}{4}\eta^{MN}\eta^{PQ}\eta^{IK}\eta^{JR}\eta^{LS}$$

$$\left.-\frac{1}{2}\eta^{MR}\eta^{NI}\eta^{PK}\eta^{QS}\eta^{JL} - \frac{1}{2}\eta^{MR}\eta^{NQ}\eta^{PK}\eta^{IL}\eta^{JS} + \frac{1}{4}\eta^{MR}\eta^{NQ}\eta^{PS}\eta^{IK}\eta^{JL} - \eta^{MI}\eta^{NQ}\eta^{PK}\eta^{JR}\eta^{LS}\right)$$

$$+\frac{5}{2}\mathcal{M}_{MN}\mathcal{M}_{PQ}\mathcal{M}_{IJ}\partial_{KLRS}d\,\eta^{MP}\eta^{NK}\eta^{QL}\eta^{IR}\eta^{JS} + \mathcal{M}_{MN}\mathcal{M}_{PQ}\partial_I\mathcal{M}_{JK}\,\partial_{LRS}d\left(\frac{5}{2}\eta^{MI}\eta^{NJ}\eta^{PL}\eta^{QR}\eta^{KS}\right.$$

$$+4\eta^{MI}\eta^{NL}\eta^{PJ}\eta^{QR}\eta^{KS} + 3\eta^{MP}\eta^{NI}\eta^{QL}\eta^{JR}\eta^{KS} + \frac{5}{2}\eta^{MP}\eta^{NL}\eta^{QR}\eta^{IJ}\eta^{KS}$$

$$\left.+\frac{5}{2}\eta^{MJ}\eta^{NL}\eta^{PR}\eta^{QS}\eta^{IK}\right) + \mathcal{M}_{MN}\mathcal{M}_{PQ}\partial_{IJ}\mathcal{M}_{KL}\,\partial_{RS}d\left(\eta^{MP}\eta^{NI}\eta^{QJ}\eta^{KR}\eta^{LS} + 3\eta^{MI}\eta^{NK}\eta^{PJ}\eta^{QR}\eta^{LS}\right.$$

$$\left.+5\eta^{MP}\eta^{NI}\eta^{QR}\eta^{JK}\eta^{LS} + 3\eta^{MI}\eta^{NR}\eta^{PK}\eta^{QS}\eta^{JL}\right)$$

$$+\mathcal{M}_{MN}\mathcal{M}_{PQ}\partial_{IJK}\mathcal{M}_{LR}\,\partial_S d\left(\frac{1}{2}\eta^{MI}\eta^{NJ}\eta^{PK}\eta^{QL}\eta^{SR} + 2\eta^{MP}\eta^{NI}\eta^{QJ}\eta^{SL}\eta^{KR}\right.$$

$$\left.+\frac{1}{2}\eta^{MS}\eta^{NI}\eta^{PJ}\eta^{QL}\eta^{KR} + \eta^{MP}\eta^{NS}\eta^{QI}\eta^{JL}\eta^{KR} + \frac{1}{2}\eta^{MS}\eta^{NL}\eta^{PI}\eta^{QJ}\eta^{KR}\right)$$

$$+\mathcal{M}_{MN}\partial_P\mathcal{M}_{QI}\,\partial_J\mathcal{M}_{KL}\,\partial_{RS}d\left(\frac{5}{2}\eta^{MQ}\eta^{NR}\eta^{PI}\eta^{JK}\eta^{LS} + 4\eta^{MQ}\eta^{NR}\eta^{PK}\eta^{IJ}\eta^{LS}\right.$$

$$+\eta^{MP}\eta^{NR}\eta^{QJ}\eta^{IK}\eta^{LS} + 2\eta^{MP}\eta^{NK}\eta^{QJ}\eta^{IR}\eta^{LS} + \eta^{MP}\eta^{NJ}\eta^{QK}\eta^{IR}\eta^{LS}$$

$$\left.+3\eta^{MP}\eta^{NR}\eta^{QK}\eta^{IS}\eta^{JL} + \eta^{MP}\eta^{NK}\eta^{QR}\eta^{IS}\eta^{JL} + \eta^{MP}\eta^{NQ}\eta^{IJ}\eta^{KR}\eta^{LS} + \frac{5}{2}\eta^{MP}\eta^{NQ}\eta^{IR}\eta^{JK}\eta^{LS}\right)$$

$$+\mathcal{M}_{MN}\partial_P\mathcal{M}_{QI}\,\partial_{JK}\mathcal{M}_{LR}\,\partial_S d\left(\frac{1}{2}\eta^{MS}\eta^{NL}\eta^{PQ}\eta^{IJ}\eta^{KR} + \frac{1}{2}\eta^{MJ}\eta^{NL}\eta^{PQ}\eta^{IK}\eta^{SR}\right.$$

$$+\frac{1}{2}\eta^{MJ}\eta^{NK}\eta^{PQ}\eta^{IL}\eta^{SR} + \frac{1}{2}\eta^{MS}\eta^{NJ}\eta^{PQ}\eta^{IL}\eta^{KR} + \eta^{MS}\eta^{NL}\eta^{PR}\eta^{QJ}\eta^{IK}$$

$$-\frac{1}{2}\eta^{MJ}\eta^{NK}\eta^{PL}\eta^{QS}\eta^{IR} + 2\eta^{MJ}\eta^{NL}\eta^{PR}\eta^{QS}\eta^{IK} - \frac{1}{3}\eta^{MS}\eta^{NJ}\eta^{PL}\eta^{QK}\eta^{IR}$$

$$+2\eta^{MQ}\eta^{NJ}\eta^{PI}\eta^{SL}\eta^{KR} + 2\eta^{MQ}\eta^{NS}\eta^{PL}\eta^{IJ}\eta^{KR} + 2\eta^{MQ}\eta^{NJ}\eta^{PL}\eta^{IK}\eta^{SR}$$

$$+\eta^{MQ}\eta^{NJ}\eta^{PL}\eta^{IS}\eta^{KR} - \frac{1}{2}\eta^{MP}\eta^{NS}\eta^{QJ}\eta^{IL}\eta^{KR} + \eta^{MP}\eta^{NL}\eta^{QS}\eta^{IJ}\eta^{KR} + \eta^{MP}\eta^{NJ}\eta^{QK}\eta^{IL}\eta^{SR}$$

$$\left.+2\eta^{MP}\eta^{NJ}\eta^{QS}\eta^{IL}\eta^{KR} + 2\eta^{MP}\eta^{NQ}\eta^{IJ}\eta^{SL}\eta^{KR}\right)$$

$$+\partial_M\mathcal{M}_{NP}\,\partial_Q\mathcal{M}_{IJ}\,\partial_K\mathcal{M}_{LR}\,\partial_S d\left(2\eta^{MN}\eta^{PI}\eta^{QL}\eta^{JK}\eta^{RS} - \frac{1}{2}\eta^{MN}\eta^{PQ}\eta^{IK}\eta^{JL}\eta^{RS}\right.$$

$$\left.+\frac{1}{2}\eta^{MN}\eta^{PQ}\eta^{IL}\eta^{JS}\eta^{KR} + 2\eta^{MI}\eta^{NQ}\eta^{PK}\eta^{JL}\eta^{RS}\right)$$

Some more coefficient's details, please!

$$L^{(6)} = \frac{1}{8}\mathcal{M}_{MN}\mathcal{M}_{PQ}\partial_{IJKLRS}\mathcal{M}_{GH}\,\eta^{MI}\eta^{NJ}\eta^{PK}\eta^{QL}\eta^{RG}\eta^{SH}$$

$$+\mathcal{M}_{MN}\partial_P\mathcal{M}_{QI}\,\partial_{JKLRS}\mathcal{M}_{GH}\,(\frac{1}{4}\eta^{MJ}\eta^{NK}\eta^{PQ}\eta^{IL}\eta^{RG}\eta^{SH}$$

$$+\frac{1}{8}\eta^{MJ}\eta^{NK}\eta^{PG}\eta^{QL}\eta^{IR}\eta^{SH}+\frac{1}{4}\eta^{MP}\eta^{NJ}\eta^{QK}\eta^{IL}\eta^{RG}\eta^{SH}\,)$$

$$+\mathcal{M}_{MN}\partial_{PQ}\mathcal{M}_{IJ}\,\partial_{KLRS}\mathcal{M}_{GH}\,(\frac{1}{4}\eta^{MK}\eta^{NL}\eta^{PI}\eta^{QG}\eta^{JR}\eta^{SH}$$

$$-\frac{1}{24}\eta^{MK}\eta^{NL}\eta^{PG}\eta^{QH}\eta^{IR}\eta^{JS}+\frac{3}{4}\eta^{MP}\eta^{NK}\eta^{QI}\eta^{JL}\eta^{RG}\eta^{SH}$$

$$+\frac{1}{4}\eta^{MP}\eta^{NK}\eta^{QG}\eta^{IL}\eta^{JR}\eta^{SH}-\frac{1}{8}\eta^{MP}\eta^{NQ}\eta^{IK}\eta^{JL}\eta^{RG}\eta^{SH}\,)$$

$$+\mathcal{M}_{MN}\partial_{PQI}\mathcal{M}_{JK}\,\partial_{LRS}\mathcal{M}_{GH}\,(\frac{1}{8}\eta^{MP}\eta^{NL}\eta^{QJ}\eta^{IK}\eta^{RG}\eta^{SH}$$

$$+\frac{1}{2}\eta^{MP}\eta^{NL}\eta^{QJ}\eta^{IG}\eta^{KR}\eta^{SH}+\frac{1}{48}\eta^{MP}\eta^{NL}\eta^{QG}\eta^{IH}\eta^{JR}\eta^{KS}$$

$$+\frac{1}{8}\eta^{MP}\eta^{NQ}\eta^{IJ}\eta^{KL}\eta^{RG}\eta^{SH}-\frac{1}{8}\eta^{MP}\eta^{NQ}\eta^{IG}\eta^{JL}\eta^{KR}\eta^{SH}\,)$$

$$+\partial_M\mathcal{M}_{NP}\,\partial_Q\mathcal{M}_{IJ}\,\partial_{KLRS}\mathcal{M}_{GH}\,(\frac{1}{8}\eta^{MN}\eta^{PK}\eta^{QI}\eta^{JL}\eta^{RG}\eta^{SH}$$

$$+\frac{1}{8}\eta^{MN}\eta^{PK}\eta^{QG}\eta^{IL}\eta^{JR}\eta^{SH}-\frac{1}{8}\eta^{MN}\eta^{PQ}\eta^{IK}\eta^{JL}\eta^{RG}\eta^{SH}$$

$$+\frac{1}{8}\eta^{MG}\eta^{NK}\eta^{PL}\eta^{QH}\eta^{IR}\eta^{JS}+\frac{1}{2}\eta^{MI}\eta^{NQ}\eta^{PK}\eta^{JL}\eta^{RG}\eta^{SH}\,)$$

$$+\partial_M\mathcal{M}_{NP}\,\partial_{QI}\mathcal{M}_{JK}\,\partial_{LRS}\mathcal{M}_{GH}\,(\frac{1}{4}\eta^{MN}\eta^{PL}\eta^{QJ}\eta^{IG}\eta^{KR}\eta^{SH}$$

$$-\frac{1}{24}\eta^{MN}\eta^{PL}\eta^{QG}\eta^{IH}\eta^{JR}\eta^{KS}+\frac{1}{8}\eta^{MN}\eta^{PQ}\eta^{IJ}\eta^{KL}\eta^{RG}\eta^{SH}$$

$$-\frac{1}{8}\eta^{MN}\eta^{PQ}\eta^{IG}\eta^{JL}\eta^{KR}\eta^{SH}+\frac{1}{2}\eta^{MG}\eta^{NL}\eta^{PR}\eta^{QJ}\eta^{IH}\eta^{KS}$$

$$-\frac{1}{4}\eta^{MJ}\eta^{NL}\eta^{PR}\eta^{QK}\eta^{IG}\eta^{SH}-\frac{1}{8}\eta^{MJ}\eta^{NL}\eta^{PR}\eta^{QG}\eta^{IH}\eta^{KS}$$

$$-\frac{1}{4}\eta^{MG}\eta^{NQ}\eta^{PI}\eta^{JL}\eta^{KR}\eta^{SH}+\frac{1}{4}\eta^{MJ}\eta^{NQ}\eta^{PI}\eta^{KL}\eta^{RG}\eta^{SH}$$

$$+\frac{1}{2}\eta^{MG}\eta^{NQ}\eta^{PL}\eta^{IJ}\eta^{KR}\eta^{SH}+\frac{1}{2}\eta^{MJ}\eta^{NQ}\eta^{PL}\eta^{IK}\eta^{RG}\eta^{SH}$$

$$+\eta^{MJ}\eta^{NQ}\eta^{PL}\eta^{IG}\eta^{KR}\eta^{SH}+\frac{1}{4}\eta^{MG}\eta^{NQ}\eta^{PL}\eta^{IH}\eta^{JR}\eta^{KS}\,)$$

$$+\partial_{MN}\mathcal{M}_{PQ}\,\partial_{IJ}\mathcal{M}_{KL}\,\partial_{RS}\mathcal{M}_{GH}\,(-\frac{1}{24}\eta^{MG}\eta^{NH}\eta^{PI}\eta^{QJ}\eta^{KR}\eta^{LS}$$

$$+\frac{1}{6}\eta^{MK}\eta^{NG}\eta^{PI}\eta^{QR}\eta^{JH}\eta^{LS}+\frac{1}{2}\eta^{MP}\eta^{NK}\eta^{QI}\eta^{JG}\eta^{LR}\eta^{SH}\,)$$

$$-\frac{1}{2}\mathcal{M}_{MN}\mathcal{M}_{PQ}\mathcal{M}_{IJ}\partial_{KLRSGH}d\,\eta^{MK}\eta^{NL}\eta^{PR}\eta^{QS}\eta^{IG}\eta^{JH}$$

$$+\mathcal{M}_{MN}\mathcal{M}_{PQ}\partial_I\mathcal{M}_{JK}\,\partial_{LRSGH}d\,(-3\eta^{MI}\eta^{NL}\eta^{PR}\eta^{QS}\eta^{JG}\eta^{KH}-\frac{3}{2}\eta^{ML}\eta^{NR}\eta^{PS}\eta^{QG}\eta^{IJ}\eta^{KH}\,)$$

Is that all, really?

$$+\mathcal{M}_{MN}\mathcal{M}_{PQ}\partial_{IJ}\mathcal{M}_{KL}\,\partial_{RS}\partial_H d\,(-\tfrac{1}{4}\eta^{MI}\eta^{NJ}\eta^{PR}\eta^{QS}\eta^{KG}\eta^{LH}$$

$$-2\eta^{MI}\eta^{NR}\eta^{PJ}\eta^{QS}\eta^{KG}\eta^{LH}-\tfrac{11}{2}\eta^{MI}\eta^{NR}\eta^{PS}\eta^{QG}\eta^{JK}\eta^{LH}\,)$$

$$+\mathcal{M}_{MN}\mathcal{M}_{PQ}\partial_{IJK}\mathcal{M}_{LR}\,\partial_S\partial_H d\,(-\eta^{MI}\eta^{NJ}\eta^{PK}\eta^{QS}\eta^{LG}\eta^{RH}$$

$$-\tfrac{3}{2}\eta^{MI}\eta^{NJ}\eta^{PS}\eta^{QG}\eta^{KL}\eta^{RH}-4\eta^{MI}\eta^{NS}\eta^{PJ}\eta^{QG}\eta^{KL}\eta^{RH}$$

$$-\tfrac{5}{4}\eta^{MI}\eta^{NS}\eta^{PG}\eta^{QH}\eta^{JL}\eta^{KR}\,)+\mathcal{M}_{MN}\mathcal{M}_{PQ}\partial_{IJKL}\mathcal{M}_{RS}\,\partial_G\partial_H d\,(-\tfrac{1}{4}\eta^{MI}\eta^{NJ}\eta^{PK}\eta^{QL}\eta^{GR}\eta^{HS}$$

$$-\tfrac{5}{2}\eta^{MG}\eta^{NI}\eta^{PJ}\eta^{QK}\eta^{HR}\eta^{LS}-\tfrac{3}{2}\eta^{MG}\eta^{NI}\eta^{PH}\eta^{QJ}\eta^{KR}\eta^{LS}\,)$$

$$+\mathcal{M}_{MN}\mathcal{M}_{PQ}\partial_{IJKLR}\mathcal{M}_{SG}\,\partial_H d\,(-\tfrac{1}{2}\eta^{MI}\eta^{NJ}\eta^{PK}\eta^{QL}\eta^{HS}\eta^{RG}$$

$$-\tfrac{1}{2}\eta^{MH}\eta^{NI}\eta^{PJ}\eta^{QK}\eta^{LS}\eta^{RG}\,)+\mathcal{M}_{MN}\partial_P\mathcal{M}_{QI}\,\partial_J\mathcal{M}_{KL}\,\partial_{RS}\partial_H d\,(-\tfrac{1}{4}\eta^{MR}\eta^{NS}\eta^{PQ}\eta^{IJ}\eta^{KG}\eta^{LH}$$

$$-\tfrac{3}{2}\eta^{MR}\eta^{NS}\eta^{PQ}\eta^{IG}\eta^{JK}\eta^{LH}-\tfrac{5}{2}\eta^{MR}\eta^{NS}\eta^{PK}\eta^{QJ}\eta^{IG}\eta^{LH}$$

$$-2\eta^{MP}\eta^{NR}\eta^{QJ}\eta^{IS}\eta^{KG}\eta^{LH}-\eta^{MP}\eta^{NJ}\eta^{QR}\eta^{IS}\eta^{KG}\eta^{LH}$$

$$-3\eta^{MP}\eta^{NR}\eta^{QS}\eta^{IG}\eta^{JK}\eta^{LH}\,)+\mathcal{M}_{MN}\partial_P\mathcal{M}_{QI}\,\partial_{JK}\mathcal{M}_{LR}\,\partial_S\partial_H d\,(-\tfrac{3}{2}\eta^{MS}\eta^{NG}\eta^{PQ}\eta^{IJ}\eta^{KL}\eta^{RH}$$

$$-\eta^{MJ}\eta^{NS}\eta^{PQ}\eta^{IK}\eta^{LG}\eta^{RH}-\tfrac{1}{4}\eta^{MJ}\eta^{NK}\eta^{PQ}\eta^{IS}\eta^{LG}\eta^{RH}$$

$$-\tfrac{11}{2}\eta^{MJ}\eta^{NS}\eta^{PQ}\eta^{IG}\eta^{KL}\eta^{RH}-\tfrac{5}{2}\eta^{MS}\eta^{NG}\eta^{PL}\eta^{QJ}\eta^{IH}\eta^{KR}$$

$$-\tfrac{5}{4}\eta^{MS}\eta^{NG}\eta^{PL}\eta^{QJ}\eta^{IK}\eta^{RH}-4\eta^{MJ}\eta^{NS}\eta^{PL}\eta^{QK}\eta^{IG}\eta^{RH}$$

$$-\tfrac{3}{2}\eta^{MJ}\eta^{NS}\eta^{PL}\eta^{QG}\eta^{IH}\eta^{KR}+\tfrac{1}{2}\eta^{MJ}\eta^{NK}\eta^{PL}\eta^{QS}\eta^{IG}\eta^{RH}$$

$$-4\eta^{MP}\eta^{NS}\eta^{QJ}\eta^{IG}\eta^{KL}\eta^{RH}-2\eta^{MP}\eta^{NJ}\eta^{QK}\eta^{IS}\eta^{LG}\eta^{RH}$$

$$-\tfrac{7}{2}\eta^{MP}\eta^{NJ}\eta^{QS}\eta^{IG}\eta^{KL}\eta^{RH}\,)$$

$$+\mathcal{M}_{MN}\partial_P\mathcal{M}_{QI}\,\partial_{JKL}\mathcal{M}_{RS}\,\partial_G\partial_H d\,(-\tfrac{1}{2}\eta^{MJ}\eta^{NK}\eta^{PQ}\eta^{IL}\eta^{GR}\eta^{HS}-\tfrac{3}{2}\eta^{MJ}\eta^{NK}\eta^{PQ}\eta^{IG}\eta^{HR}\eta^{LS}$$

$$-\tfrac{5}{2}\eta^{MG}\eta^{NJ}\eta^{PQ}\eta^{IK}\eta^{HR}\eta^{LS}-\tfrac{5}{4}\eta^{MG}\eta^{NJ}\eta^{PQ}\eta^{IH}\eta^{KR}\eta^{LS}-\tfrac{1}{4}\eta^{MJ}\eta^{NK}\eta^{PR}\eta^{QG}\eta^{IH}\eta^{LS}$$

$$-\tfrac{3}{2}\eta^{MG}\eta^{NJ}\eta^{PR}\eta^{QK}\eta^{IL}\eta^{HS}-3\eta^{MG}\eta^{NJ}\eta^{PR}\eta^{QH}\eta^{IK}\eta^{LS}-\tfrac{1}{2}\eta^{MP}\eta^{NG}\eta^{QJ}\eta^{IK}\eta^{HR}\eta^{LS}$$

$$-\tfrac{3}{2}\eta^{MP}\eta^{NG}\eta^{QH}\eta^{IJ}\eta^{KR}\eta^{LS}-\tfrac{1}{2}\eta^{MP}\eta^{NJ}\eta^{QK}\eta^{IL}\eta^{GR}\eta^{HS}-4\eta^{MP}\eta^{NJ}\eta^{QG}\eta^{IK}\eta^{HR}\eta^{LS}$$

$$-\tfrac{1}{2}\eta^{MP}\eta^{NJ}\eta^{QG}\eta^{IH}\eta^{KR}\eta^{LS}\,)+\mathcal{M}_{MN}\partial_P\mathcal{M}_{QI}\,\partial_{JKLR}\mathcal{M}_{SG}\,\partial_H d\,(-\eta^{MJ}\eta^{NK}\eta^{PQ}\eta^{IL}\eta^{HS}\eta^{RG}$$

$$-\tfrac{1}{2}\eta^{MH}\eta^{NJ}\eta^{PQ}\eta^{IK}\eta^{LS}\eta^{RG}-\tfrac{1}{4}\eta^{MJ}\eta^{NK}\eta^{PS}\eta^{QL}\eta^{IR}\eta^{HG}-\tfrac{1}{2}\eta^{MJ}\eta^{NK}\eta^{PS}\eta^{QH}\eta^{IL}\eta^{RG}$$

$$-\tfrac{1}{4}\eta^{MH}\eta^{NJ}\eta^{PS}\eta^{QK}\eta^{IL}\eta^{RG}+\tfrac{1}{4}\eta^{MP}\eta^{NH}\eta^{QJ}\eta^{IK}\eta^{LS}\eta^{RG}-\eta^{MP}\eta^{NJ}\eta^{QK}\eta^{IL}\eta^{HS}\eta^{RG}$$

$$-\tfrac{3}{2}\eta^{MP}\eta^{NJ}\eta^{QH}\eta^{IK}\eta^{LS}\eta^{RG}\,)$$

$$+\mathcal{M}_{MN}\partial_{PQ}\mathcal{M}_{IJ}\,\partial_{KL}\mathcal{M}_{RS}\,\partial_G\partial_H d\,(-3\eta^{MP}\eta^{NG}\eta^{QI}\eta^{JK}\eta^{LR}\eta^{SH}-\tfrac{3}{2}\eta^{MP}\eta^{NK}\eta^{QI}\eta^{JL}\eta^{RG}\eta^{SH}$$

$$-3\eta^{MP}\eta^{NK}\eta^{QI}\eta^{JG}\eta^{LR}\eta^{SH}-3\eta^{MP}\eta^{NG}\eta^{QR}\eta^{IK}\eta^{JH}\eta^{LS}-\tfrac{1}{2}\eta^{MP}\eta^{NG}\eta^{QR}\eta^{IK}\eta^{JL}\eta^{SH}$$

Just as I have thought!
Go on!!

$$-\eta^{MP}\eta^{NK}\eta^{QR}\eta^{IL}\eta^{JG}\eta^{SH} + \frac{1}{4}\eta^{MP}\eta^{NQ}\eta^{IK}\eta^{JL}\eta^{RG}\eta^{SH} + \frac{1}{2}\eta^{MP}\eta^{NQ}\eta^{IK}\eta^{JG}\eta^{LR}\eta^{SH}\,)$$

$$+\mathcal{M}_{MN}\partial_{PQ}\mathcal{M}_{IJ}\,\partial_{KLR}\mathcal{M}_{SG}\,\partial_H d\;(\,-\frac{1}{2}\eta^{MH}\eta^{NK}\eta^{PI}\eta^{QS}\eta^{JL}\eta^{RG} + \frac{1}{12}\eta^{MH}\eta^{NK}\eta^{PS}\eta^{QG}\eta^{IL}\eta^{JR}$$

$$-\frac{1}{2}\eta^{MK}\eta^{NL}\eta^{HI}\eta^{PJ}\eta^{QS}\eta^{RG} + \frac{1}{4}\eta^{MK}\eta^{NL}\eta^{HI}\eta^{PS}\eta^{QG}\eta^{JR} - \frac{1}{2}\eta^{MK}\eta^{NL}\eta^{HS}\eta^{PI}\eta^{QG}\eta^{JR}$$

$$-\eta^{MP}\eta^{NK}\eta^{HI}\eta^{QJ}\eta^{LS}\eta^{RG} - 2\eta^{MP}\eta^{NK}\eta^{HI}\eta^{QS}\eta^{JL}\eta^{RG} - 3\eta^{MP}\eta^{NK}\eta^{HS}\eta^{QI}\eta^{JL}\eta^{RG}$$

$$-\frac{1}{2}\eta^{MP}\eta^{NK}\eta^{HS}\eta^{QG}\eta^{IL}\eta^{JR} - \frac{1}{4}\eta^{MP}\eta^{NQ}\eta^{HI}\eta^{JK}\eta^{LS}\eta^{RG} + \frac{1}{2}\eta^{MP}\eta^{NQ}\eta^{HS}\eta^{IK}\eta^{JL}\eta^{RG}$$

$$-\frac{1}{4}\eta^{MH}\eta^{NP}\eta^{QI}\eta^{JK}\eta^{LS}\eta^{RG} + \frac{1}{4}\eta^{MH}\eta^{NP}\eta^{QS}\eta^{IK}\eta^{JL}\eta^{RG}\,)$$

$$+\partial_M\mathcal{M}_{NP}\,\partial_Q\mathcal{M}_{IJ}\,\partial_K\mathcal{M}_{LR}\,\partial_{SGH}d\;(\,-\frac{1}{2}\eta^{MN}\eta^{PS}\eta^{QI}\eta^{JG}\eta^{KL}\eta^{RH} - \frac{5}{2}\eta^{MN}\eta^{PS}\eta^{QL}\eta^{IK}\eta^{JG}\eta^{RH}$$

$$+\frac{1}{2}\eta^{MN}\eta^{PQ}\eta^{IK}\eta^{JS}\eta^{LG}\eta^{RH} - \frac{1}{4}\eta^{MN}\eta^{PQ}\eta^{IS}\eta^{JG}\eta^{KL}\eta^{RH} - 3\eta^{MI}\eta^{NQ}\eta^{PK}\eta^{JS}\eta^{LG}\eta^{RH}\,)$$

$$+\partial_M\mathcal{M}_{NP}\,\partial_Q\mathcal{M}_{IJ}\,\partial_{KL}\mathcal{M}_{RS}\,\partial_{GH}d\;(\,-\frac{1}{4}\eta^{MN}\eta^{PK}\eta^{QI}\eta^{JL}\eta^{RG}\eta^{SH} - \frac{3}{2}\eta^{MN}\eta^{PK}\eta^{QI}\eta^{JG}\eta^{LR}\eta^{SH}$$

$$-\frac{5}{2}\eta^{MN}\eta^{PG}\eta^{QR}\eta^{IK}\eta^{JH}\eta^{LS} - \frac{5}{4}\eta^{MN}\eta^{PG}\eta^{QR}\eta^{IK}\eta^{JL}\eta^{SH} - \frac{1}{4}\eta^{MN}\eta^{PK}\eta^{QR}\eta^{IG}\eta^{JH}\eta^{LS}$$

$$+\frac{1}{4}\eta^{MN}\eta^{PQ}\eta^{IK}\eta^{JL}\eta^{RG}\eta^{SH} + \frac{1}{2}\eta^{MN}\eta^{PQ}\eta^{IK}\eta^{JG}\eta^{LR}\eta^{SH} - \frac{1}{2}\eta^{MR}\eta^{NK}\eta^{PL}\eta^{QS}\eta^{IG}\eta^{JH}$$

$$-2\eta^{MR}\eta^{NK}\eta^{PG}\eta^{QS}\eta^{IL}\eta^{JH} - 2\eta^{MR}\eta^{NQ}\eta^{PK}\eta^{IL}\eta^{JG}\eta^{SH} - \eta^{MR}\eta^{NQ}\eta^{PK}\eta^{IG}\eta^{JH}\eta^{LS}$$

$$+\eta^{MR}\eta^{NQ}\eta^{PG}\eta^{IK}\eta^{JL}\eta^{SH} - \eta^{MI}\eta^{NQ}\eta^{PK}\eta^{JL}\eta^{RG}\eta^{SH} - 5\eta^{MI}\eta^{NQ}\eta^{PK}\eta^{JG}\eta^{LR}\eta^{SH}\,)$$

$$+\partial_M\mathcal{M}_{NP}\,\partial_Q\mathcal{M}_{IJ}\,\partial_{KLR}\mathcal{M}_{SG}\,\partial_H d\;(\,-\frac{1}{2}\eta^{MN}\eta^{PK}\eta^{QI}\eta^{JL}\eta^{HS}\eta^{RG} - \frac{1}{4}\eta^{MN}\eta^{PK}\eta^{QS}\eta^{IL}\eta^{JR}\eta^{HG}$$

$$-\frac{1}{2}\eta^{MN}\eta^{PK}\eta^{QS}\eta^{IH}\eta^{JL}\eta^{RG} + \frac{1}{2}\eta^{MN}\eta^{PQ}\eta^{IK}\eta^{JL}\eta^{HS}\eta^{RG} - \frac{1}{4}\eta^{MN}\eta^{PQ}\eta^{IH}\eta^{JK}\eta^{LS}\eta^{RG}$$

$$-\eta^{MS}\eta^{NH}\eta^{PK}\eta^{QG}\eta^{IL}\eta^{JR} - \eta^{MS}\eta^{NQ}\eta^{PK}\eta^{IH}\eta^{JL}\eta^{RG} + \frac{1}{2}\eta^{MS}\eta^{NQ}\eta^{PH}\eta^{IK}\eta^{JL}\eta^{RG}$$

$$-2\eta^{MI}\eta^{NQ}\eta^{PK}\eta^{JL}\eta^{HS}\eta^{RG} - \eta^{MI}\eta^{NQ}\eta^{PH}\eta^{JK}\eta^{LS}\eta^{RG}\,)$$

$$+\partial_M\mathcal{M}_{NP}\,\partial_{QI}\mathcal{M}_{JK}\,\partial_{LR}\mathcal{M}_{SG}\,\partial_H d\;(\,-\frac{1}{2}\eta^{MN}\eta^{PQ}\eta^{HJ}\eta^{IS}\eta^{KL}\eta^{RG} - \frac{1}{2}\eta^{MN}\eta^{PQ}\eta^{HS}\eta^{IJ}\eta^{KL}\eta^{RG}$$

$$+\frac{1}{4}\eta^{MN}\eta^{PQ}\eta^{HS}\eta^{IG}\eta^{JL}\eta^{KR} - 2\eta^{MJ}\eta^{NH}\eta^{PQ}\eta^{IS}\eta^{KL}\eta^{RG} + \frac{1}{2}\eta^{MS}\eta^{NQ}\eta^{PI}\eta^{HJ}\eta^{KL}\eta^{RG}$$

$$-\eta^{MJ}\eta^{NQ}\eta^{PI}\eta^{HS}\eta^{KL}\eta^{RG} + \frac{1}{2}\eta^{MS}\eta^{NQ}\eta^{PI}\eta^{HG}\eta^{JL}\eta^{KR} - \eta^{MJ}\eta^{NQ}\eta^{PL}\eta^{HK}\eta^{IS}\eta^{RG}$$

$$-2\eta^{MJ}\eta^{NQ}\eta^{PL}\eta^{HS}\eta^{IG}\eta^{KR} - 2\eta^{MJ}\eta^{NQ}\eta^{PL}\eta^{HS}\eta^{IK}\eta^{RG}\,)$$

$$+3\,\mathcal{M}_{MN}\mathcal{M}_{PQ}\mathcal{M}_{IJ}\partial_K d\,\partial_{LRSGH}d\,\eta^{MK}\eta^{NL}\eta^{PR}\eta^{QS}\eta^{IG}\eta^{JH}$$

$$+11\,\mathcal{M}_{MN}\mathcal{M}_{PQ}\mathcal{M}_{IJ}\partial_{KL}d\,\partial_{RSGH}d\,\eta^{MK}\eta^{NR}\eta^{PL}\eta^{QS}\eta^{IG}\eta^{JH}$$

$$+\mathcal{M}_{MN}\mathcal{M}_{PQ}\mathcal{M}_{IJ}\partial_{KLR}d\,\partial_{SGH}d\;(\,4\eta^{MK}\eta^{NL}\eta^{PR}\eta^{QS}\eta^{IG}\eta^{JH}$$

$$+4\eta^{MK}\eta^{NS}\eta^{PL}\eta^{QG}\eta^{IR}\eta^{JH}\,) + \mathcal{M}_{MN}\mathcal{M}_{PQ}\partial_I\mathcal{M}_{JK}\,\partial_L d\,\partial_{RSGH}d\;(\,\frac{1}{2}\eta^{MI}\eta^{NL}\eta^{PR}\eta^{QS}\eta^{JG}\eta^{KH}$$

$$+6\eta^{MI}\eta^{NR}\eta^{PL}\eta^{QS}\eta^{JG}\eta^{KH} + 11\eta^{MI}\eta^{NR}\eta^{PS}\eta^{QG}\eta^{JL}\eta^{KH} + 6\eta^{ML}\eta^{NR}\eta^{PS}\eta^{QG}\eta^{IJ}\eta^{KH}\,)$$

$$+\mathcal{M}_{MN}\mathcal{M}_{PQ}\partial_I\mathcal{M}_{JK}\,\partial_{LR}d\,\partial_{SGH}d\;(\,10\eta^{MI}\eta^{NL}\eta^{PR}\eta^{QS}\eta^{JG}\eta^{KH}$$

$$+11\eta^{MI}\eta^{NL}\eta^{PS}\eta^{QG}\eta^{JR}\eta^{KH} + 24\eta^{MI}\eta^{NS}\eta^{PL}\eta^{QG}\eta^{JR}\eta^{KH} + \frac{5}{2}\eta^{MI}\eta^{NS}\eta^{PG}\eta^{QH}\eta^{JL}\eta^{KR}$$

$$+11\eta^{ML}\eta^{NS}\eta^{PR}\eta^{QG}\eta^{IJ}\eta^{KH} + 8\eta^{ML}\eta^{NS}\eta^{PG}\eta^{QH}\eta^{IJ}\eta^{KR}\,)$$

$$+\frac{1}{2}\eta^{MP}\eta^{NG}\eta^{QH}\eta^{IJ}\eta^{KR}\eta^{LS} + \eta^{MP}\eta^{NJ}\eta^{QK}\eta^{IL}\eta^{GR}\eta^{HS} + 6\eta^{MP}\eta^{NJ}\eta^{QG}\eta^{IK}\eta^{HR}\eta^{LS}$$

$$+\eta^{MP}\eta^{NJ}\eta^{QG}\eta^{IH}\eta^{KR}\eta^{LS}\,) + \mathcal{M}_{MN}\partial_{PQ}\mathcal{M}_{IJ}\,\partial_{KL}\mathcal{M}_{RS}\,\partial_G d\,\partial_H d\;(\,\eta^{MP}\eta^{NQ}\eta^{GI}\eta^{HR}\eta^{JK}\eta^{LS}$$

$$-\frac{1}{2}\eta^{MP}\eta^{NQ}\eta^{GR}\eta^{HS}\eta^{IK}\eta^{JL} + 3\eta^{MP}\eta^{NK}\eta^{GI}\eta^{HJ}\eta^{QR}\eta^{LS}$$

$$+2\eta^{MP}\eta^{NK}\eta^{GI}\eta^{HR}\eta^{QS}\eta^{JL} + 2\eta^{MP}\eta^{NK}\eta^{GI}\eta^{HR}\eta^{QJ}\eta^{LS} + \eta^{MG}\eta^{NP}\eta^{HI}\eta^{QR}\eta^{JK}\eta^{LS}$$

$$+\eta^{MG}\eta^{NP}\eta^{HR}\eta^{QI}\eta^{JK}\eta^{LS} - \frac{1}{2}\eta^{MG}\eta^{NP}\eta^{HR}\eta^{QS}\eta^{IK}\eta^{JL}\,)$$

$$+\partial_M\mathcal{M}_{NP}\,\partial_Q\mathcal{M}_{IJ}\,\partial_K\mathcal{M}_{LR}\,\partial_{S}d\,\partial_{GH}d\;(\,5\eta^{MN}\eta^{PG}\eta^{QL}\eta^{IK}\eta^{JS}\eta^{RH}$$

$$+\frac{1}{2}\eta^{MN}\eta^{PQ}\eta^{IK}\eta^{JS}\eta^{LG}\eta^{RH} - \eta^{MN}\eta^{PQ}\eta^{IK}\eta^{JG}\eta^{LS}\eta^{RH}$$

$$+3\eta^{MN}\eta^{PQ}\eta^{IS}\eta^{JG}\eta^{KL}\eta^{RH} + 2\eta^{MI}\eta^{NQ}\eta^{PK}\eta^{JS}\eta^{LG}\eta^{RH} + 10\eta^{MI}\eta^{NQ}\eta^{PK}\eta^{JG}\eta^{LS}\eta^{RH}\,)$$

$$+\partial_M\mathcal{M}_{NP}\,\partial_Q\mathcal{M}_{IJ}\,\partial_{KL}\mathcal{M}_{RS}\,\partial_G d\,\partial_H d\;(\,\frac{1}{2}\eta^{MN}\eta^{PK}\eta^{QI}\eta^{JL}\eta^{GR}\eta^{HS} + \frac{1}{2}\eta^{MN}\eta^{PK}\eta^{QR}\eta^{IG}\eta^{JH}\eta^{LS}$$

$$+\eta^{MN}\eta^{PK}\eta^{QR}\eta^{IG}\eta^{JL}\eta^{HS} - \frac{1}{2}\eta^{MN}\eta^{PQ}\eta^{IK}\eta^{JL}\eta^{GR}\eta^{HS} + \eta^{MN}\eta^{PQ}\eta^{IG}\eta^{JK}\eta^{HR}\eta^{LS}$$

$$+2\eta^{MR}\eta^{NG}\eta^{PK}\eta^{QS}\eta^{IH}\eta^{JL} + \eta^{MR}\eta^{NG}\eta^{PH}\eta^{QS}\eta^{IK}\eta^{JL} + 2\eta^{MR}\eta^{NQ}\eta^{PK}\eta^{IG}\eta^{JH}\eta^{LS}$$

$$+2\eta^{MR}\eta^{NQ}\eta^{PK}\eta^{IG}\eta^{JL}\eta^{HS} - \eta^{MR}\eta^{NQ}\eta^{PG}\eta^{IK}\eta^{JL}\eta^{HS} + 2\eta^{MI}\eta^{NQ}\eta^{PK}\eta^{JL}\eta^{GR}\eta^{HS}$$

$$+4\eta^{MI}\eta^{NQ}\eta^{PG}\eta^{JK}\eta^{HR}\eta^{LS}\,) - 6\,\mathcal{M}_{MN}\mathcal{M}_{PQ}\mathcal{M}_{IJ}\partial_K d\,\partial_L d\,\partial_{RSGH}d\,\eta^{MK}\eta^{NR}\eta^{PL}\eta^{QS}\eta^{IG}\eta^{JH}$$

$$+\mathcal{M}_{MN}\mathcal{M}_{PQ}\mathcal{M}_{IJ}\partial_K d\,\partial_{LR}d\,\partial_{SGH}d\;(\,-16\eta^{MK}\eta^{NL}\eta^{PR}\eta^{QS}\eta^{IG}\eta^{JH}$$

$$-22\eta^{MK}\eta^{NS}\eta^{PL}\eta^{QG}\eta^{IR}\eta^{JH}\,) - 12\,\mathcal{M}_{MN}\mathcal{M}_{PQ}\mathcal{M}_{IJ}\partial_{KL}d\,\partial_{RS}d\,\partial_{GH}d\,\eta^{MK}\eta^{NR}\eta^{PL}\eta^{QG}\eta^{IS}\eta^{JH}$$

$$+\mathcal{M}_{MN}\mathcal{M}_{PQ}\partial_I\mathcal{M}_{JK}\,\partial_L d\,\partial_R d\,\partial_{SGH}d\;(\,-\eta^{MI}\eta^{NL}\eta^{PR}\eta^{QS}\eta^{JG}\eta^{KH}$$

$$-6\eta^{MI}\eta^{NL}\eta^{PS}\eta^{QG}\eta^{JR}\eta^{KH} - 22\eta^{MI}\eta^{NS}\eta^{PL}\eta^{QG}\eta^{JR}\eta^{KH} - 5\eta^{MI}\eta^{NS}\eta^{PG}\eta^{QH}\eta^{JL}\eta^{KR}$$

$$-6\eta^{ML}\eta^{NS}\eta^{PR}\eta^{QG}\eta^{IJ}\eta^{KH}\,) + \mathcal{M}_{MN}\mathcal{M}_{PQ}\partial_I\mathcal{M}_{JK}\,\partial_L d\,\partial_{RS}d\,\partial_{GH}d\;(\,-4\eta^{MI}\eta^{NL}\eta^{PR}\eta^{QG}\eta^{JS}\eta^{KH}$$

$$-5\eta^{MI}\eta^{NR}\eta^{PL}\eta^{QS}\eta^{JG}\eta^{KH} - 22\eta^{MI}\eta^{NR}\eta^{PL}\eta^{QG}\eta^{JS}\eta^{KH} - 36\eta^{MI}\eta^{NR}\eta^{PS}\eta^{QG}\eta^{JL}\eta^{KH}$$

$$-16\eta^{ML}\eta^{NR}\eta^{PS}\eta^{QG}\eta^{IJ}\eta^{KH}\,) + \mathcal{M}_{MN}\mathcal{M}_{PQ}\partial_{IJ}\mathcal{M}_{KL}\,\partial_R d\,\partial_S d\,\partial_{GH}d\;(\,-6\eta^{MI}\eta^{NG}\eta^{PJ}\eta^{QH}\eta^{RK}\eta^{SL}$$

$$-10\eta^{MR}\eta^{NI}\eta^{PJ}\eta^{QG}\eta^{SK}\eta^{LH} - \eta^{MR}\eta^{NI}\eta^{PS}\eta^{QJ}\eta^{KG}\eta^{LH} - 6\eta^{MR}\eta^{NI}\eta^{PS}\eta^{QG}\eta^{JK}\eta^{LH}$$

$$-6\eta^{MR}\eta^{NG}\eta^{PI}\eta^{QJ}\eta^{SK}\eta^{LH} - 10\eta^{MR}\eta^{NG}\eta^{PI}\eta^{QH}\eta^{SK}\eta^{JL}\,)$$

$$+\mathcal{M}_{MN}\mathcal{M}_{PQ}\partial_{IJK}\mathcal{M}_{LR}\,\partial_S d\,\partial_G d\,\partial_H d\;(\,-2\eta^{MS}\eta^{NI}\eta^{PJ}\eta^{QK}\eta^{GL}\eta^{HR} - 2\eta^{MS}\eta^{NI}\eta^{PG}\eta^{QJ}\eta^{HL}\eta^{KR}\,)$$

$$+\mathcal{M}_{MN}\partial_P\mathcal{M}_{QI}\,\partial_J\mathcal{M}_{KL}\,\partial_R d\,\partial_S d\,\partial_{GH}d\;(\,-6\eta^{MR}\eta^{NG}\eta^{PQ}\eta^{IJ}\eta^{KS}\eta^{LH} - 10\eta^{MR}\eta^{NG}\eta^{PK}\eta^{QJ}\eta^{IS}\eta^{LH}$$

$$-12\eta^{MP}\eta^{NG}\eta^{QJ}\eta^{IR}\eta^{KS}\eta^{LH} - \eta^{MP}\eta^{NR}\eta^{QJ}\eta^{IS}\eta^{KG}\eta^{LH}$$

$$+2\eta^{MP}\eta^{NR}\eta^{QJ}\eta^{IG}\eta^{KS}\eta^{LH} - 6\eta^{MP}\eta^{NG}\eta^{QJ}\eta^{IH}\eta^{KR}\eta^{LS}$$

$$-2\eta^{MP}\eta^{NJ}\eta^{QR}\eta^{IS}\eta^{KG}\eta^{LH} - 12\eta^{MP}\eta^{NJ}\eta^{QR}\eta^{IG}\eta^{KS}\eta^{LH}$$

$$-5\eta^{MP}\eta^{NG}\eta^{QR}\eta^{IS}\eta^{JK}\eta^{LH} - 6\eta^{MP}\eta^{NR}\eta^{QS}\eta^{IG}\eta^{JK}\eta^{LH}\,)$$

$$+\mathcal{M}_{MN}\partial_P\mathcal{M}_{QI}\,\partial_{JK}\mathcal{M}_{LR}\,\partial_S d\,\partial_G d\,\partial_H d\;(\,-2\eta^{MS}\eta^{NJ}\eta^{PQ}\eta^{IK}\eta^{GL}\eta^{HR}$$

$$-\eta^{MJ}\eta^{NK}\eta^{PL}\eta^{QS}\eta^{IG}\eta^{HR} - \eta^{MS}\eta^{NJ}\eta^{PL}\eta^{QG}\eta^{IH}\eta^{KR}$$

… and other complices?

$$-2\,\eta^{MS}\eta^{NJ}\eta^{PL}\eta^{QG}\eta^{IK}\eta^{HR} + \eta^{MP}\eta^{NS}\eta^{QJ}\eta^{IK}\eta^{GL}\eta^{HR}$$

$$-2\,\eta^{MP}\eta^{NS}\eta^{QG}\eta^{IJ}\eta^{HL}\eta^{KR} - 6\,\eta^{MP}\eta^{NJ}\eta^{QS}\eta^{IK}\eta^{GL}\eta^{HR}$$

$$-4\,\eta^{MP}\eta^{NJ}\eta^{QS}\eta^{IG}\eta^{HL}\eta^{KR}\,) + \partial_M\mathcal{M}_{NP}\,\partial_Q\mathcal{M}_{IJ}\,\partial_K\mathcal{M}_{LR}\,\partial_S d\,\partial_G d\,\partial_H d\,(-\eta^{MN}\eta^{PQ}\eta^{IK}\eta^{JS}\eta^{LG}\eta^{RH}$$

$$-4\,\eta^{MI}\eta^{NQ}\eta^{PK}\eta^{JS}\eta^{LG}\eta^{RH}\,) + 4\,\mathcal{M}_{MN}\mathcal{M}_{PQ}\mathcal{M}_{IJ}\partial_K d\,\partial_L d\,\partial_R d\,\partial_{SGH} d\,\eta^{MK}\eta^{NS}\eta^{PL}\eta^{QG}\eta^{IR}\eta^{JH}$$

$$+16\,\mathcal{M}_{MN}\mathcal{M}_{PQ}\mathcal{M}_{IJ}\partial_K d\,\partial_L d\,\partial_{RS} d\,\partial_{GH} d\,\eta^{MK}\eta^{NR}\eta^{PL}\eta^{QG}\eta^{IS}\eta^{JH}$$

$$+\mathcal{M}_{MN}\mathcal{M}_{PQ}\partial_I\mathcal{M}_{JK}\,\partial_L d\,\partial_R d\,\partial_S d\,\partial_{GH} d\,(12\,\eta^{MI}\eta^{NL}\eta^{PR}\eta^{QG}\eta^{JS}\eta^{KH}$$

$$+10\,\eta^{MI}\eta^{NG}\eta^{PL}\eta^{QH}\eta^{JR}\eta^{KS}\,) + 2\,\mathcal{M}_{MN}\mathcal{M}_{PQ}\partial_{IJ}\mathcal{M}_{KL}\,\partial_R d\,\partial_S d\,\partial_G d\,\partial_H d\,\eta^{MR}\eta^{NI}\eta^{PS}\eta^{QJ}\eta^{GK}\eta^{HL}$$

$$+\mathcal{M}_{MN}\partial_P\mathcal{M}_{QI}\,\partial_J\mathcal{M}_{KL}\,\partial_R d\,\partial_S d\,\partial_G d\,\partial_H d\,(2\,\eta^{MP}\eta^{NR}\eta^{QJ}\eta^{IS}\eta^{KG}\eta^{LH}$$

$$+2\,\eta^{MP}\eta^{NJ}\eta^{QR}\eta^{IS}\eta^{KG}\eta^{LH}\,) \tag{8.2.5}$$

Break for today, till later on!