

Stiffening of matter in quark-hadron continuity

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Refs) Baym-Hatsuda-TK-Powell-Song-Takatsuka, "QHC", review on neutron stars (2018)

TK, "Stiffening of matter in quark-hadron continuity" PRD (2021)

Fujimoto-TK-McLerran, "IdylliQ matter model" PRL (2024)

Chiba-TK, PRD(2024); TK-Suenaga-Chiba, "isospin QCD" (2024)

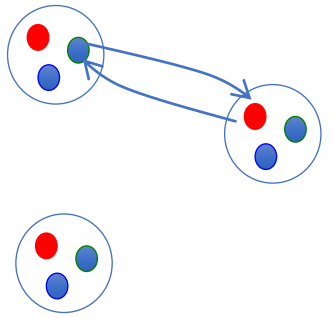
State of matter: **overview**

$(n_0 = 0.16 \text{ fm}^{-3})$

[Masuda+ '12; TK+ '14]

- few meson exchange

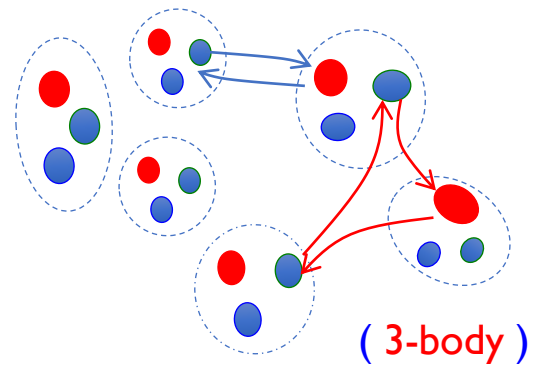
- nucleons **only**



- many-quark exchange

- structural change,...

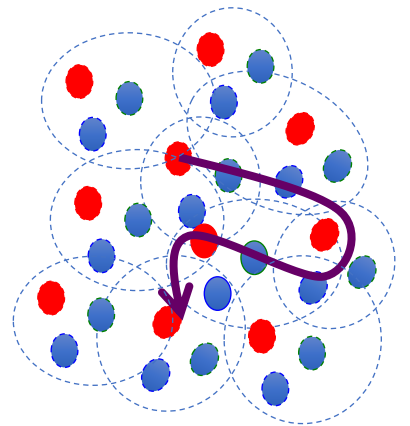
- hyperons, Δ , ...



most difficult
(d.o.f ??)

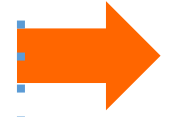
- Baryons overlap

- Quark Fermi sea



strongly correlated
(d.o.f : quasi-particles??)

not explored well



pQCD(?)

[Freedman-McLerran, Kurkela+, Fujimoto+...]

ab-initio nuclear cal.
laboratory experiments
steady progress

$\sim 1.4 M_{\odot}$

$\sim 2 M_{\odot}$

n_B

$\sim 2n_0$

Hints from NS

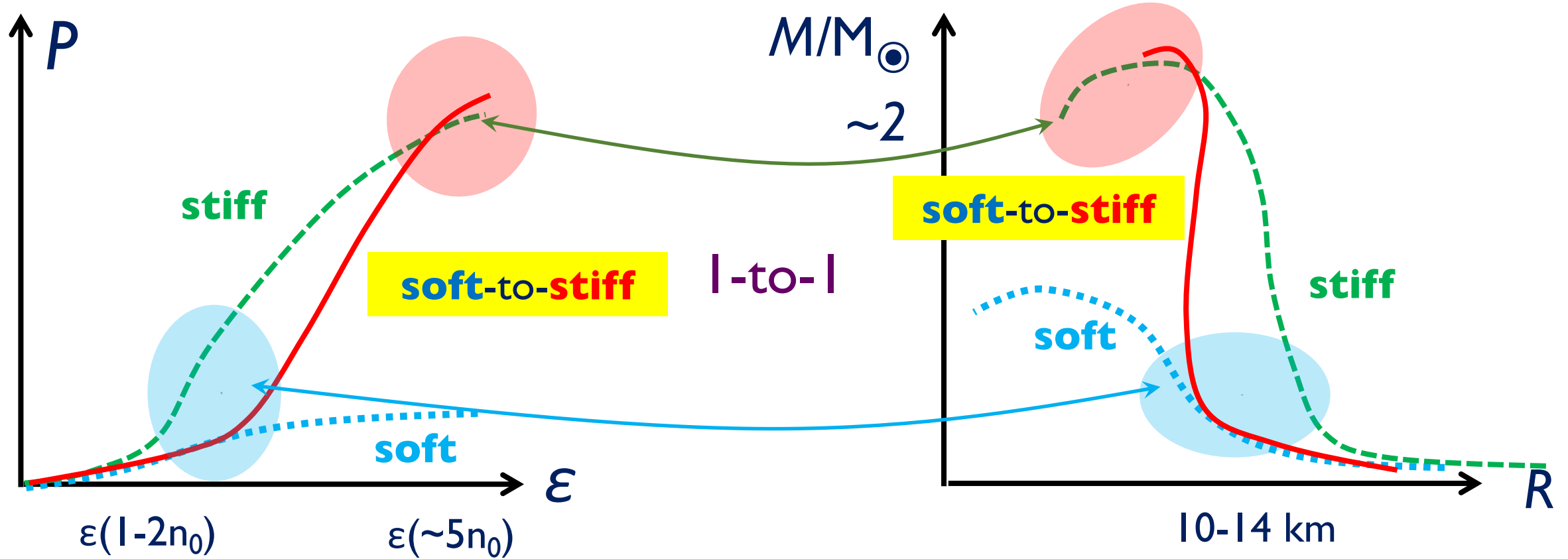
$\sim 5n_0$

$\sim 40n_0$



EoS & Neutron Star M-R relation

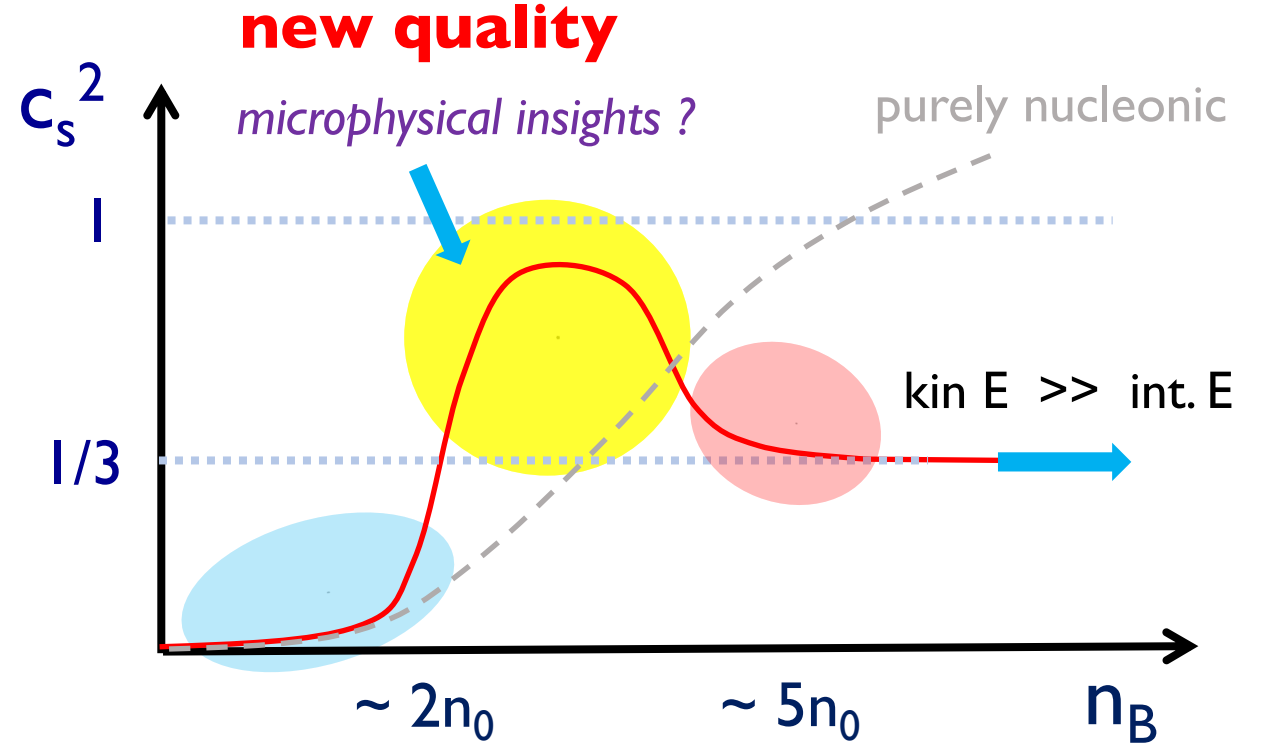
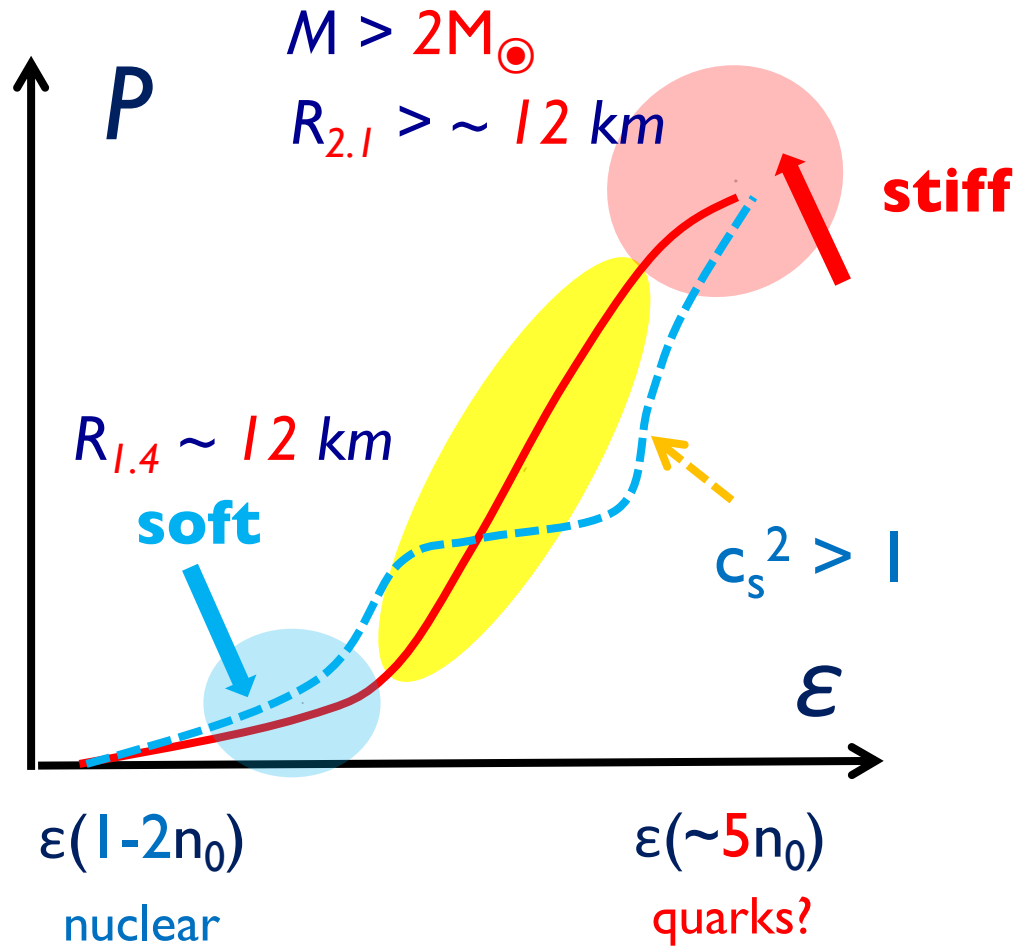
Einstein eq.: $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ QCD (+EW) EoS



Soft to *stiff* is challenging:

sound speed: $c_s^2 = dP/d\varepsilon < 1$ (*causality*)

nuclear & quark physics constrain each other



baseline: quark-hadron continuity (QHC)

Contents

1, Introduction

2, Early vs late stiffening

3, A model of quark-hadron duality

4, Hints from isospin QCD

Pressure from $\varepsilon(n_B)$

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B} \left(\frac{\varepsilon}{n_B} \right)$$

energy per particle

e.g.)

gas of **heavy** particles (**massive** limit)

$$\varepsilon(n_B) = m_N n_B \quad \longrightarrow \quad \varepsilon/n_B = m_N \quad \longrightarrow \quad P = 0 \quad \text{soft}$$

gas of **relativistic** particles (**massless** limit)

$$\varepsilon(n_B) = a n_B^{4/3} \quad \longrightarrow \quad \varepsilon/n_B = a n_B^{1/3} \quad \longrightarrow \quad P = \frac{\varepsilon}{3} \quad \text{stiff}$$

Nucleonic models & many-body forces

$$\varepsilon(n_B) = \underbrace{m_N n_B}_{\text{large (!)}} + \underbrace{a \frac{n_B^{5/3}}{m_N}}_{\text{small (!)}} + \underbrace{b n_B^\alpha}_{\text{small (!)}} \quad \longrightarrow \quad P = \underbrace{\frac{2}{3} a \frac{n_B^{5/3}}{m_N}}_{\text{small (!)}} + \underbrace{b(\alpha - 1) n_B^\alpha}_{\text{small (!)}}$$

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B} \left(\frac{\varepsilon}{n_B} \right)$$

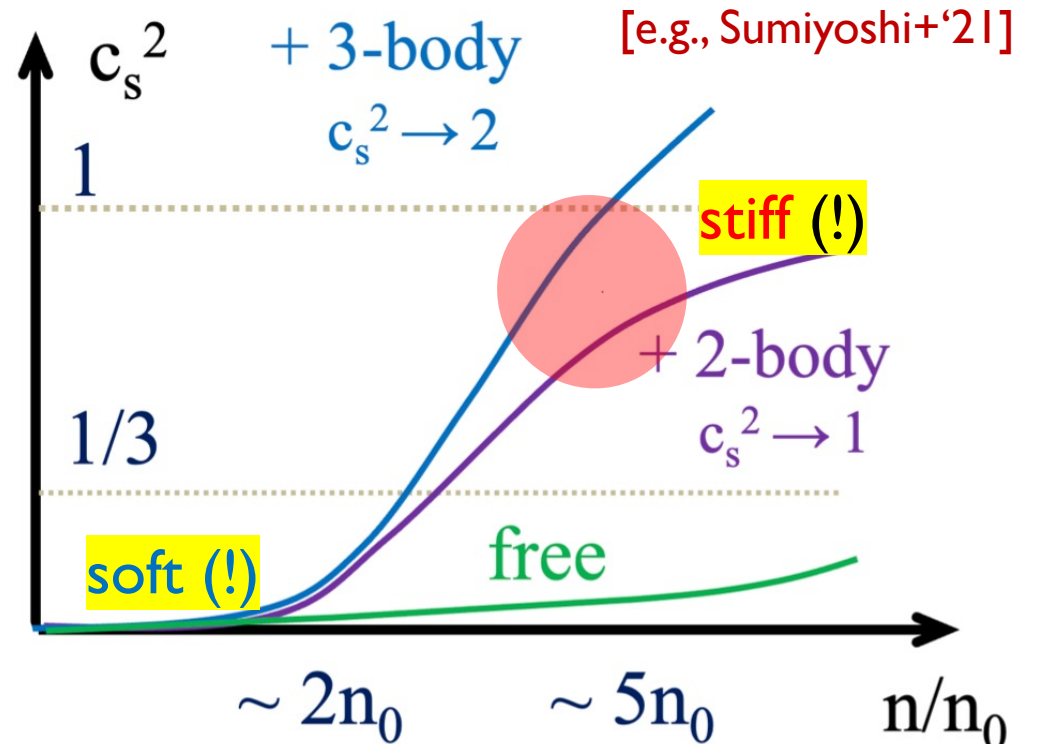
$\longrightarrow \quad p \ll \varepsilon \quad (\text{at least in dilute regime})$

If interactions dominate (at large n_B):

$$P \sim (\alpha - 1)\varepsilon \rightarrow c_s^2 \sim (\alpha - 1)$$

2-body int. $\rightarrow \alpha = 2$ 3-body int. $\rightarrow \alpha = 3$

- causality & convergence ??
- stiffening occurs *slowly* (power growth)

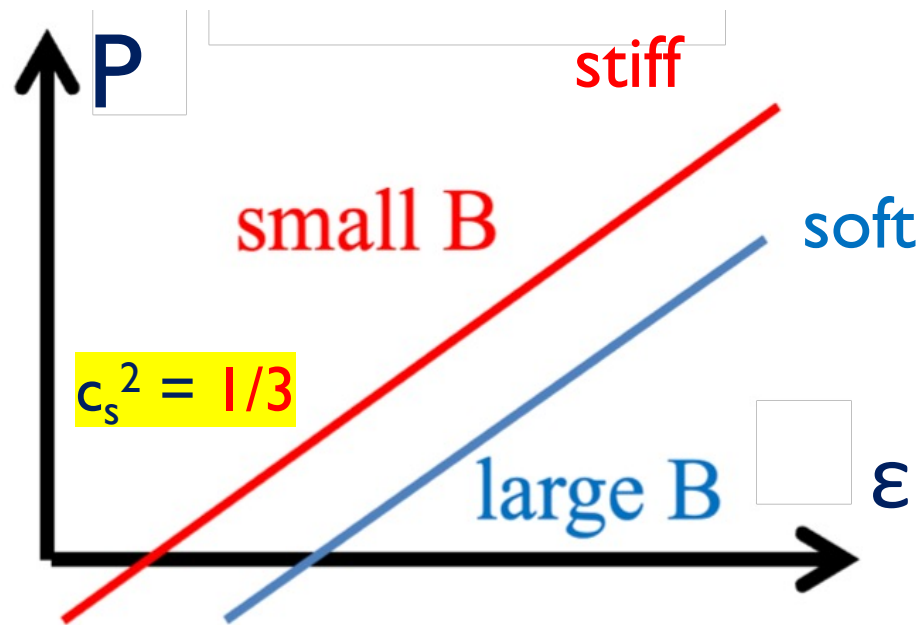


alternative: quark EOS

e.g.) free massless quarks

$$P = \frac{\epsilon}{3} - B'$$

normalization

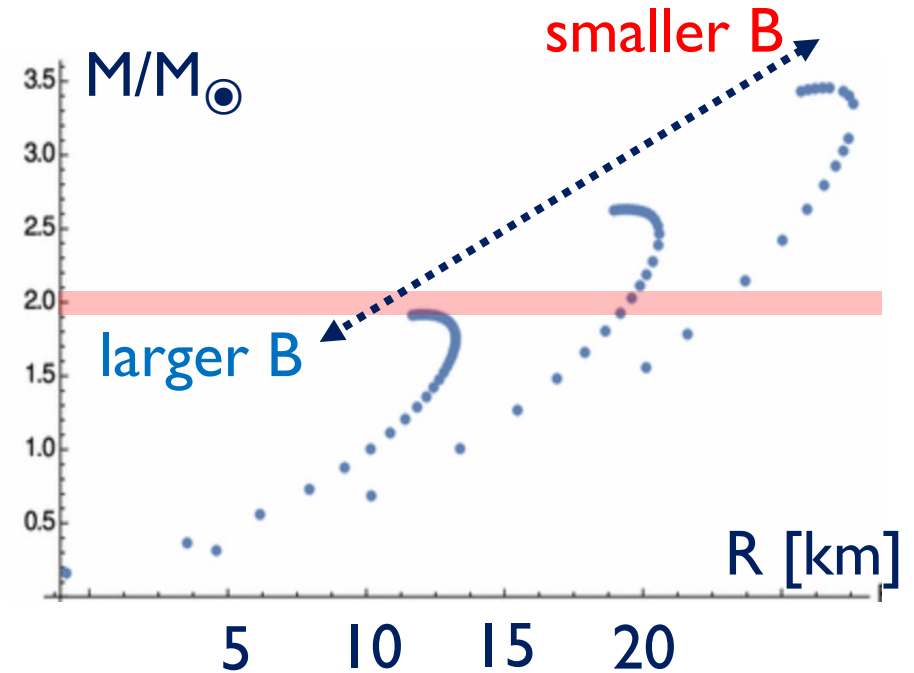


relativistic pressure \rightarrow stiff EOS ?

quark kin. pressure \gg baryon kin. pressure

$O(N_c)$

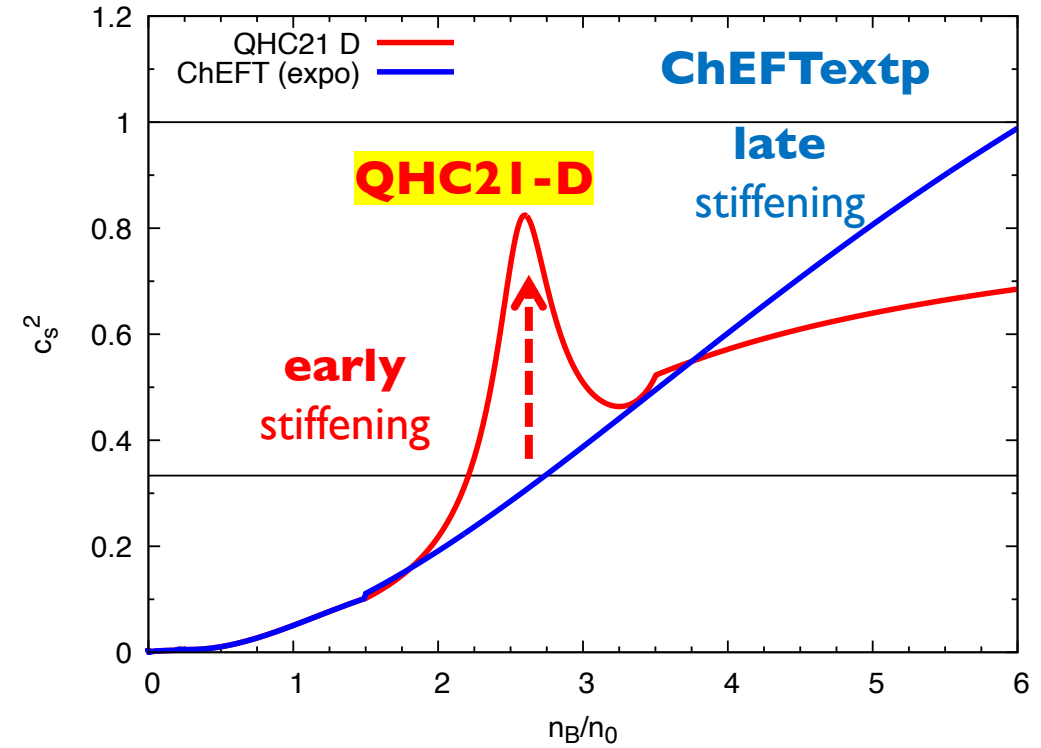
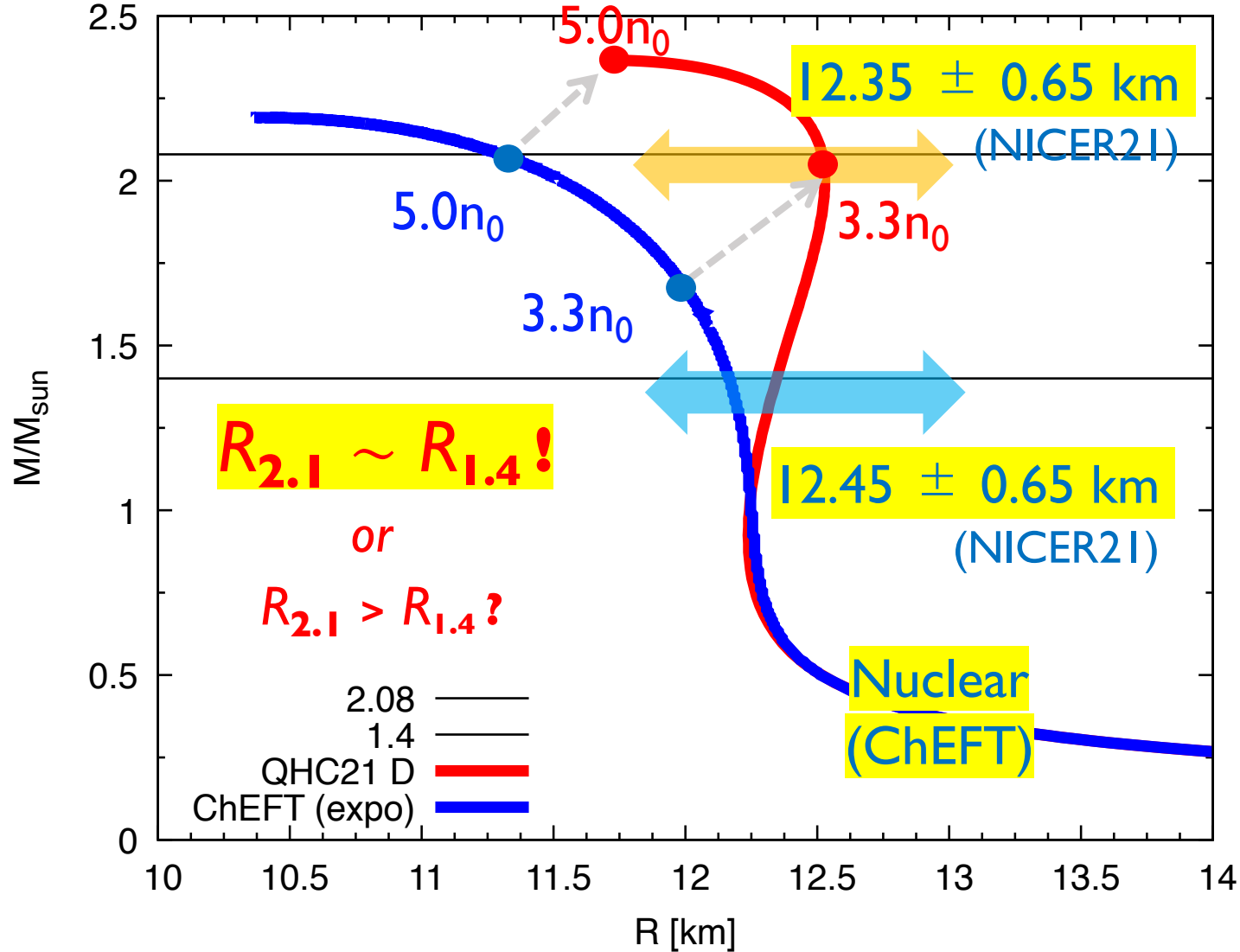
$O(1/N_c)$



depends on **where** to start...

Early vs late stiffening: QHC vs ChEFTexp

[TK-Hatsuda-Baym '21]



2-3n₀ : already beyond purely nucleonic regime?

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2, Early vs late stiffening

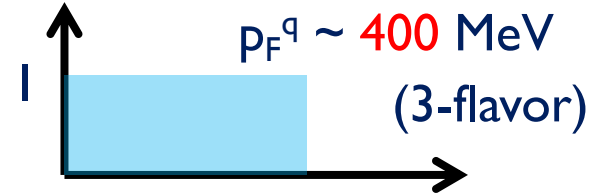
3, A model of quark-hadron duality

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Claim

- naive estimate for quark matter formation density: ($R_B \sim 0.5\text{-}0.8$ fm)

$$n_B^{\text{overlap}} \sim 1 / (4\pi R_B^3 / 3) \sim 4\text{-}7n_0$$



- we claim the existence of **another scale**, characterizing:

- breakdown of many-body expansion
- soft-deconfinement** [not explained today, see Fukushima-TK-Weise '20]
- quark saturation** [TK '21; Fujimoto-TK-McLerran '24]

$$n_B^{\text{q-sat}} \sim 0.5 \times n_B^{\text{overlap}} \sim 2\text{-}3n_0$$

Sum rules for occupation probabilities

cf) [TK '21, TK-Suenaga '21]

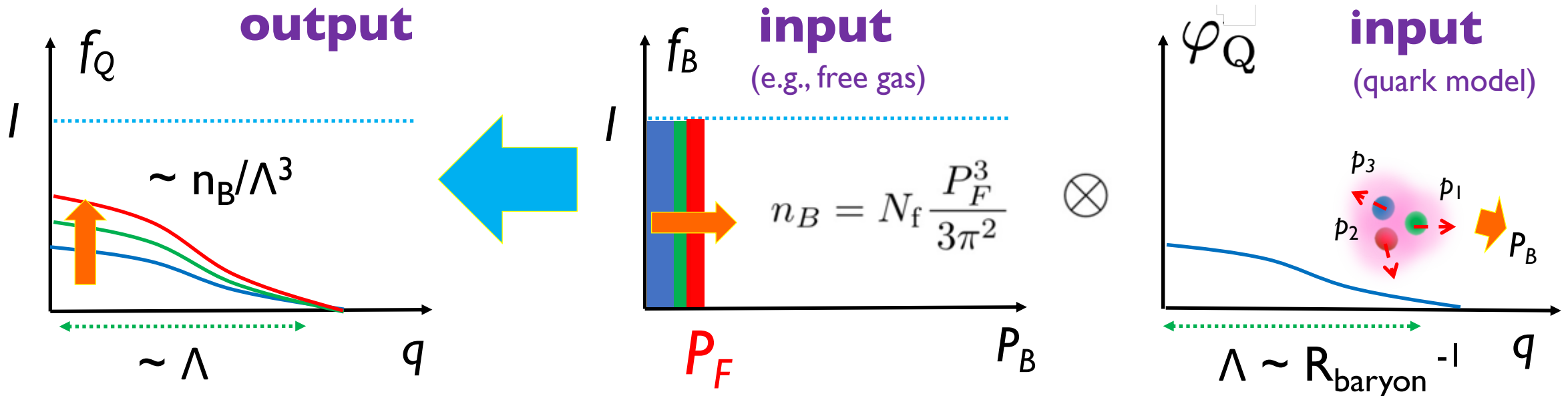
occupation **probability**
of **quark** state with p

occupation **probability**
of **baryon** state with P_B

quark mom. distribution
in a baryon

$$\underline{f_Q(\mathbf{q})} = \int_{P_B} \underline{f_B(\mathbf{P}_B)} \underline{\varphi_Q^B(\mathbf{q} - \mathbf{P}_B/N_c)}$$

e.g.) in **ideal** baryonic matter



An ideal model

[Fujimoto-TK-McLerran, PRL'24]

1) neglect interactions *except* confining forces

e.g.) 2-flavor hamiltonian: $\varepsilon_B[f_B] = 4 \int_k E_B(k) f_B(k)$

isospin, spin
↓

2) quark distributions in a baryon remains the same (confinement persists)

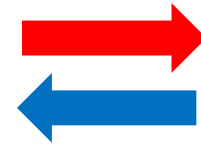
3) use a special quark distribution \rightarrow models become analytically **solvable**

$$\varphi_{3d}(\mathbf{q}) = \frac{2\pi^2}{\Lambda^3} \frac{e^{-q/\Lambda}}{q/\Lambda} \quad \hat{L} = -\nabla^2 + \frac{1}{\Lambda^2} \quad \hat{L}[\varphi(\mathbf{p} - \mathbf{q})] = \frac{(2\pi)^3}{\Lambda^2} \delta(\mathbf{p} - \mathbf{q})$$

nontrivial output

$$f_Q(\mathbf{q}) = \int_{\mathbf{P}_B} f_B(\mathbf{P}_B) \varphi_Q^B(\mathbf{q} - \mathbf{P}_B/N_c)$$

natural at **low** density



nontrivial output

$$f_B(N_c \mathbf{q}) = \frac{\Lambda^2}{N_c^3} \hat{L}[f_Q(\mathbf{q})]$$

natural at **high** density

useful for studies of the *transient regime* (d.o.f are not clear-cut)

Variational problem **with** sum rule constraints 14/24

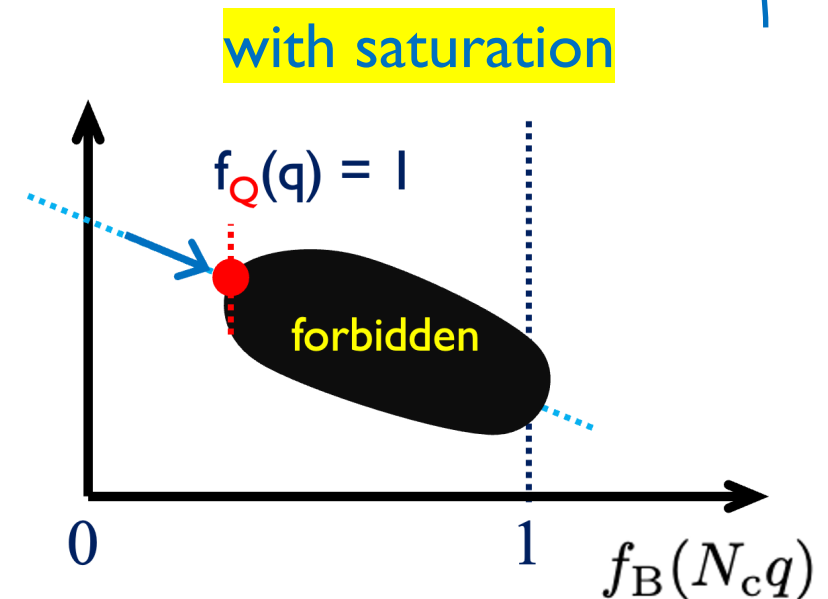
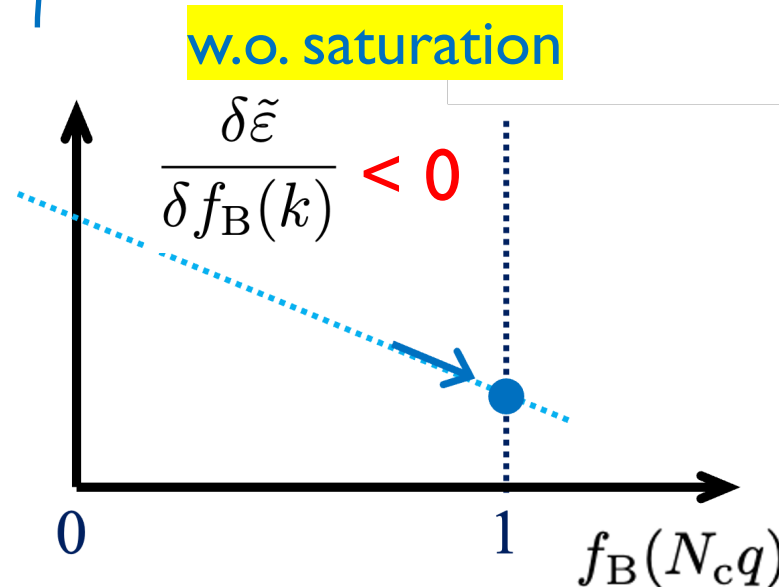
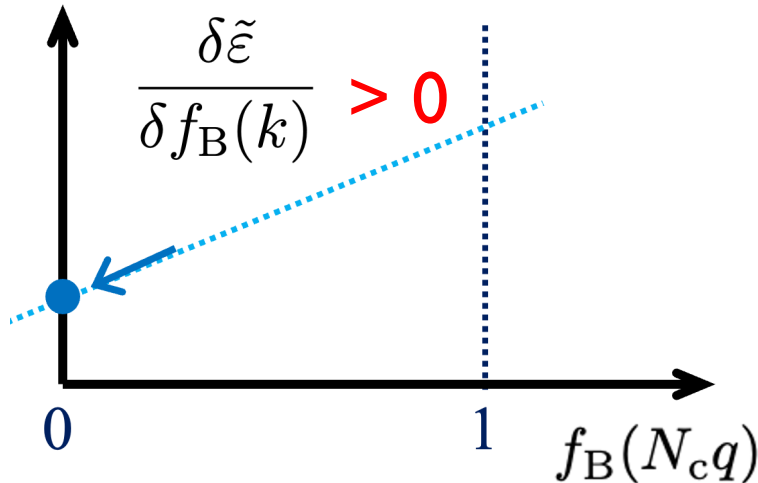
[Fujimoto-TK-McLerran, PRL'24]

$$\tilde{\varepsilon} = \varepsilon_B[f_B] - \lambda_B n_B \quad \text{constraint to fix } n_B \quad E_B(k) = \sqrt{M_B^2 + k^2} \quad n_B = 4 \int_k f_B(k)$$

optimization: $\frac{\delta \tilde{\varepsilon}}{\delta f_B(k)} = E_B(k) - \lambda_B$ **at a given k**

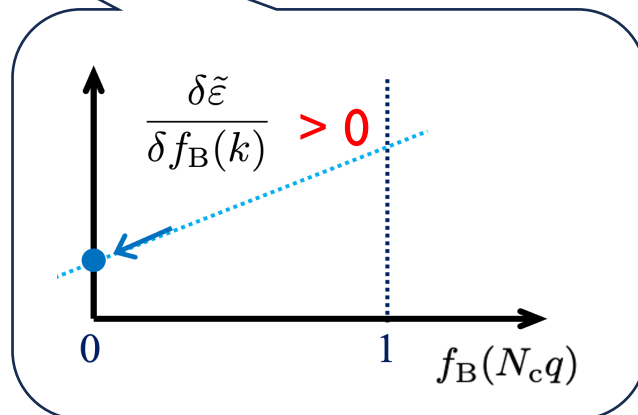
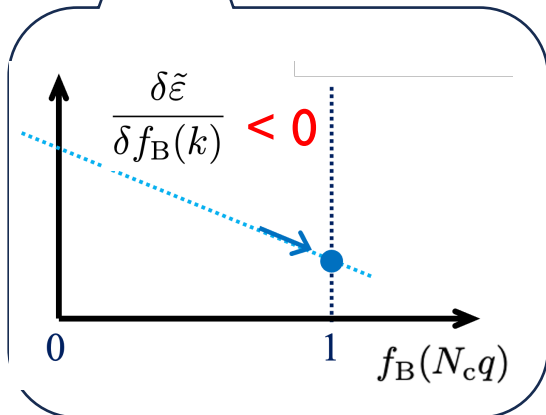
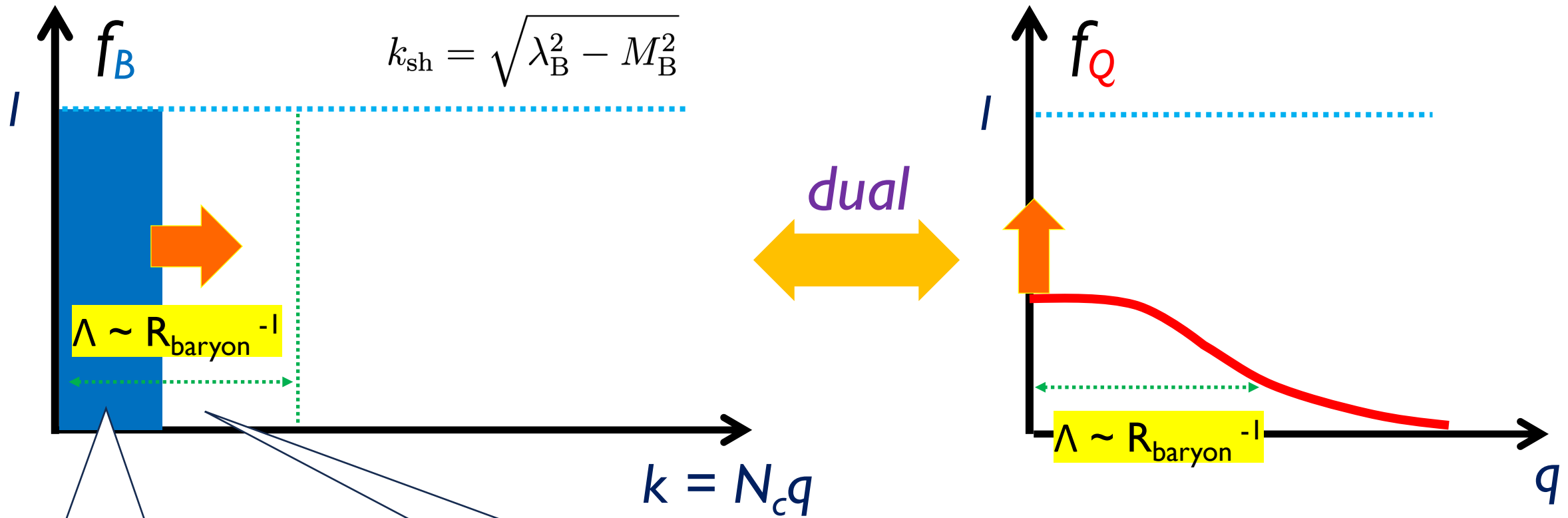
$$E_B(k) > \lambda_B$$

$$E_B(k) < \lambda_B$$



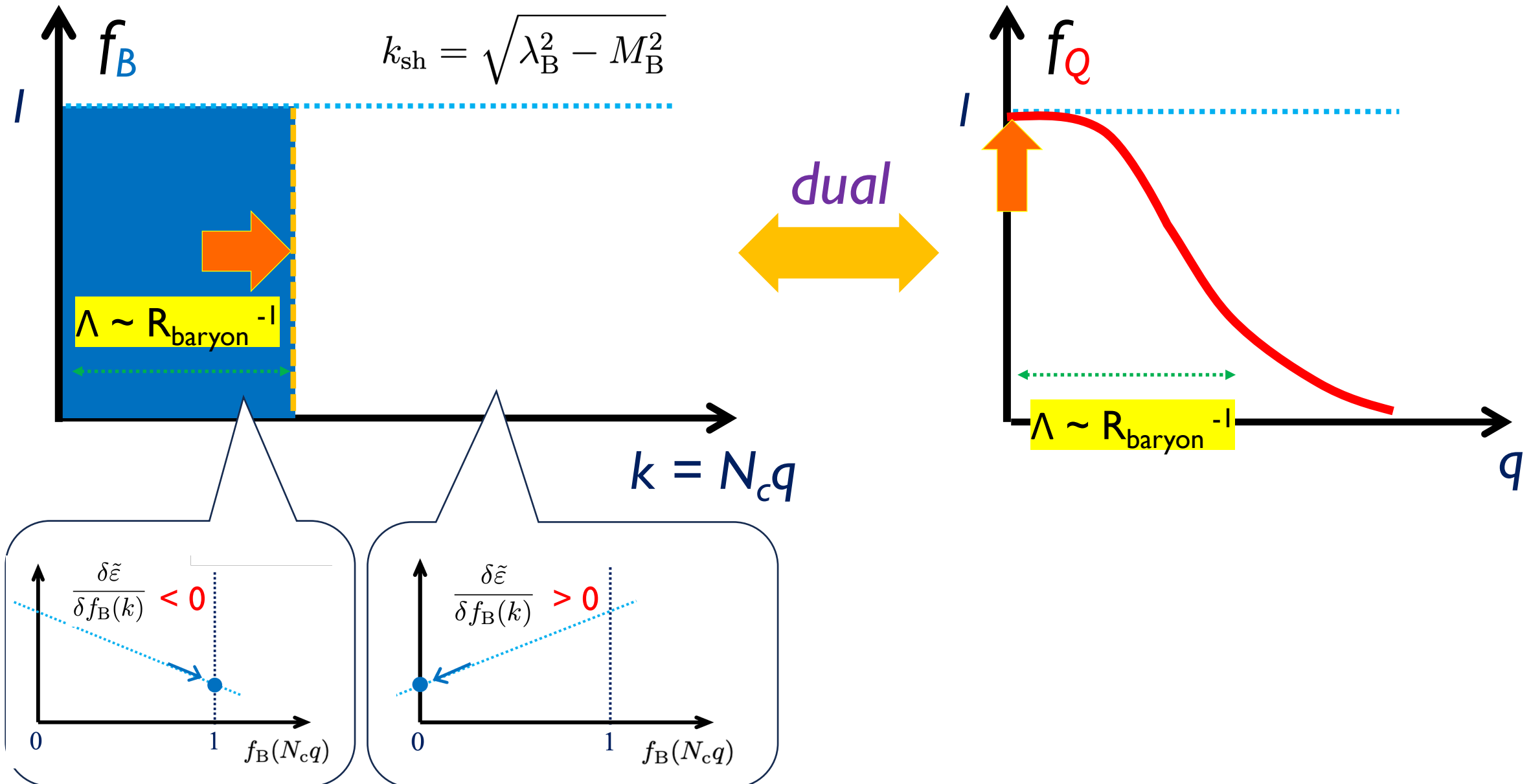
Solution (**dilute** regime)

[Fujimoto-TK-McLerran, PRL'24]



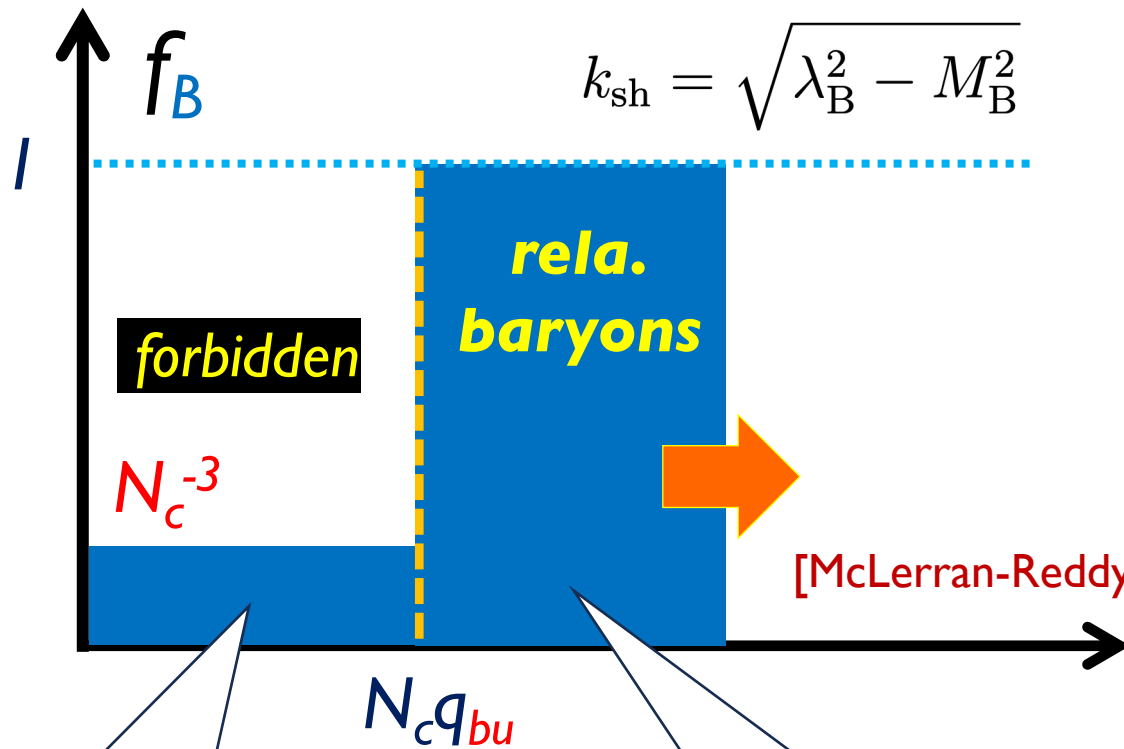
Solution (at saturation)

[Fujimoto-TK-McLerran, PRL'24]

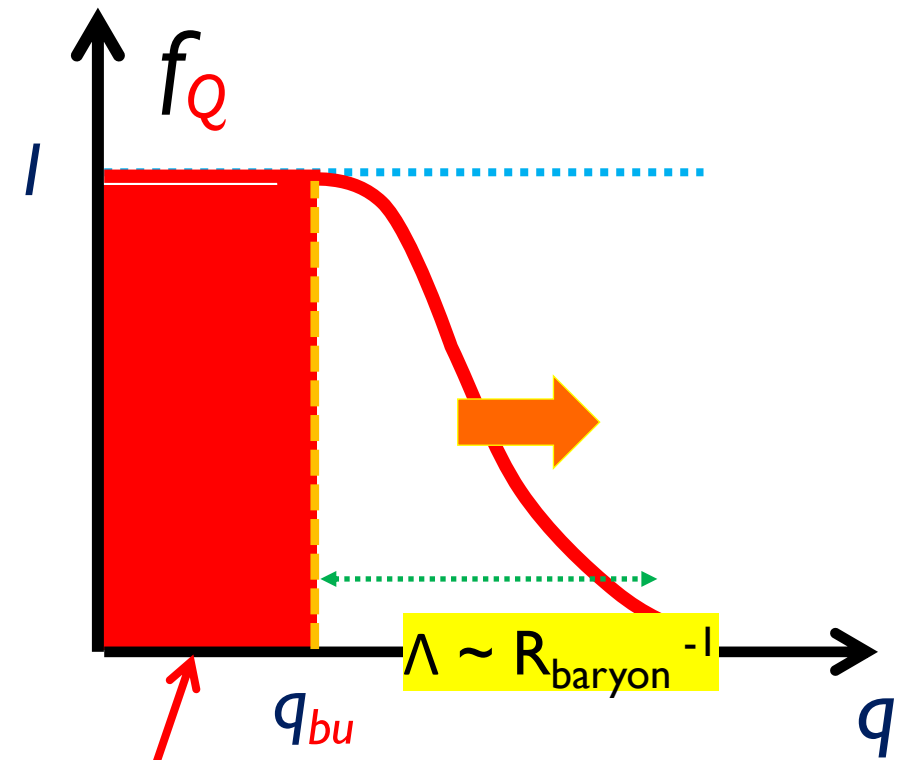


Solution (post saturation)

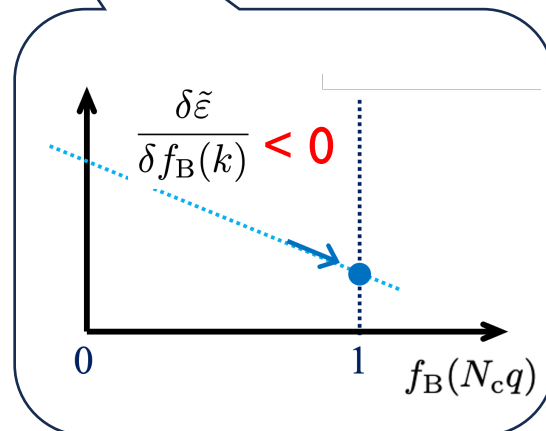
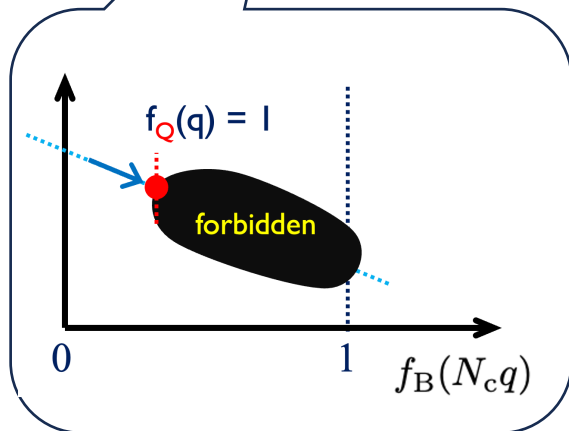
[Fujimoto-TK-McLerran, PRL'24]



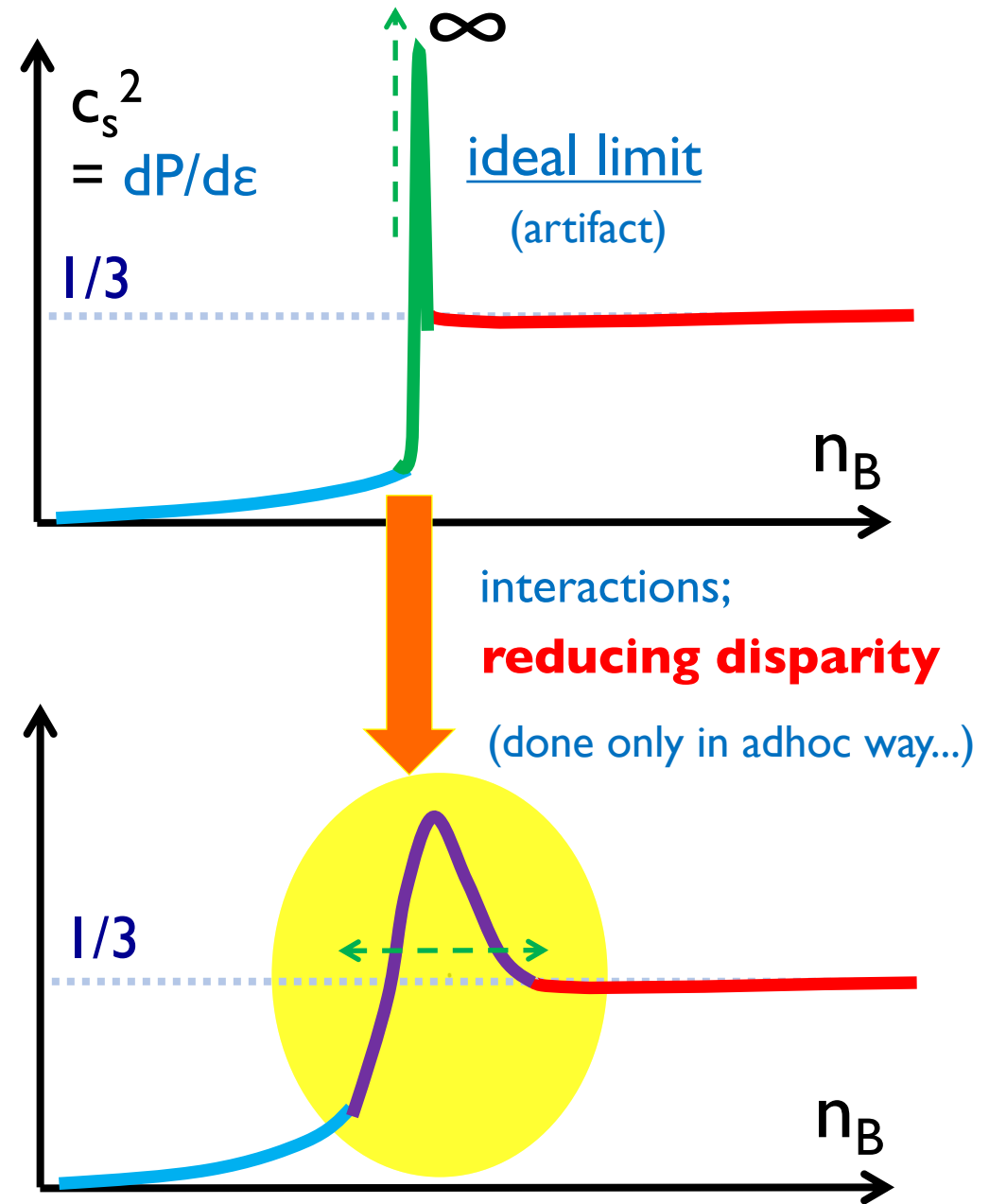
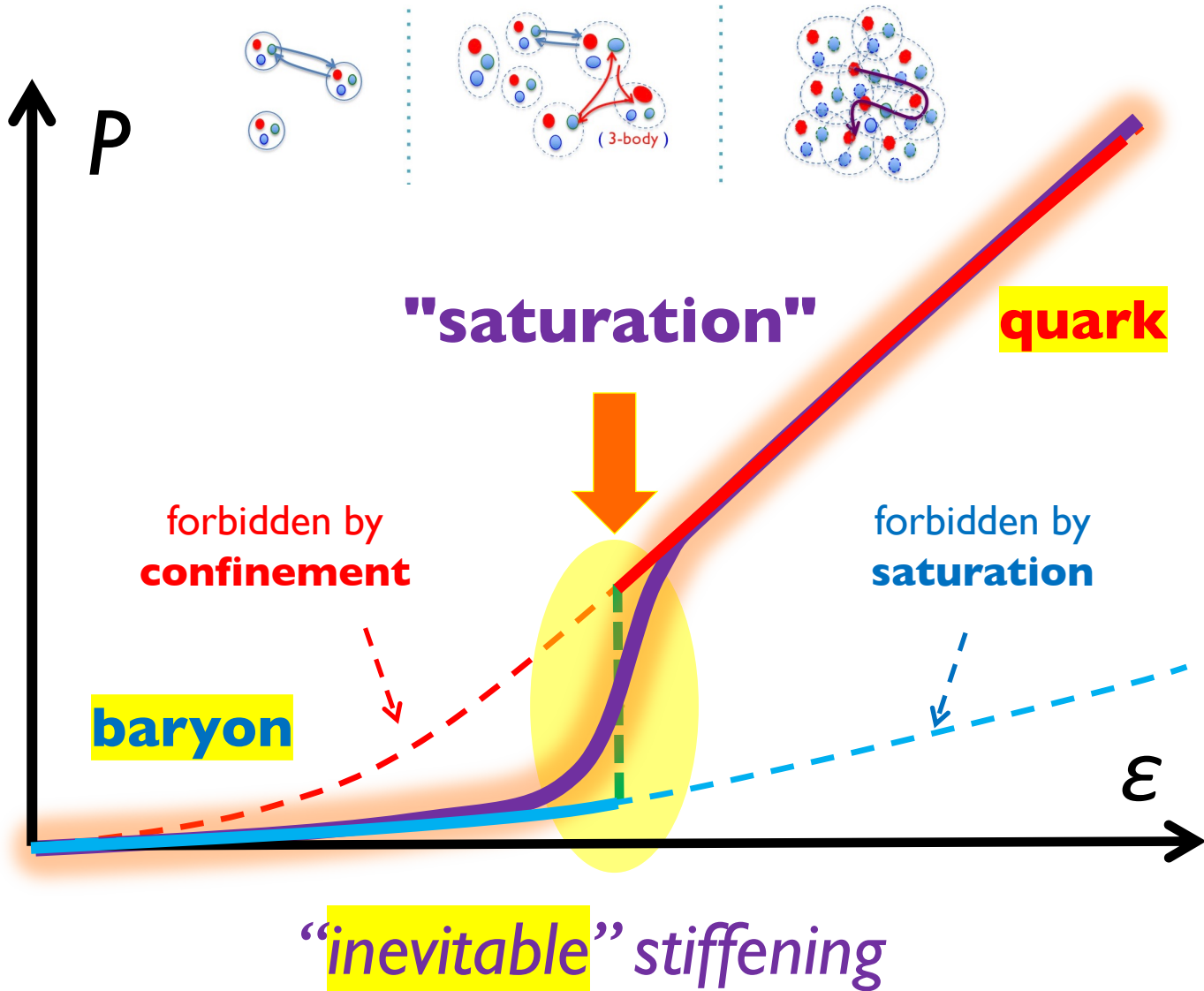
dual



“inevitable” formation of the quark Fermi sea



Peak in sound velocity



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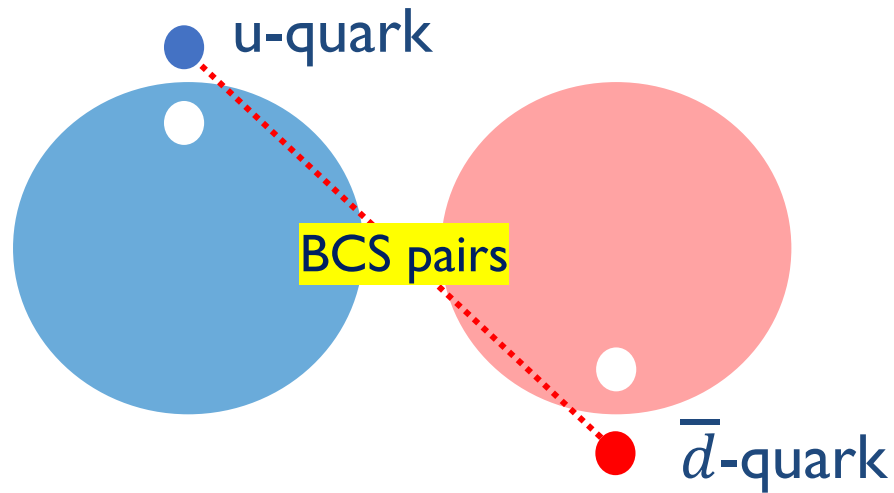
Isospin QCD

isospin chemical pot. $\mu_l = \mu_u = -\mu_d$

$n_0 = 0.16 \text{ fm}^{-3}$

- onset of finite density \rightarrow begin with pions (instead of nucleons)

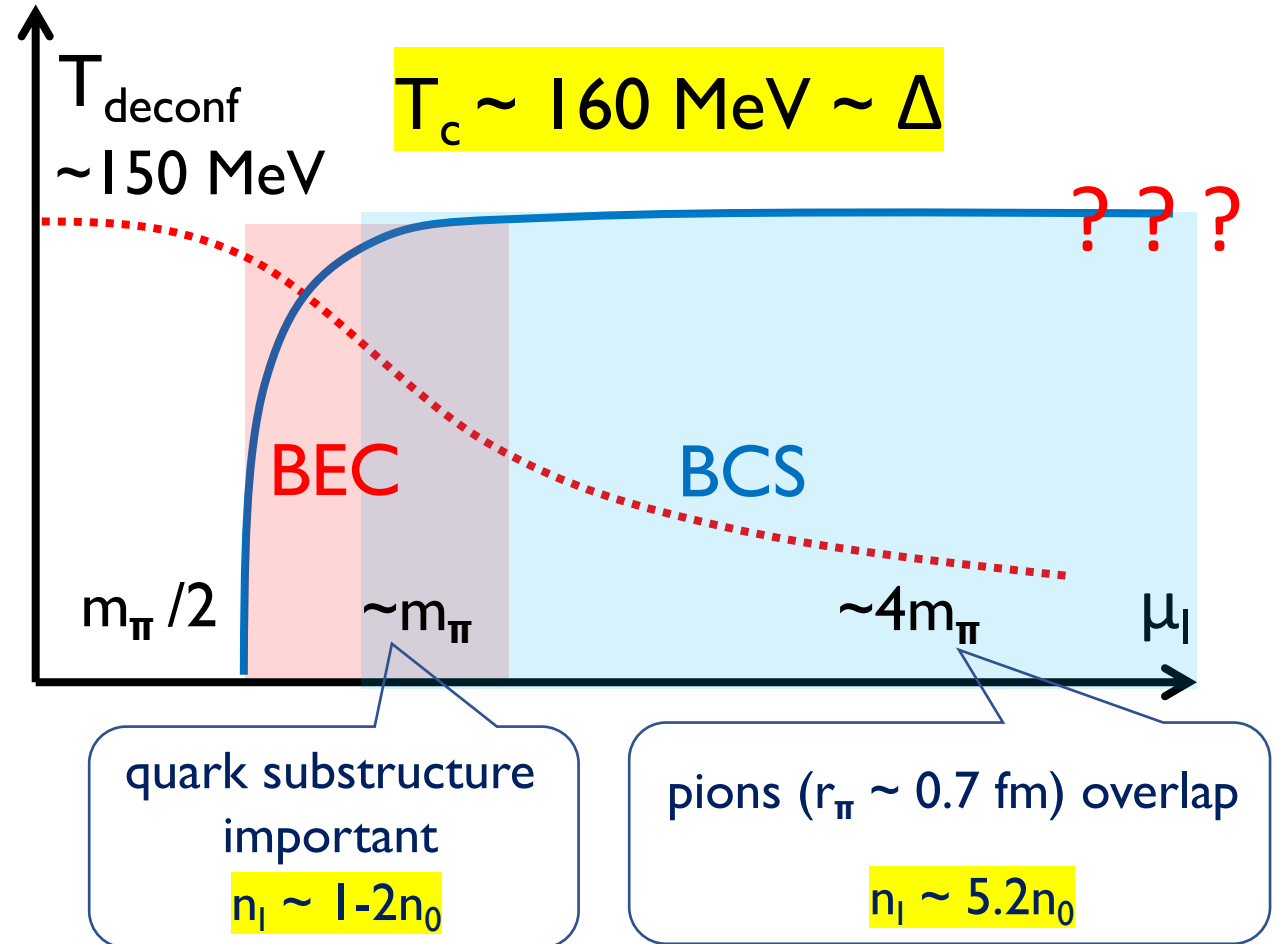
high density



pairing \rightarrow pion cond. π^+ (p-a) & π^- (h-ah)
 \rightarrow quark mass gap Δ

$\sim T_c / 0.57 \sim 280 \text{ MeV}$

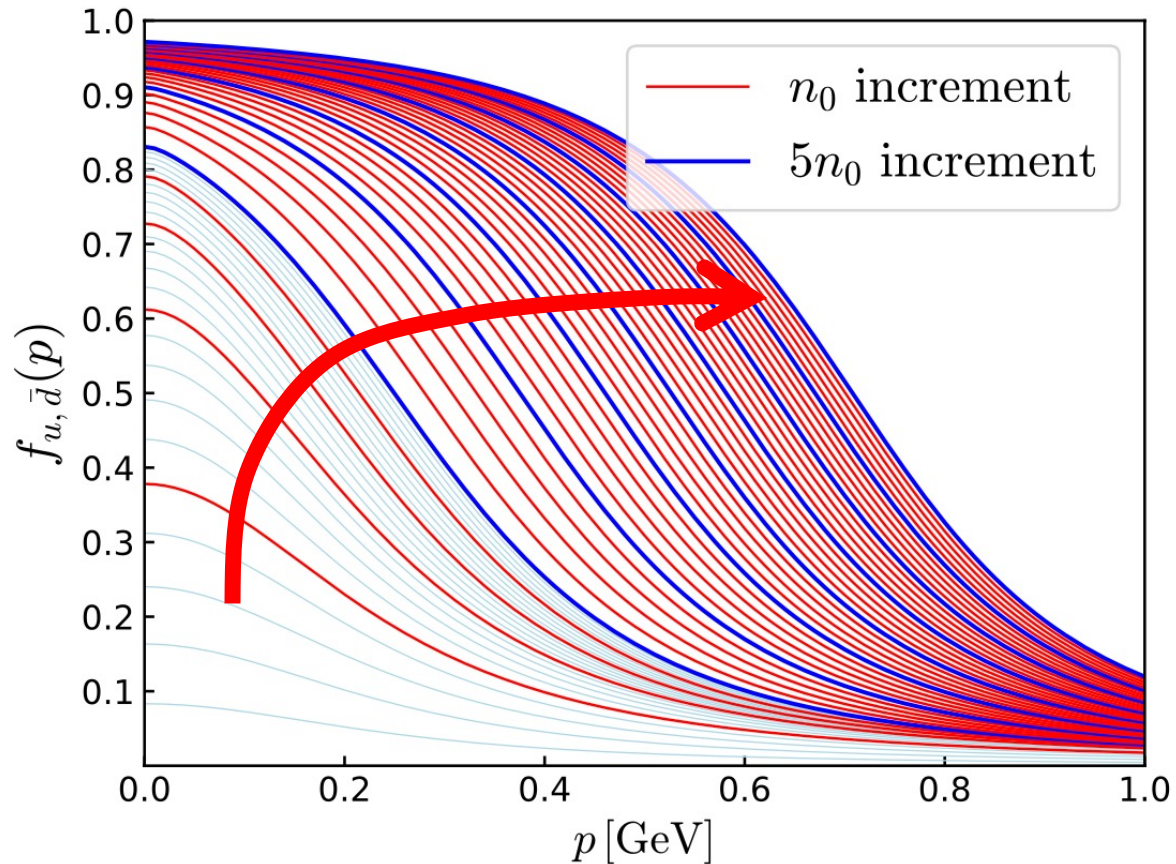
[see also Fujimoto '24]



Hints for new scale & saturation

- BCS occ. probability:

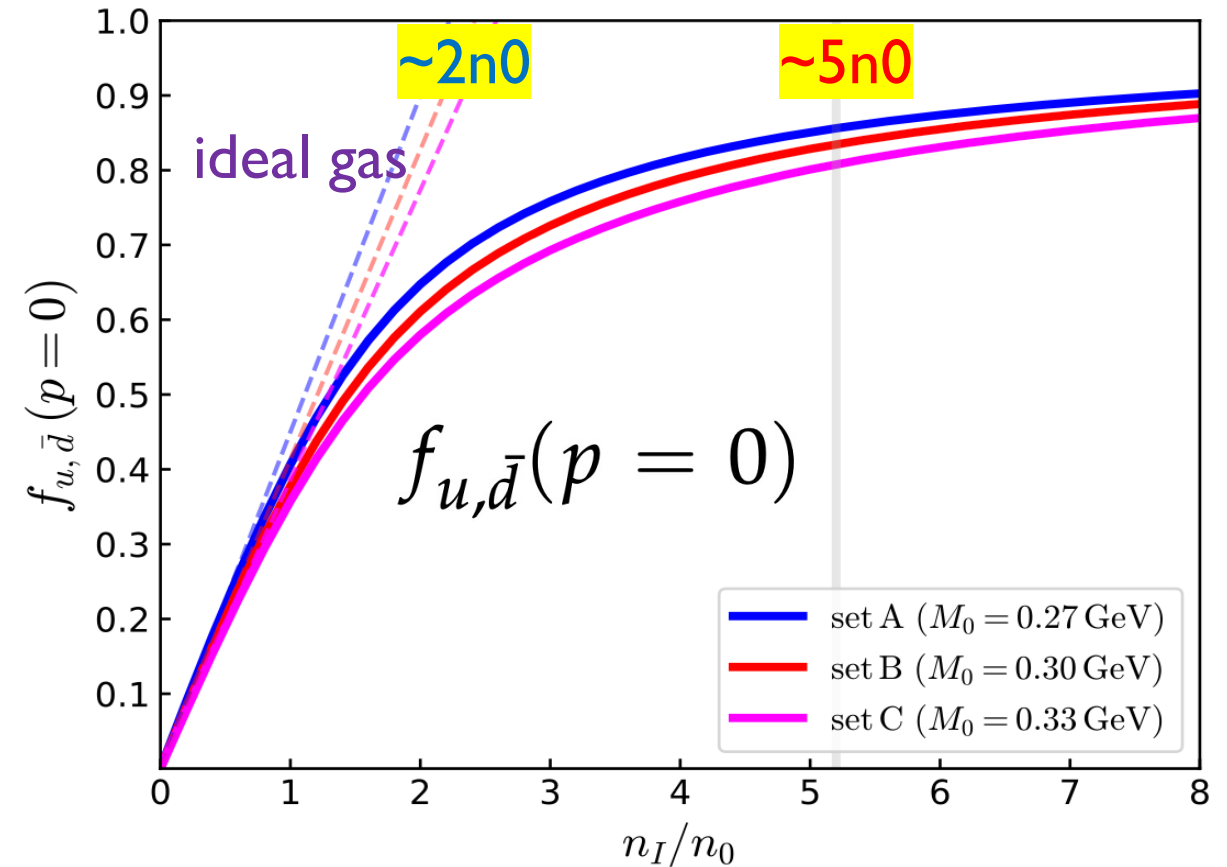
$$f_{u,\bar{d}}(p;n_I) = \frac{1}{2} \left(1 - \frac{E_l - \mu_I}{\sqrt{(E_l - \mu_I)^2 + \Delta^2}} \right)$$



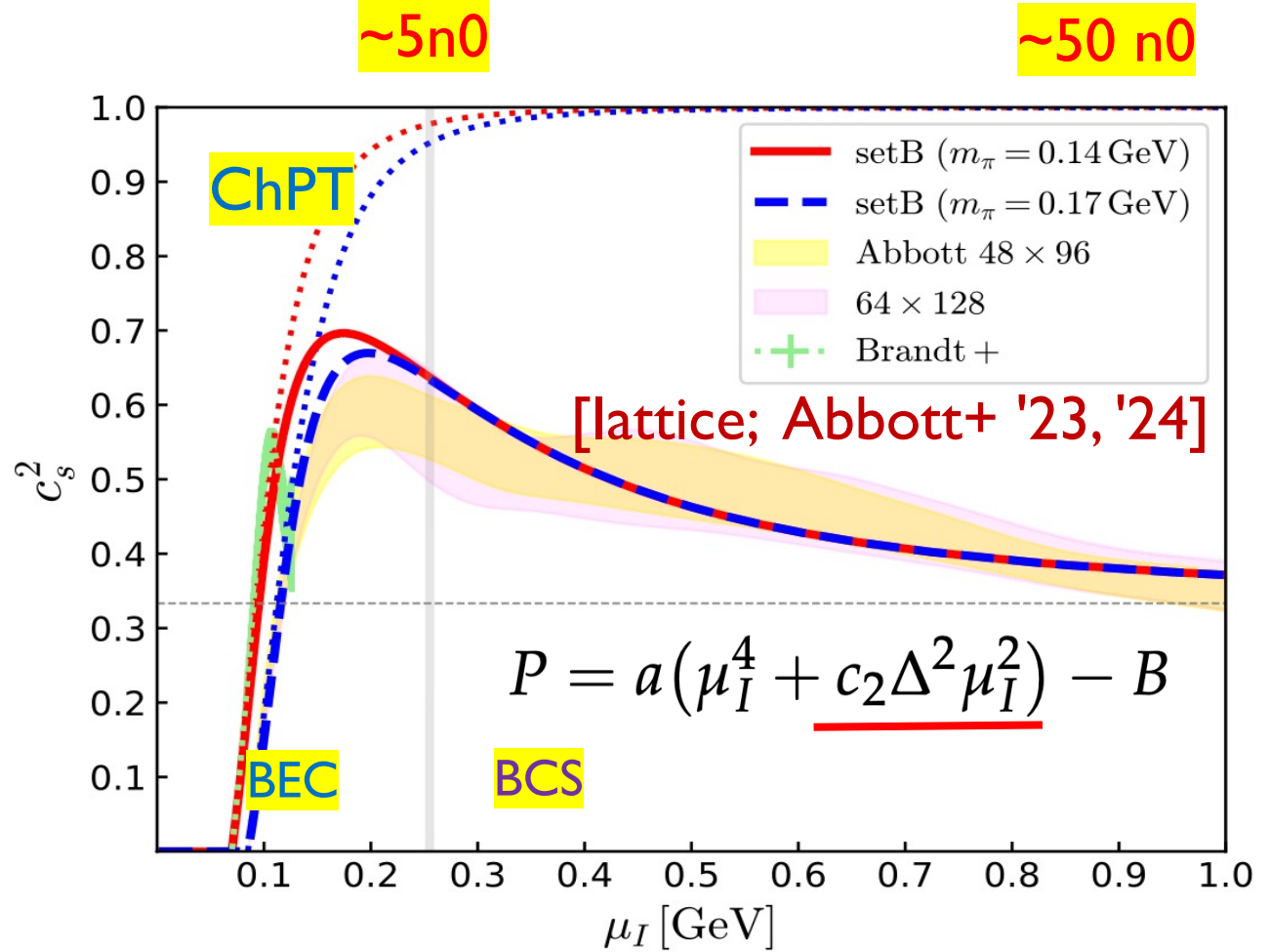
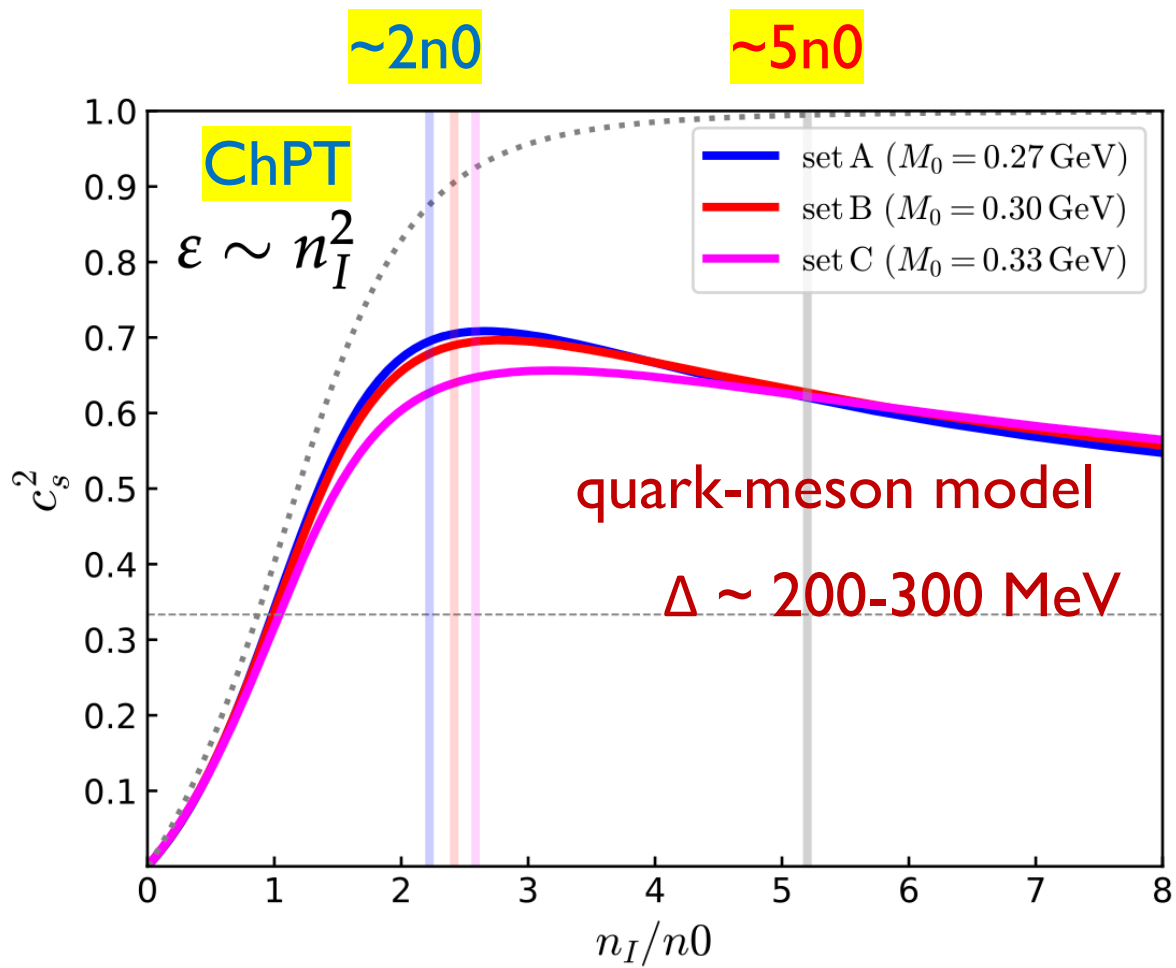
[quark-meson model study, Chiba+ '23; TK+ '24; ...]

ideal pion gas pic.
definitely violated

pions with
 $r \sim 0.66$ fm overlap



Sound speed: quark-meson model, ChPT, and Lattice



see also two-color QCD, Iida-Itou-Murakami-... ('22, '23, '24)

[model study; TK-Suenaga-Chiba '24]

Summary (of claims)

- *nuclear* + $R_{1.4} \sim R_{2.1}$ → *early* stiffening at $\sim 0.5 n^{\text{overlap}}$
- *inevitable* quark Fermi sea formation at $\sim 0.5 n^{\text{overlap}}$
- *quark saturation* changes the trend of EOS; *disparity* → c_s^2 peak
- Quarks can be agents of rapid stiffening:
 - can be microscopic origin of many-body repulsion
 - nontrivial constraints on the baryon dynamics
- Quarks can be important near nuclear saturation densities, $1-2n_0$
 - Larry's talk tomorrow

Back Up

Summary (of claims)

- Quarks can be agents of rapid stiffening:
 - can be microscopic origin of many-body repulsion
 - nontrivial constraints on the baryon dynamics
- Quarks can be important near nuclear saturation densities, $1-2n_0$
 - see Larry's talk tomorrow
- Quark substructure of hadrons are important

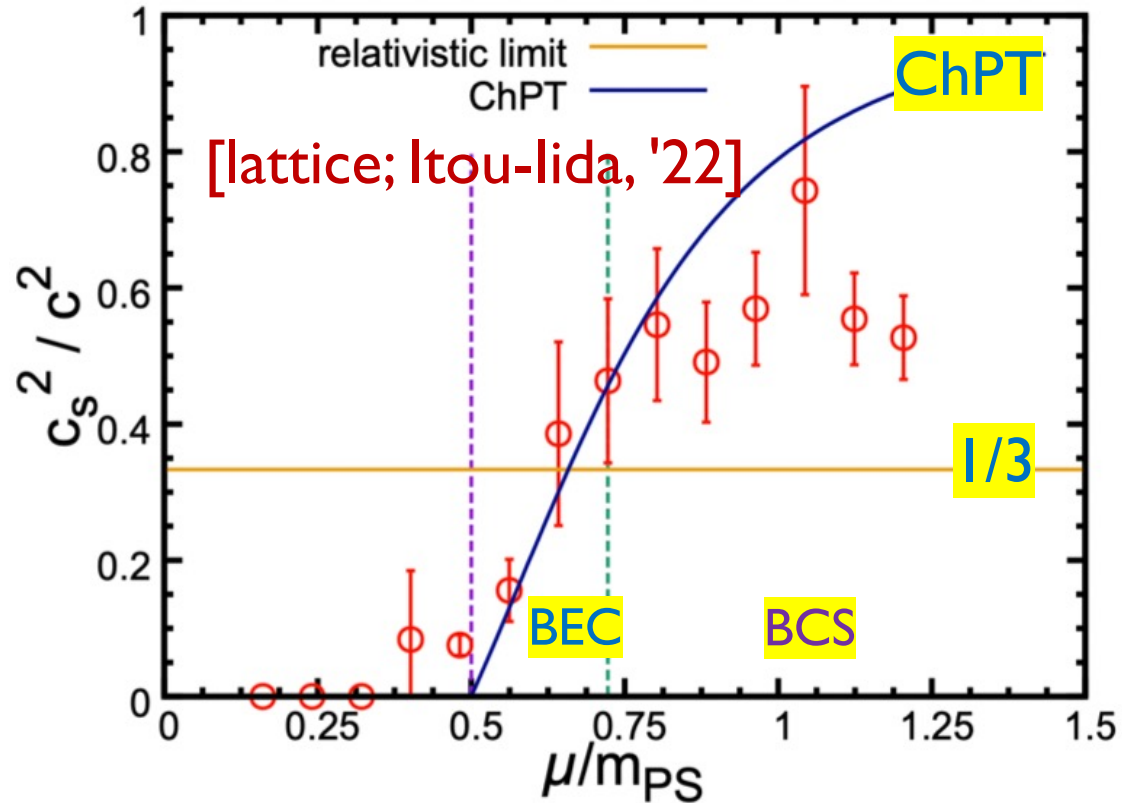
Summary (of claims)

- *nuclear* + $R_{1.4} \sim R_{2.1}$ \rightarrow *early* stiffening at $\sim 0.5 n^{\text{overlap}}$
(beyond simple nuclear regime)
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- *quark saturation* changes the trend of EOS; *disparity* $\rightarrow c_s^2$ peak
- *attractive* correlations near the Fermi surface stiffen EOS
- *unreasonable*(?) effectiveness of *quasi-particle* model in isospin QCD
(as in constituent quark models in hadron spectroscopy)

Examples from QCD-like theories

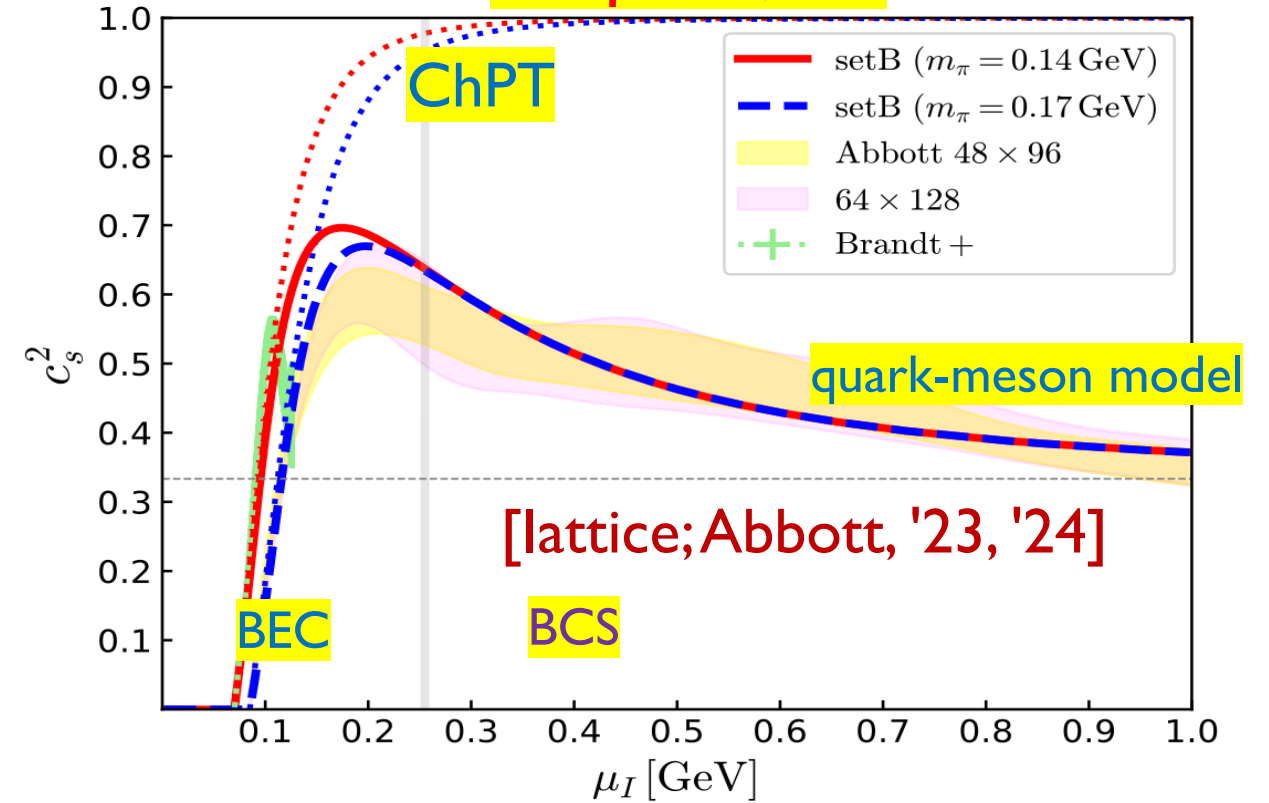
tests on the **lattice**

2-color QCD



[model study, TK-Suenaga '21]

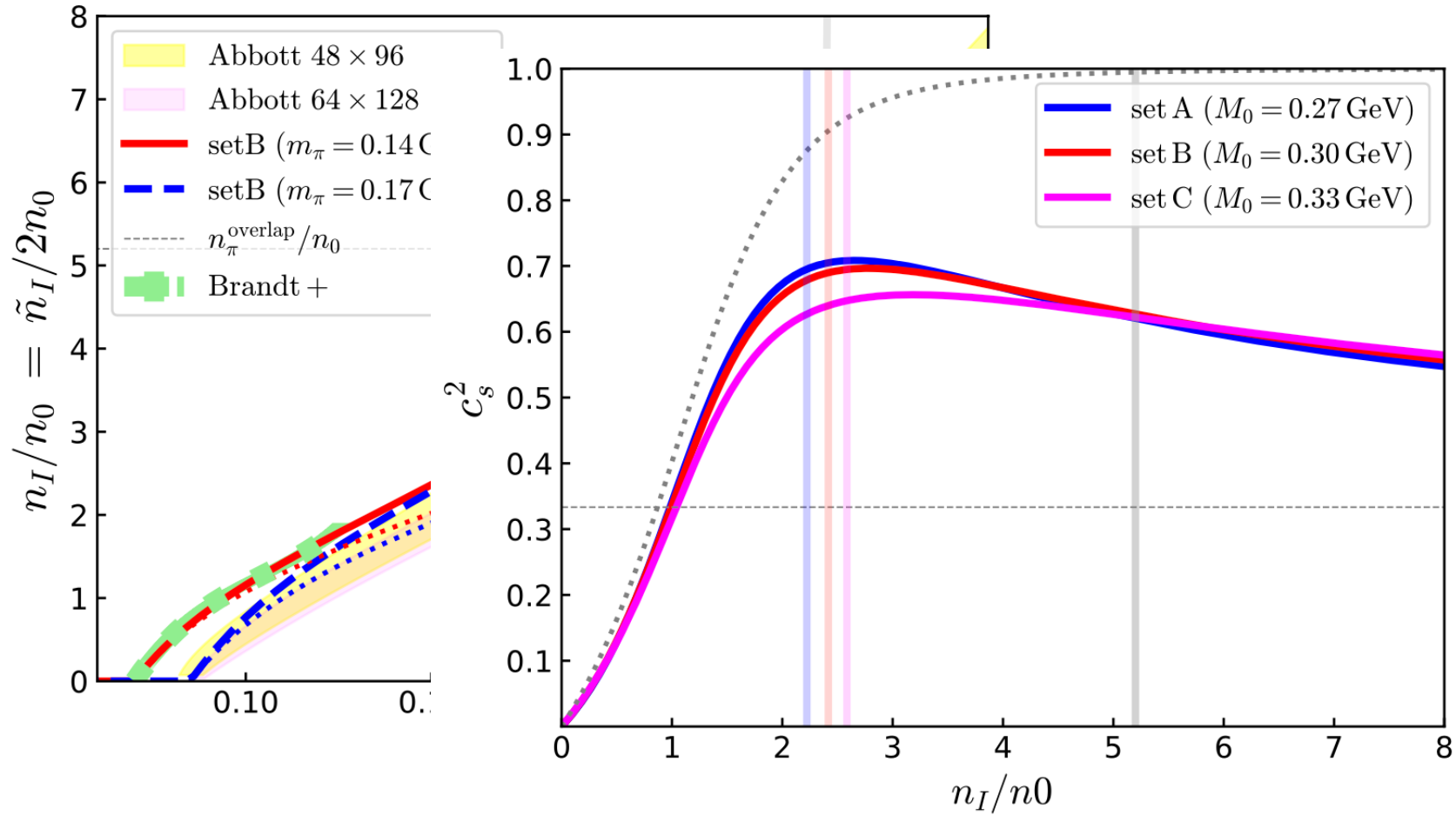
isospin QCD



[model study, Chiba-TK '23; TK-Suenaga-Chiba '24]

Peak in the BEC-BCS type crossover

Hints for new scale & saturation



Back Up

Stiff quark matter

The appearance of c_s^2 peak is **characteristic** in the QHC scenarios:

good baseline, but **NOT necessarily sufficient** for $\sim 2.1-2.3M_\odot$ NS.

(just after the crossover, quarks are **not fully relativistic**.)

Can the **chiral restoration** stiffens EOS by making quarks relativistic?

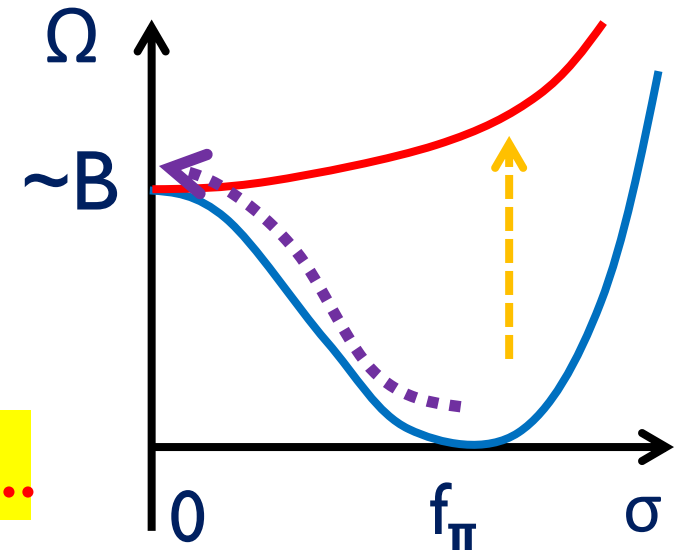
Unlikely: “the bag constant” from the Dirac sea

$$\varepsilon \rightarrow \varepsilon + B$$

$$P \rightarrow P - B$$

significant softening!

At this stage, we begin to discuss **interactions...**



Parametric analyses

[TK-Powell-Song-Baym, '14; QHCl8, 19, 21]

$$\begin{array}{c} \text{rela. kin. energy} \\ \varepsilon(n) = an^{4/3} + \text{interactions} \\ \quad \quad \quad \underline{bn^\alpha} \end{array} \quad \longrightarrow \quad \begin{array}{c} \text{ideal} \\ P = \frac{\varepsilon}{3} + \text{interactions} \\ \quad \quad \quad \underline{b} \left(\underline{\alpha} - \frac{4}{3} \right) n^\alpha \end{array}$$

(n : quark density)

For **stiff** EOS:

(for large P)

for $\alpha > 4/3$:

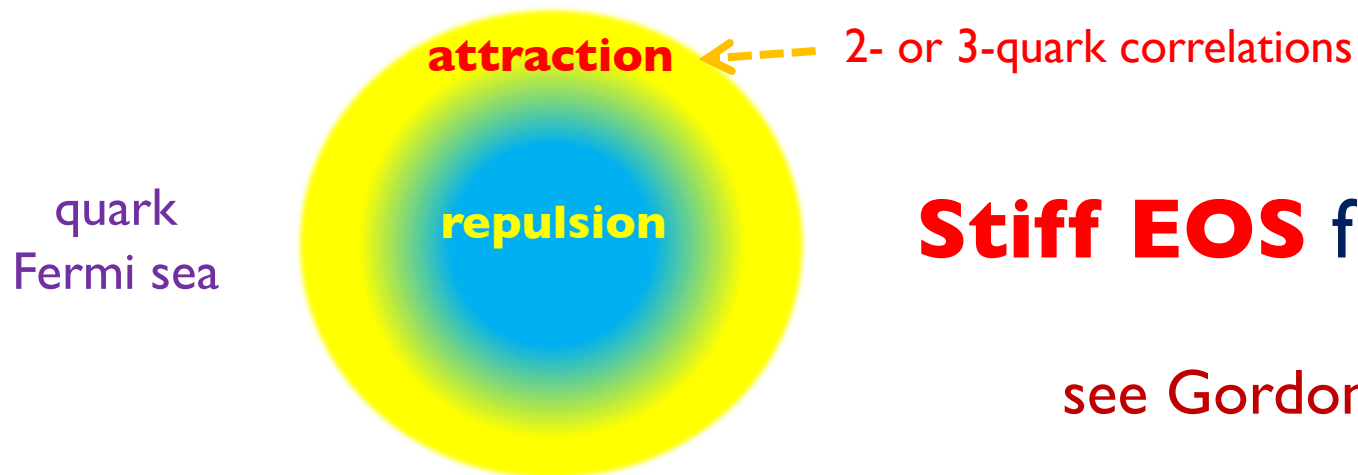
$b > 0$

(e.g. bulk **repulsion**, $\sim + n_B^2/\Lambda^2$)

for $\alpha < 4/3$:

$b < 0$

(e.g. surface **pairings**, $\sim - \Lambda^2 n_B^{2/3}$)



Stiff EOS from **attractive forces**

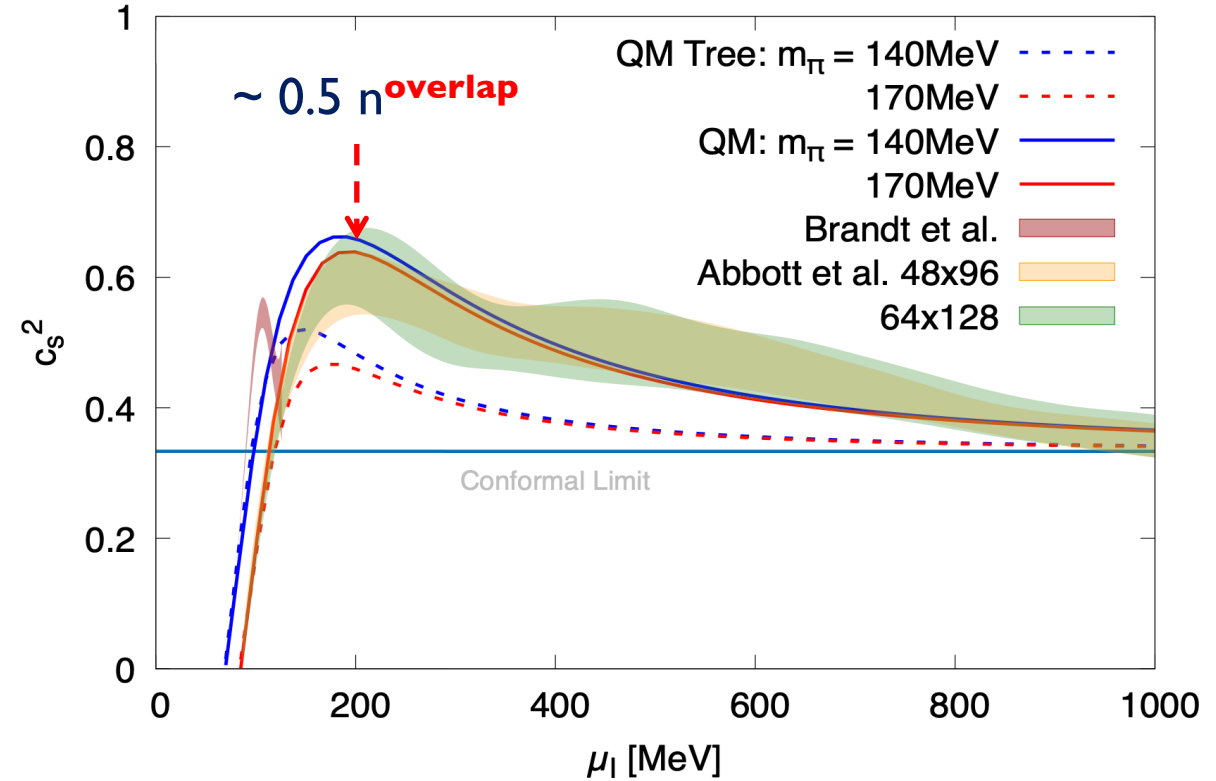
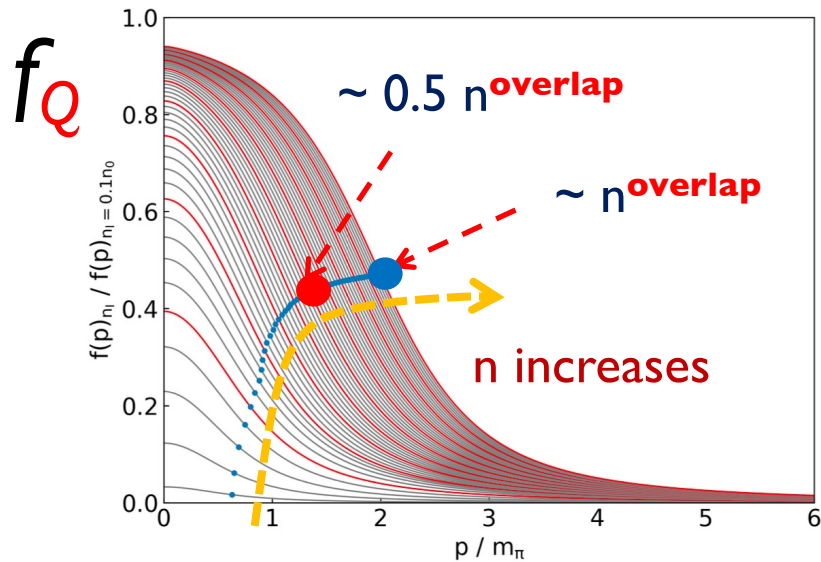
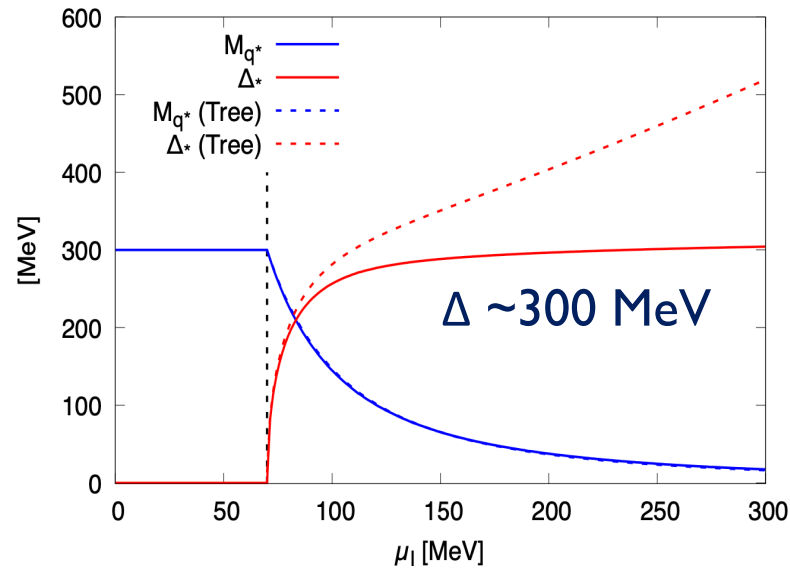
see Gordon-san's slides in the last week

e.g. isospin QCD

quark-meson model vs lattice

[Adhikari+ '17; Chiba-TK '23, ...]

[Brandt+ '22; Abbott+ '23, ...]



- $\Delta \sim 300$ MeV (!) $T_c^{\text{QM}} \sim 180$ MeV vs $T_c^{\text{lat}} \sim 170$ MeV
- $\Delta > \sim 100$ MeV $\rightarrow c_s^2 > 1/3$ at $\mu_q \sim 1$ GeV and beyond
- c_s^2 peak found at $n \sim 0.5 n$ overlap

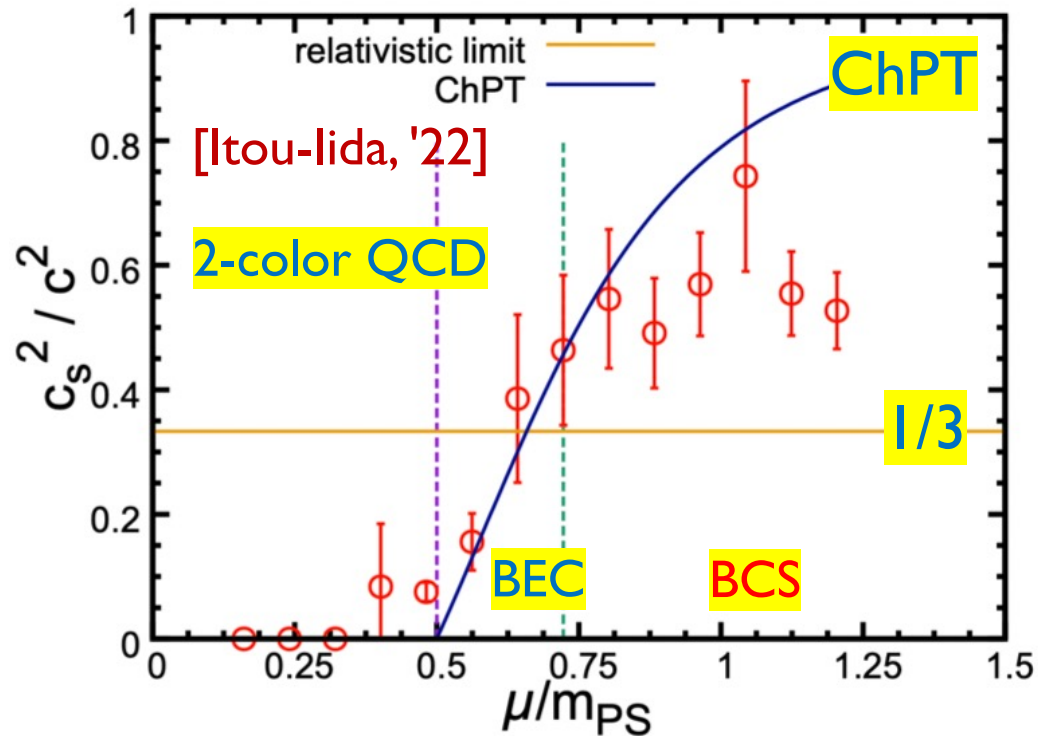
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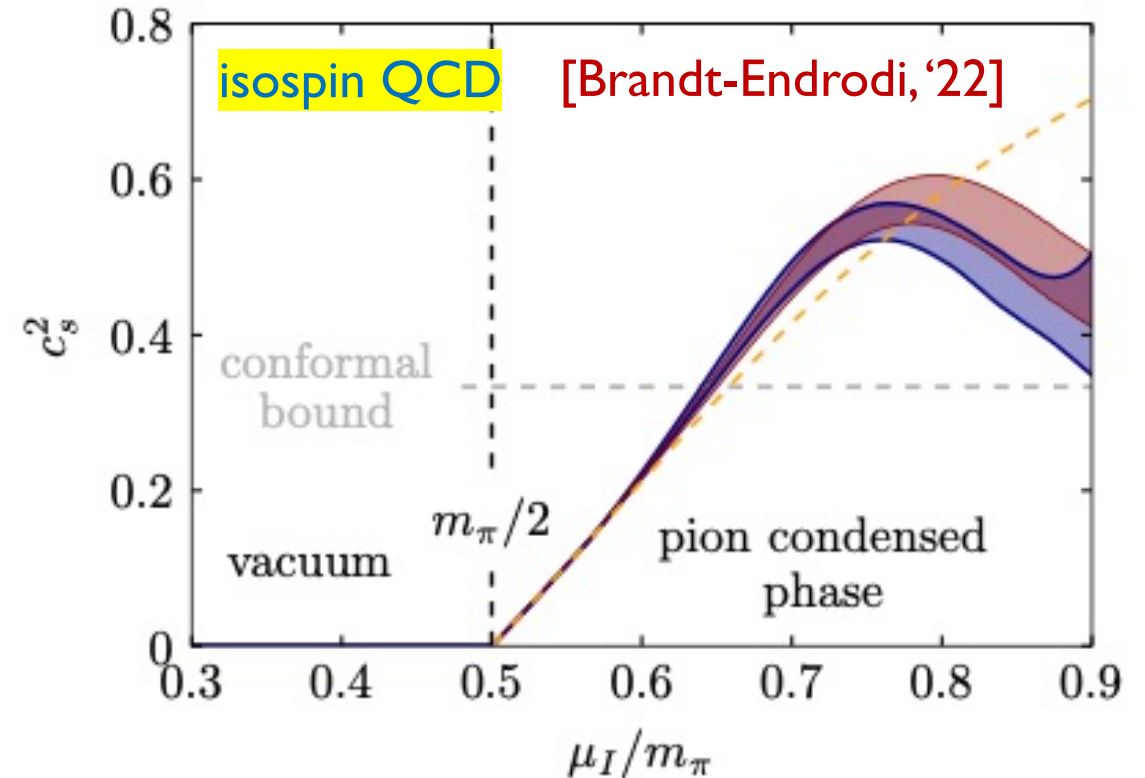
Updated

Examples from QCD-like theories

tests on the **lattice**



[model study, TK-Suenaga '21]



Peak in the BEC-BCS type crossover

EoS with interactions

cf) [TK '21, TK-Suenaga '21]

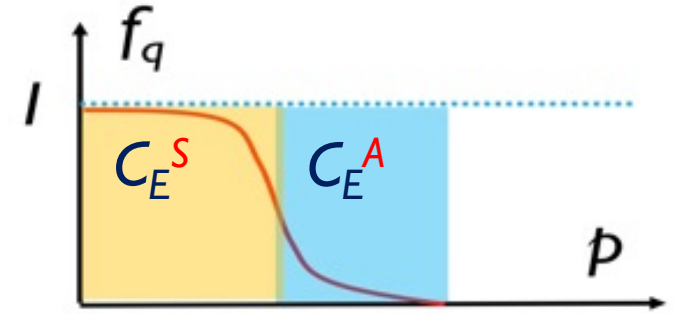
e.g.,
$$\mathcal{V}[f_Q] = -\underline{C_E^A} [1 - (f_Q)^n] + \underline{C_E^S} (f_Q)^n$$

→ I (dilute)

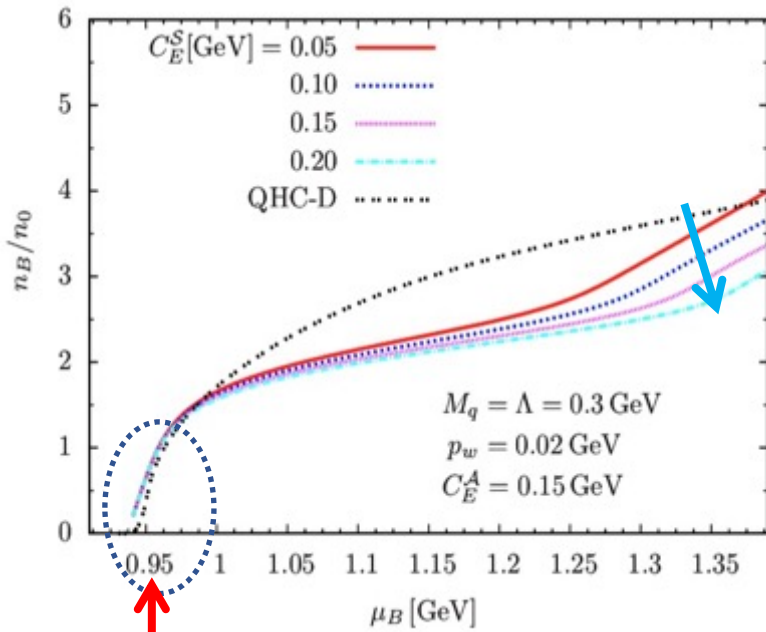
→ 0 (dilute)

→ 0 (dense)

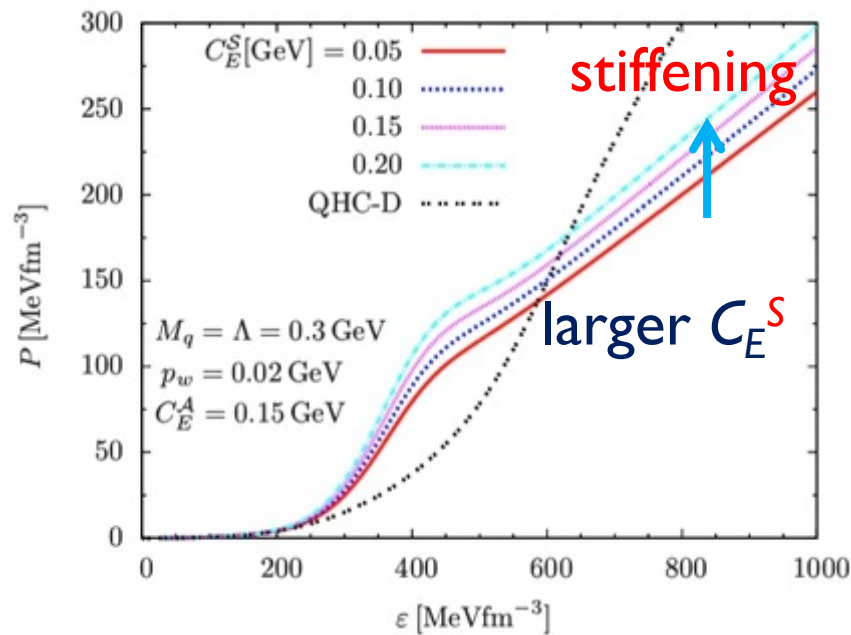
→ I (dense)



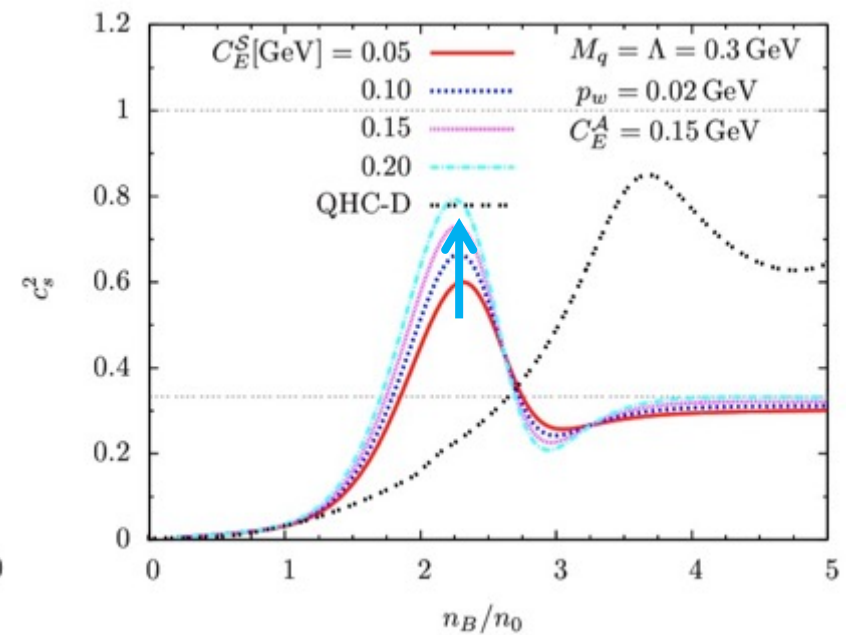
repulsive attractive



adjust C_E^A (fit $M_B = 939$ MeV)



high density stiffening



stronger peak in c_s

Underlying picture (guess)

- *Gluons remain non-perturbative at $5-10n_0$*

(see, e.g., lattice results for 2-color & isospin QCD)

- *Chiral restoration occurs mildly*

implicitly included
in IdylliQ type models

- *Continuity:* interactions in quark matter should have
natural counterpart in hadron physics

Short range correlations in a baryon:

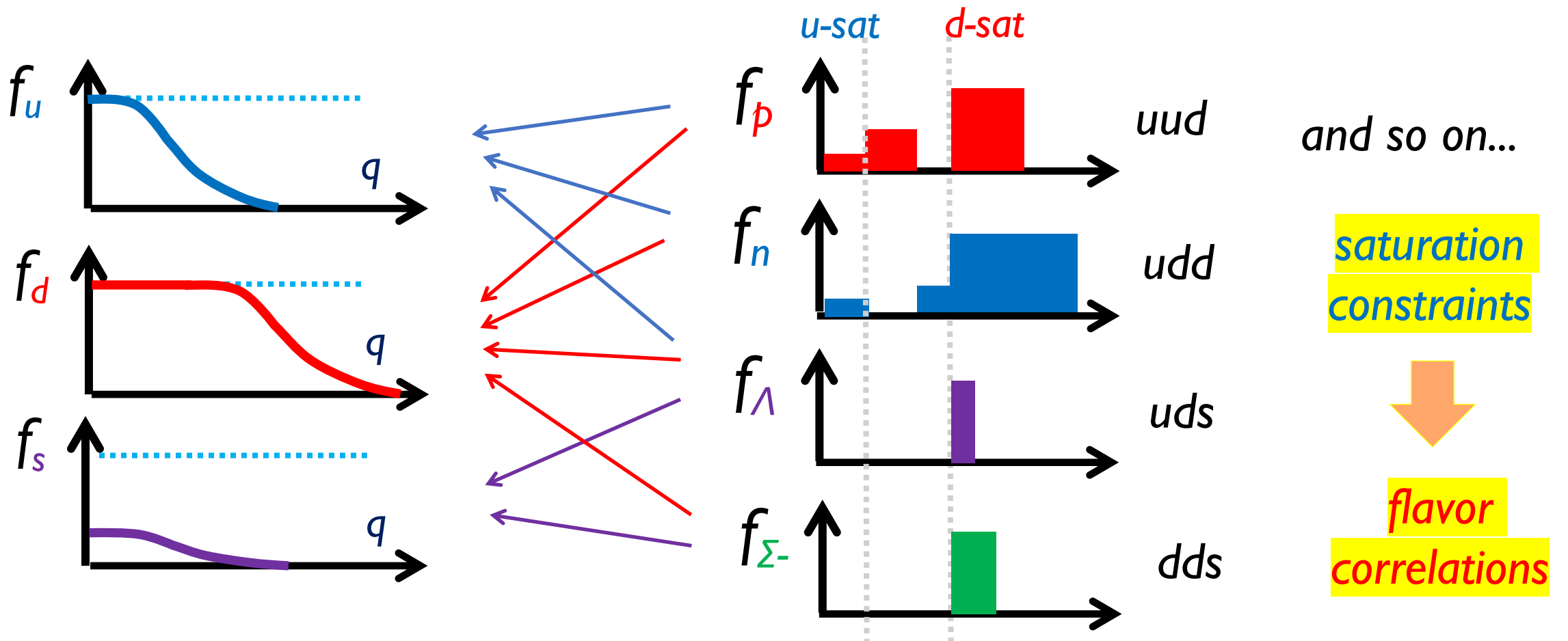
my favorite: *color-electric & magnetic interactions*

Multi-hadron extension

[Fujimoto-TK-McLerran, in prep.]

$$f_Q(\mathbf{q}) = \sum_{B=p,n,\Sigma,\dots} N_Q^B \int_{\mathbf{k}} f_B(\mathbf{k}) \varphi\left(\mathbf{q} - \frac{\mathbf{k}}{N_c}\right)$$

$Q = u, d, s$



Summary

- **Soft-to-stiff** EOS
- quark-exchange and **soft-deconfinement**
- **quark saturation effects**, can be relevant at **$2-3n_0$** (!)
- **not addressed** (please ask in Q&A or personally) :

relevant interactions at high density

hyperon puzzle

finite temperature effects

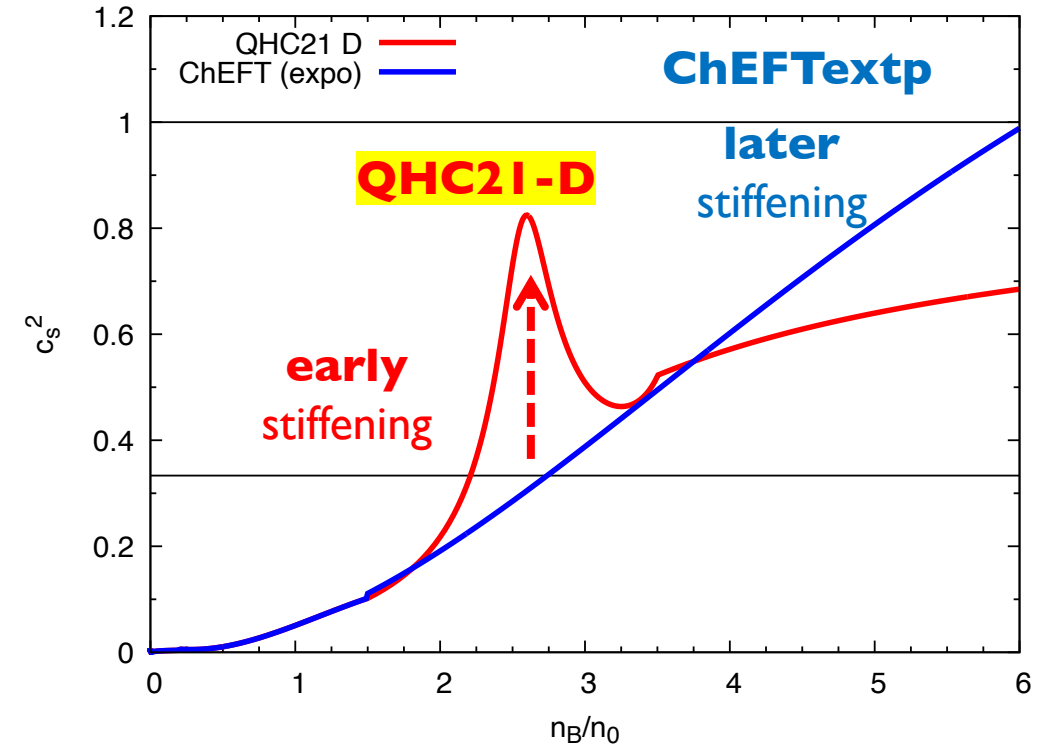
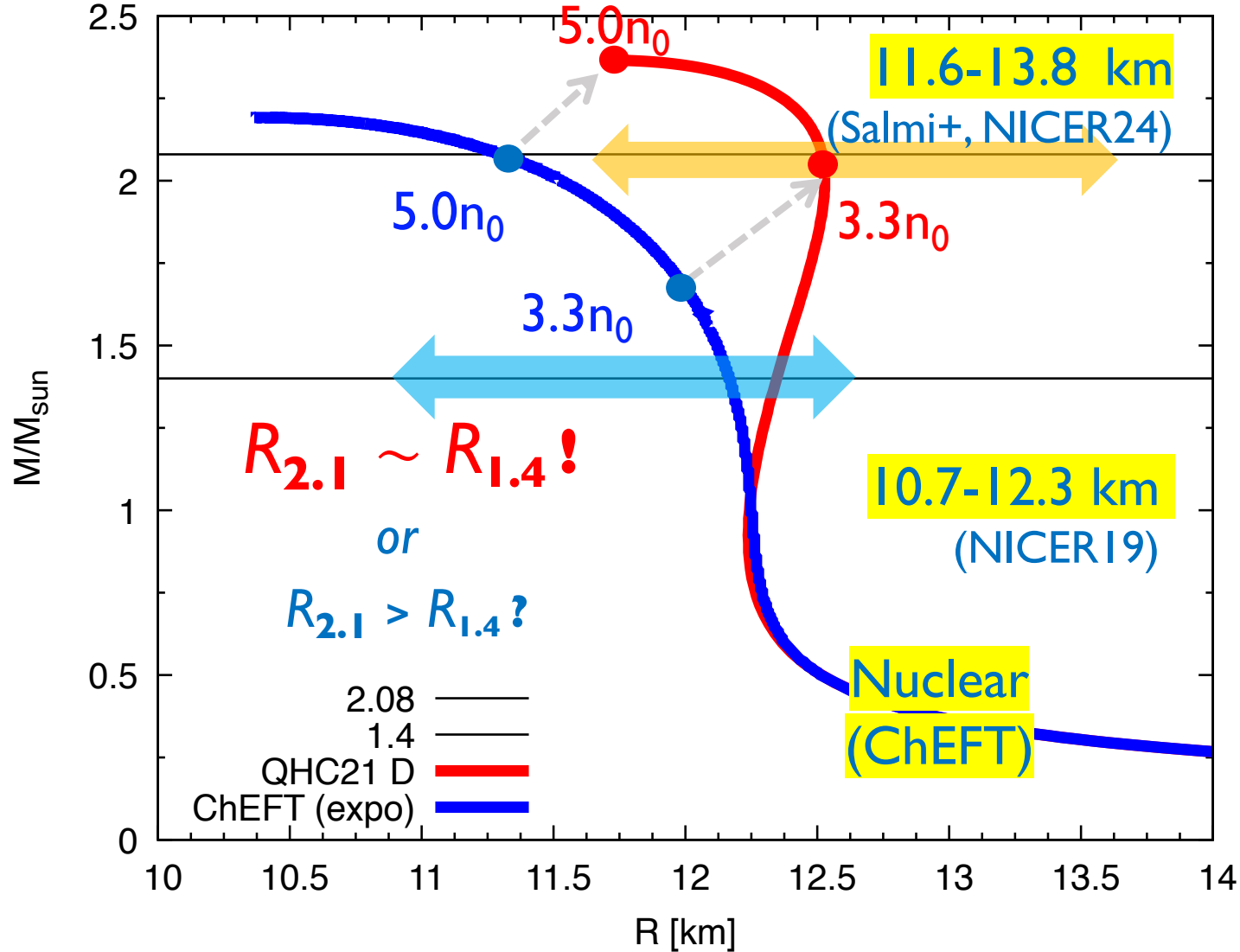
proof of concept: isospin QCD in lattice simulations

....

Early vs later stiffening:

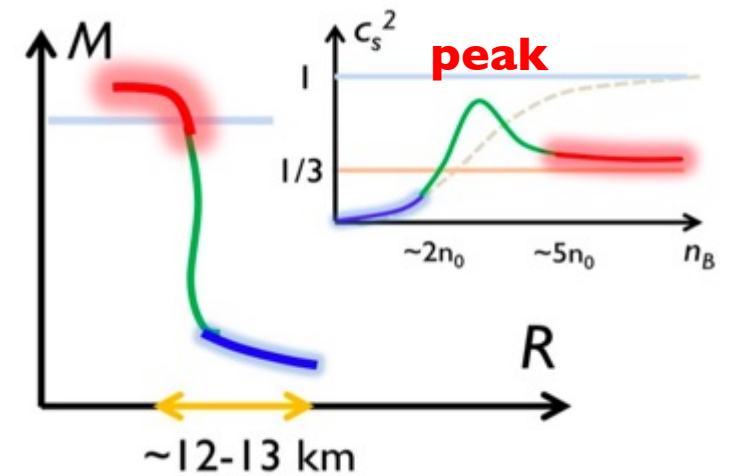
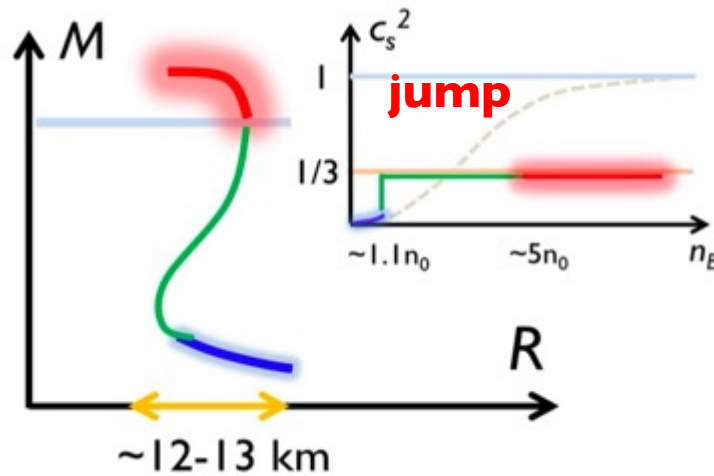
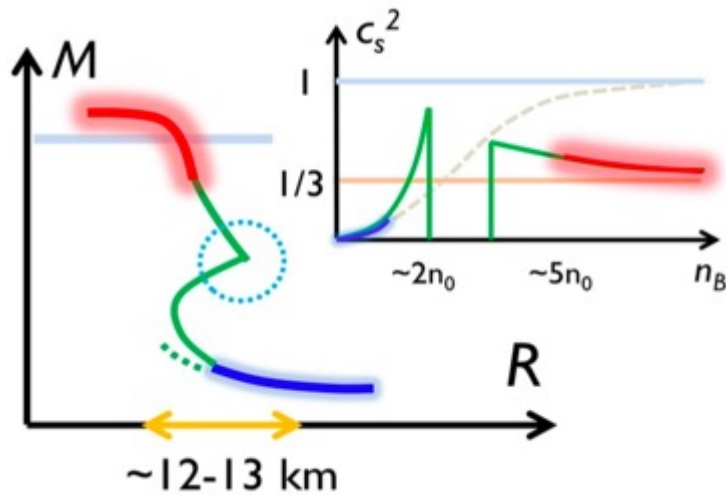
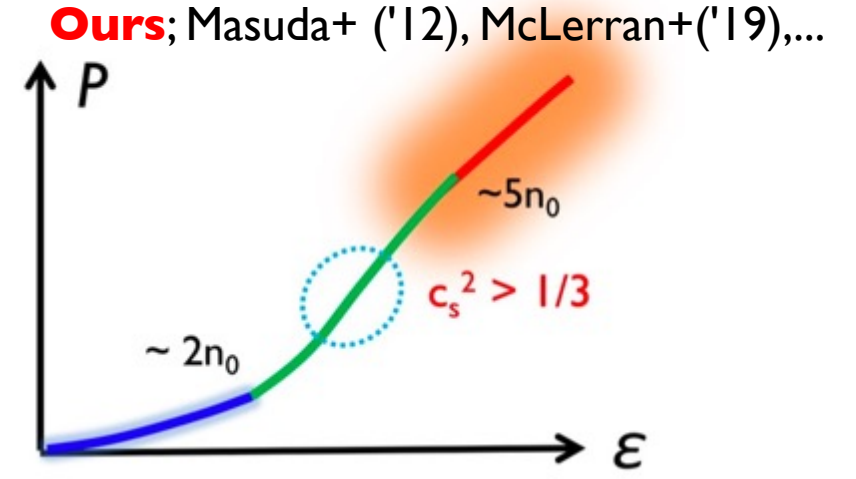
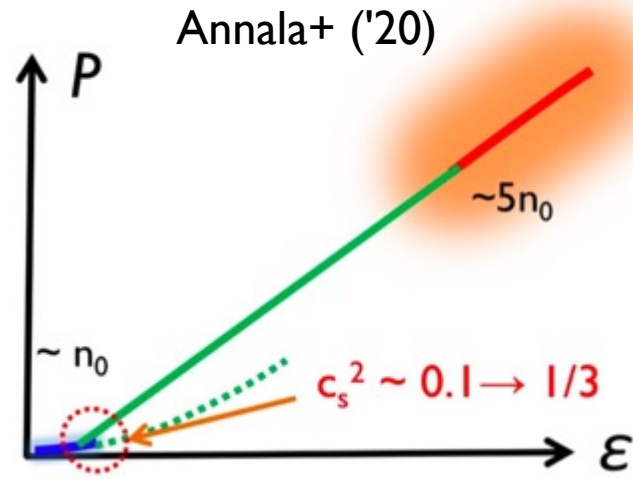
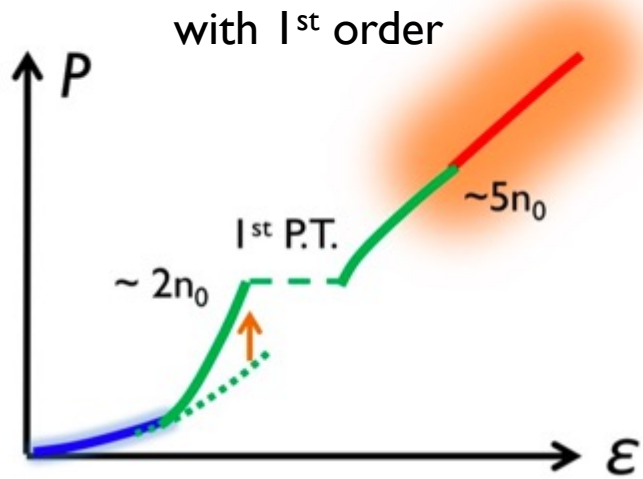
QHC21 vs ChEFTexp

[TK-Hatsuda-Baym '21]



2-3 n_0 : already beyond
 purely nucleonic regime?

Three possible scenarios



$\rightarrow R_{1.4}$ and $R_{2.1}$?

\rightarrow nuclear physics ?

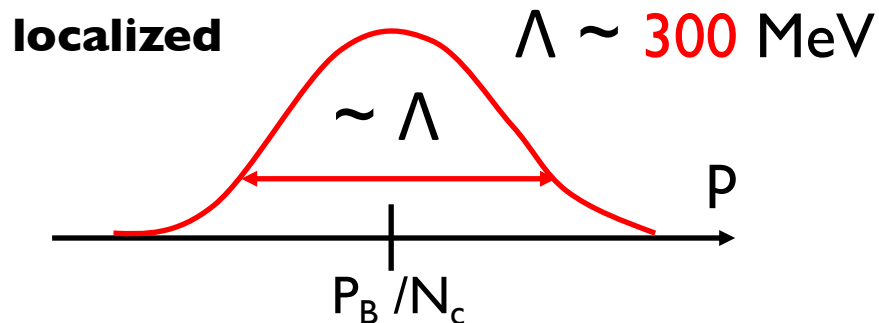
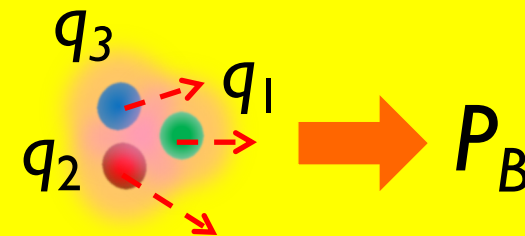
\rightarrow my favorite

Quarks in a baryon

$N_c (=3)$: number of colors

probability density:

$$\varphi(\mathbf{q}; \mathbf{P}_B) = \mathcal{N} e^{-\frac{1}{\Lambda^2} \left(\mathbf{q} - \frac{\mathbf{P}_B}{N_c} \right)^2}$$



variance: $\left\langle \left(\mathbf{p} - \frac{\mathbf{P}_B}{N_c} \right)^2 \right\rangle \sim \Lambda^2$ **energetic !**

→ large **“mechanical”** pressure

$$\langle E_q(\mathbf{p}) \rangle_{\mathbf{P}_B} = \mathcal{N} \int_{\mathbf{p}} E_q(\mathbf{p}) e^{-\frac{1}{\Lambda^2} \left(\mathbf{p} - \frac{\mathbf{P}_B}{N_c} \right)^2} \simeq \underbrace{\langle E_q(\mathbf{p}) \rangle_{\mathbf{P}_B=0}}_{\times N_c} + \frac{1}{6} \underbrace{\left\langle \frac{\partial^2 E_q}{\partial p_i \partial p_i} \right\rangle_{\mathbf{P}_B=0}}_{\times N_c} \left(\frac{\mathbf{P}_B}{N_c} \right)^2 + \dots$$

average energy (quark)

↓ $\times N_c$

$$\sim N_c (M_q + E_{\text{kin}})$$

baryon mass

\gg

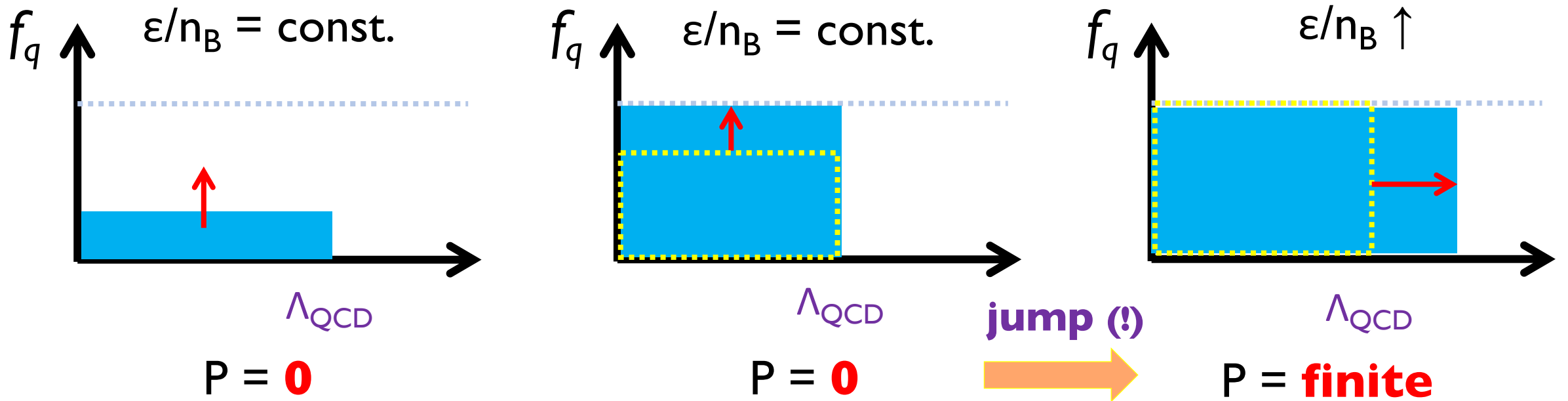
↓ $\times N_c$

$$\sim P_B^2 / (N_c E_q)$$

baryon kin. energy

Jump in pressure : schematic picture

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B} \left(\frac{\varepsilon}{n_B} \right) \quad \text{energy per particle}$$



f_q continuous \rightarrow ε, n_B are continuous

Quarks do contribute to ε even before saturation; but *to P only after the saturation!!*

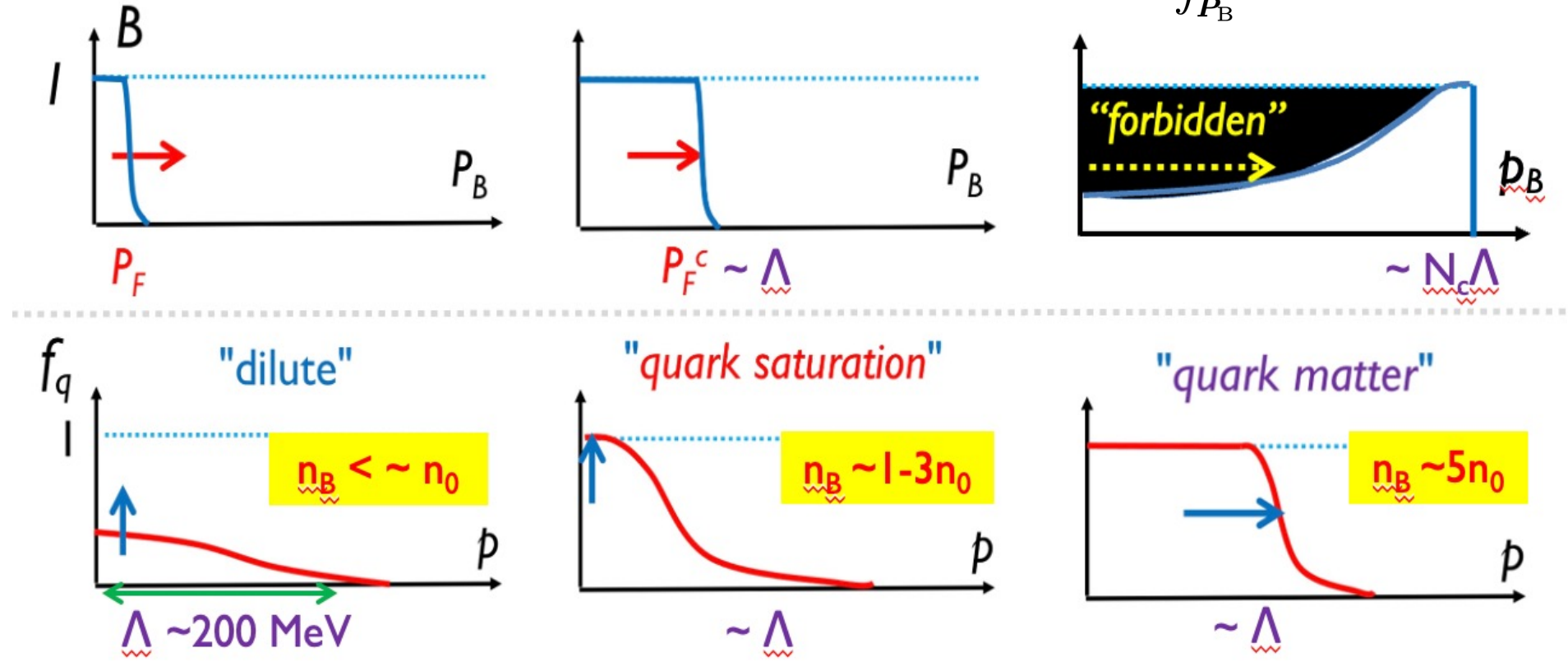
Evolution of occ. probabilities

$$f_Q(\mathbf{q}; n_B) = \int_{P_B} f_B(P_B; n_B) \varphi_Q^B(\mathbf{q}; P_B)$$

baryon bases

dual

quark bases



"quark saturation" constraint

→ **relativistic baryons at low density, $n_B \sim 1-3n_0$!**

cf) McLerran-Reddy model (2019); microscopic description, TK (2021)

Quantum numbers ?

quark quantum numbers; N_c , N_f , 2-spins (for a given spatial w.f.)

how many **baryon species** are needed to saturate quark states?

→ need only **$2N_f = 6$** species for $N_f = 3$

(full members of singlet, octet, decuplet are **NOT** necessary)

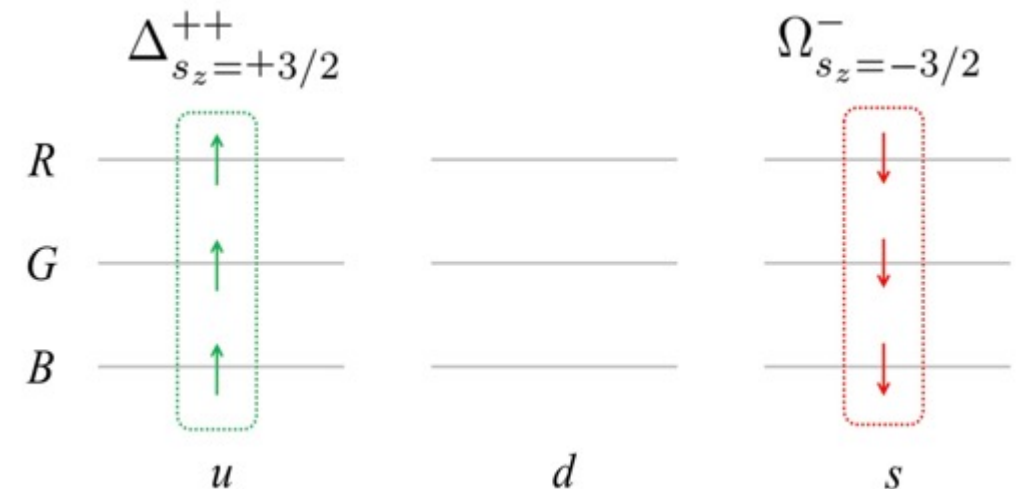
convenient **color-flavor-spin** bases

[neglect N- Δ splitting etc. for simplicity]

$$\Delta_{s_z=\pm 3/2}^{++} = [u_R \uparrow u_G \uparrow u_B \uparrow], [u_R \downarrow u_G \downarrow u_B \downarrow],$$

$$\Delta_{s_z=\pm 3/2}^- = [d_R \uparrow d_G \uparrow d_B \uparrow], [d_R \downarrow d_G \downarrow d_B \downarrow],$$

$$\Omega_{s_z=\pm 3/2}^- = [s_R \uparrow s_G \uparrow s_B \uparrow], [s_R \downarrow s_G \downarrow s_B \downarrow],$$



Underlying picture (guess)

- *Gluons remain non-perturbative at $5-10n_0$*

(see, e.g., lattice results for 2-color & isospin QCD)

- *Chiral restoration occurs mildly*

implicitly included
in IdylliQ type models

- *Continuity:* interactions in quark matter should have
natural counterpart in hadron physics

Short range correlations in a baryon:

my favorite: *color-electric & magnetic interactions*

Color-magnetic interaction play **many** roles

1) **Coupling** \propto **velocity** $\sim p/E$

become important in **relativistic regime & high density**

2) **Pairing**: strongly channel dependent

hadron mass ordering: N- Δ , etc. [DeRujula+ (1975), Isgur-Karl (1978), ...]

color-super-conductivity [Alford, Wilczek, Rajagopal, Schafer, ... 1998-]

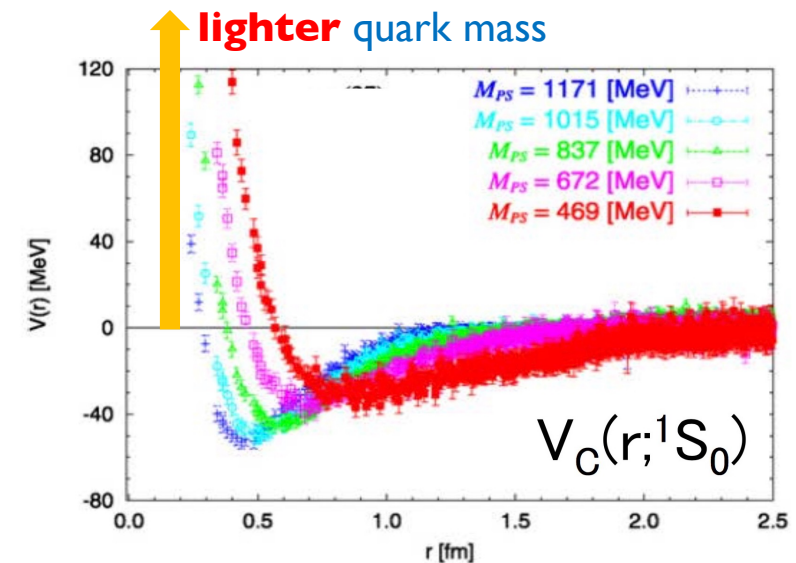
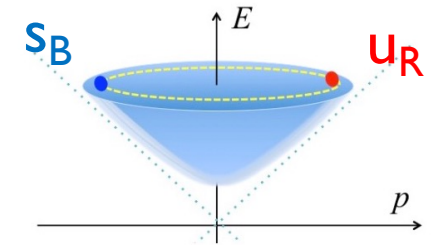
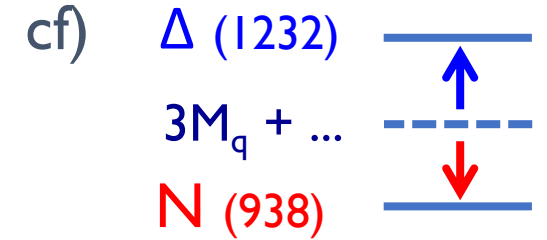
3) **Baryon-Baryon int.**: **short-range** correlation

(**Pauli + color-mag.**) [Oka-Yazaki (1980), ...]

channel dep. \rightarrow **non-universal** hard core (some are **attractive!**)

mass dep. \rightarrow **stronger** hard core in **relativistic** quarks

\rightarrow **consistent with the lattice QCD** [HAL-collaboration]

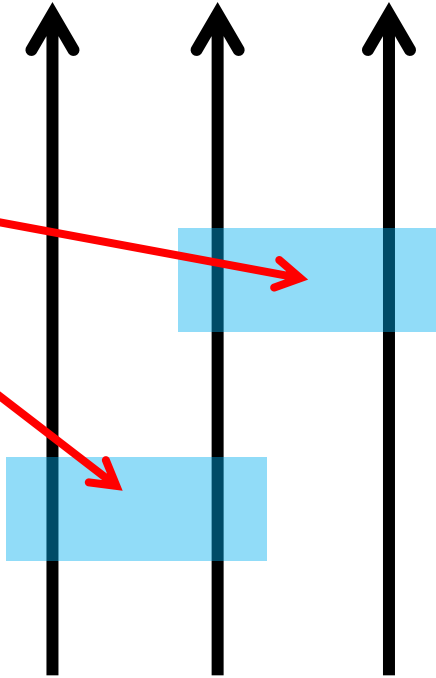


a baryon in dilute regime

(color-singlet)

(always) color-antisymmetric

(attractive electric int.)



e.g., nucleons

$$M_N \sim 3M_q + \underbrace{\text{kin.}} + \text{color-EM}$$

~ 940MeV

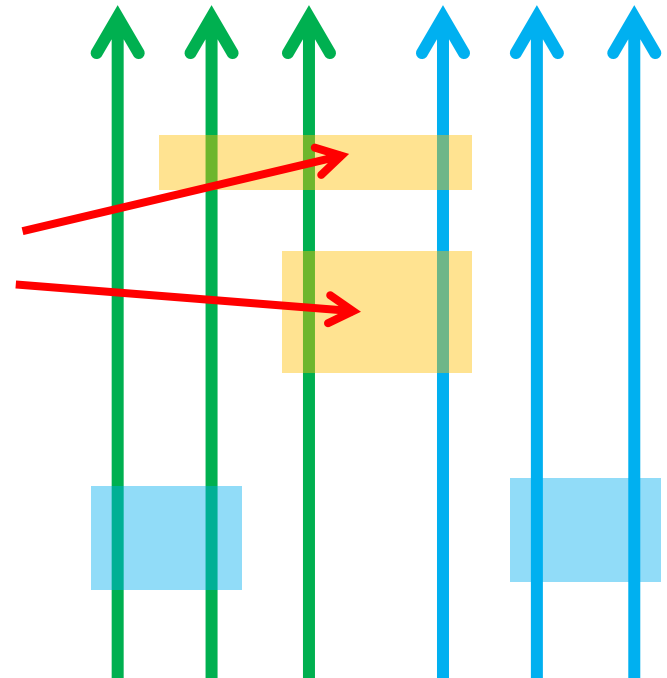
~ 1100MeV

~ -150-200MeV

in dense regime

sometimes color-symmetric

(repulsive)



more chances to feel repulsion

EoS with interactions

cf) [TK '21, TK-Suenaga '21]

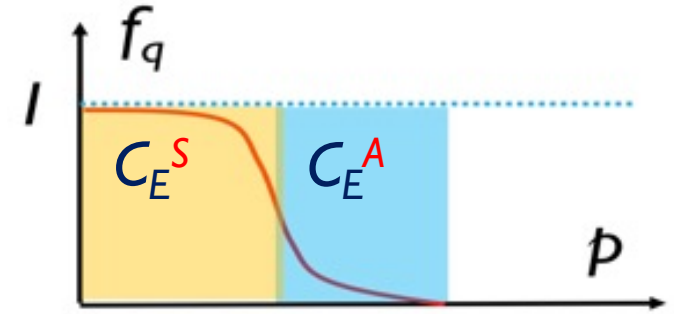
e.g.,
$$\mathcal{V}[f_Q] = -\underline{C_E^A} [1 - (f_Q)^n] + \underline{C_E^S} (f_Q)^n$$

→ I (dilute)

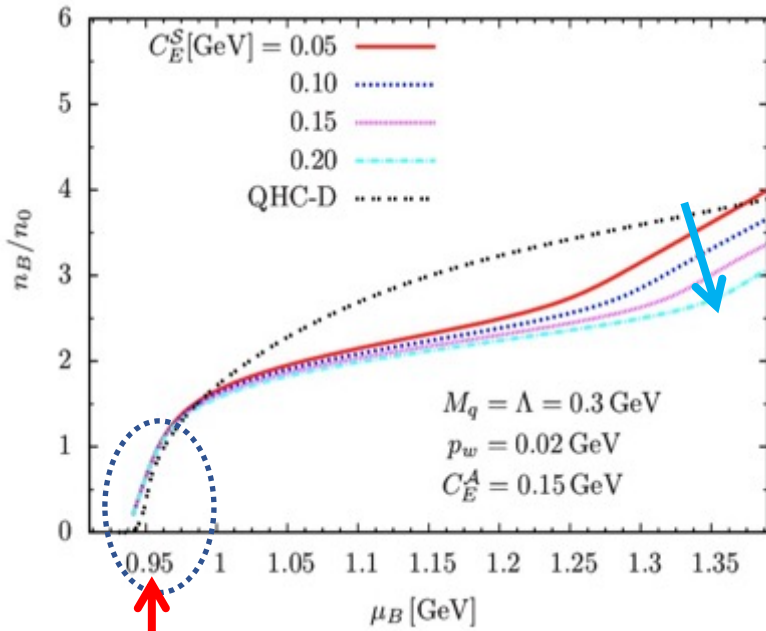
→ 0 (dilute)

→ 0 (dense)

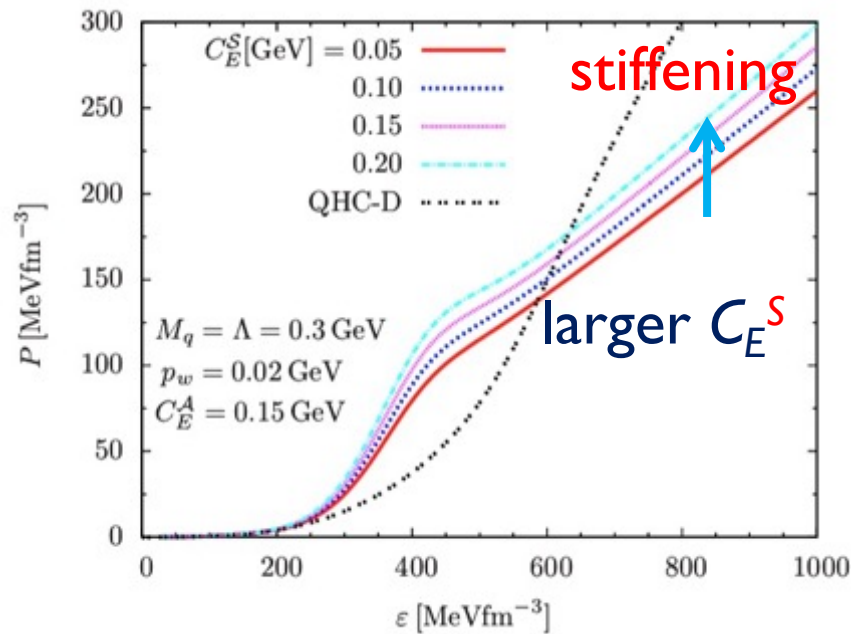
→ I (dense)



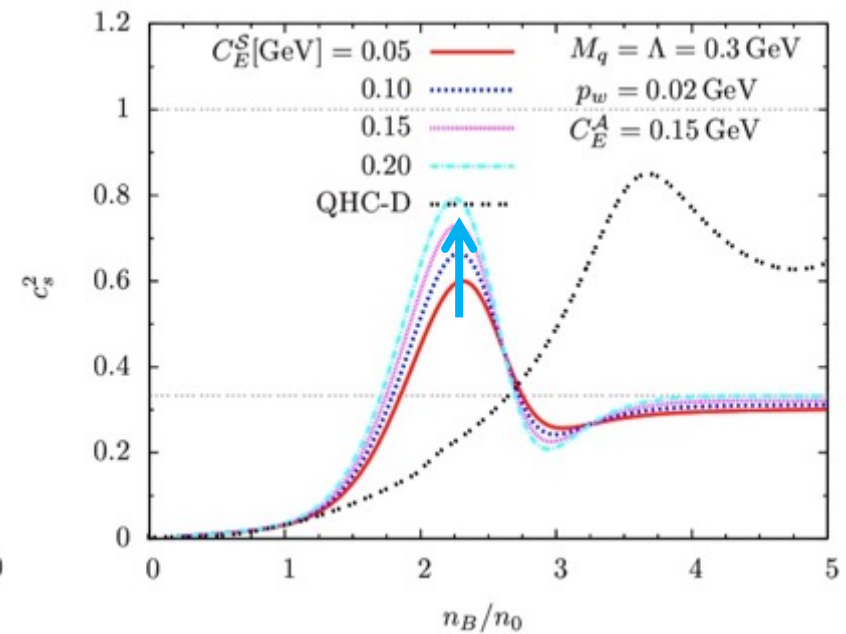
repulsive attractive



adjust C_E^A (fit $M_B = 939$ MeV)



high density stiffening



stronger peak in c_s

Important relations

sum rule

single baryon contain single R- or G- or B- quark

$$n_q^{R,G,B} = \int_{\mathbf{p}} f_q(p) = \int_{\mathbf{p}} \left(\int_{\mathbf{P}_B} \mathcal{B}(P_B) \underline{Q_{\text{in}}(\mathbf{p}; \mathbf{P}_B)} \right) = \int_{\mathbf{P}_B} \mathcal{B}(P_B) = n_B$$

energy density

$$E_B(P_B) \equiv N_c \int_{\mathbf{p}} E_q(\mathbf{p}) Q_{\text{in}}(\mathbf{p}; \mathbf{P}_B)$$

$$\varepsilon = \int_{\mathbf{P}_B} \underline{E_B(P_B)} \mathcal{B}(P_B) = N_c \int_{\mathbf{P}_B} \left(\int_{\mathbf{p}} E_q(\mathbf{p}) \underline{Q_{\text{in}}(\mathbf{p}; \mathbf{P}_B)} \right) \mathcal{B}(P_B) = N_c \int_{\mathbf{p}} E_q(\mathbf{p}) \underline{f_q(p)}$$

Dual expression: one can freely switch descriptions

No double counting

Finite-T model

Hadron Resonance Gas model for quark distribution

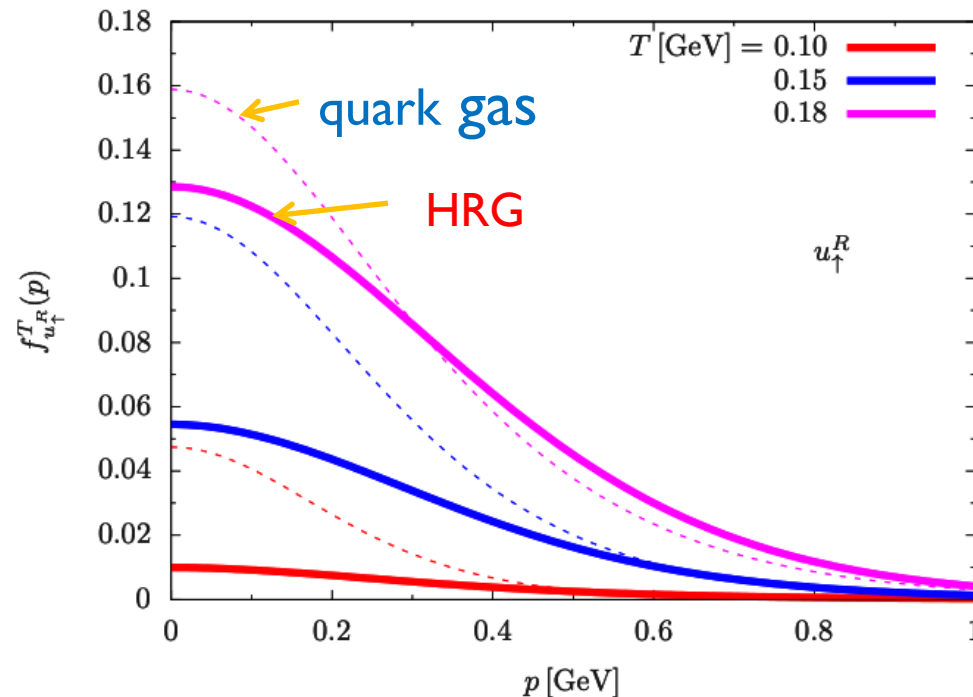
see [TK-Suenaga, '22]

$$f_{\mathbf{q}}^T(\mathbf{p}) = \sum_h \int_{\mathbf{P}_h} n_h^T(\mathbf{P}_h) Q_{\text{in}}^{hq}(\mathbf{p}; \mathbf{P}_h)$$

$$n_h^T(\mathbf{P}_h) = [e^{E_h(\mathbf{P}_h)/T} - 1]^{-1}$$

- calculate quark w.f. for mesons up to $L = 3$, $n_r = 4$; $E < \sim 2.5$ GeV

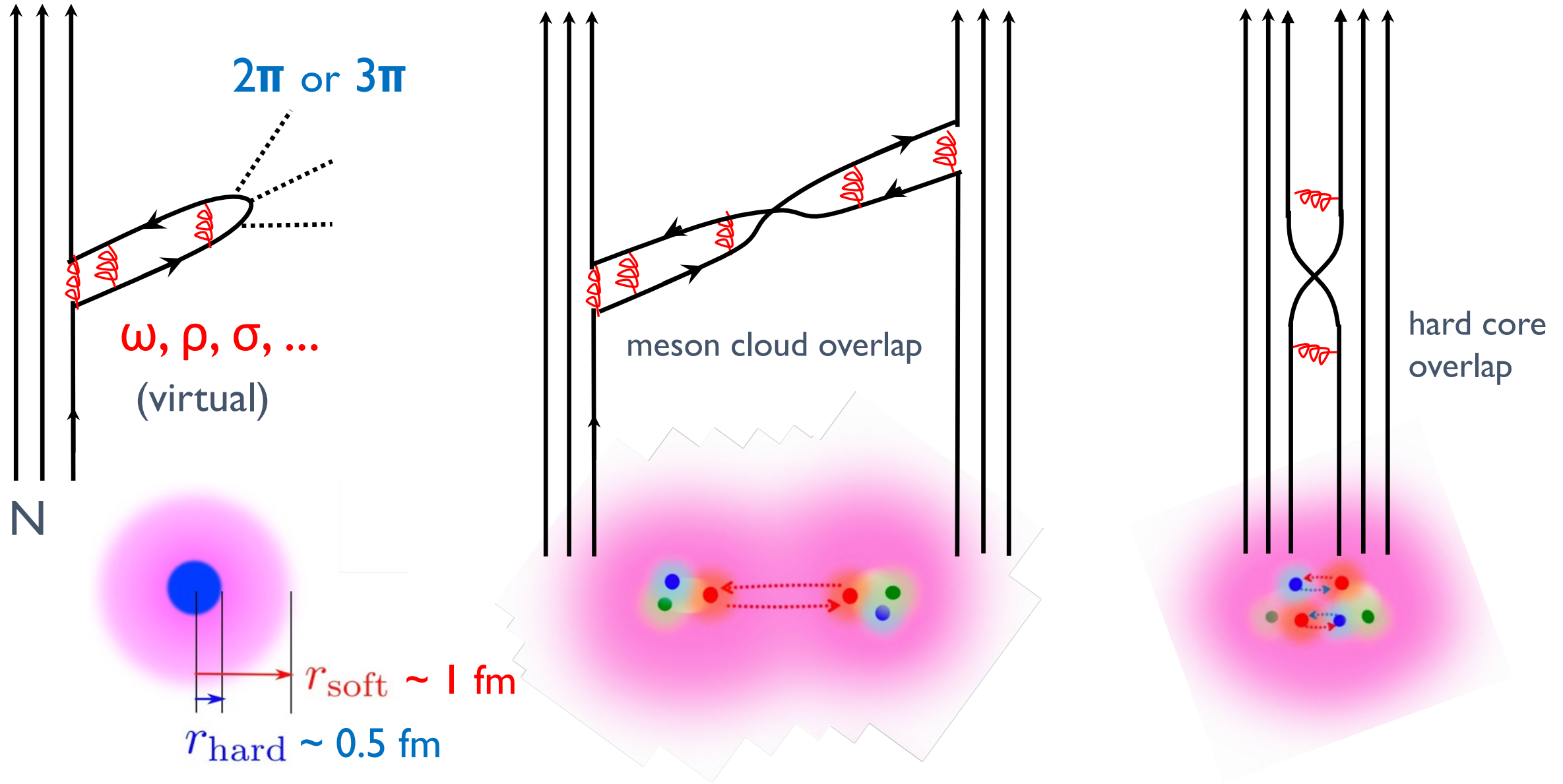
	$n_r^{2S+1} L_J$	M_{exp}	M_{cal}	\bar{P}^2	$\sqrt{\langle r^2 \rangle}$	f_S	α_s
π	$1^1 S_0$	0.14	0.16	0.47	0.50	0.70	0.80
	$2^1 S_0$	1.30	1.28	0.43	0.98		
	$3^1 S_0$	1.81	1.82	0.55	1.38		
	$4^1 S_0$	2.07**	2.22	0.67	1.66		
ρ	$1^3 S_1$	0.78	0.76	0.21	0.66	0.74	0.80
	$2^3 S_1$	1.47	1.44	0.35	1.17		
	$3^3 S_1$	1.91*	1.87	0.48	1.55		
	$4^1 S_1$	2.27**	2.22	0.61	1.83		
K	$1^1 S_0$	0.49	0.49	0.42	0.49	0.72	0.77
	$2^1 S_0$	1.46*	1.46	0.45	0.98		
K^*	$1^3 S_1$	0.89	0.91	0.24	0.63	0.75	0.77
	$2^3 S_1$	1.41	1.54	0.39	1.10		



quark gas ~ HRG

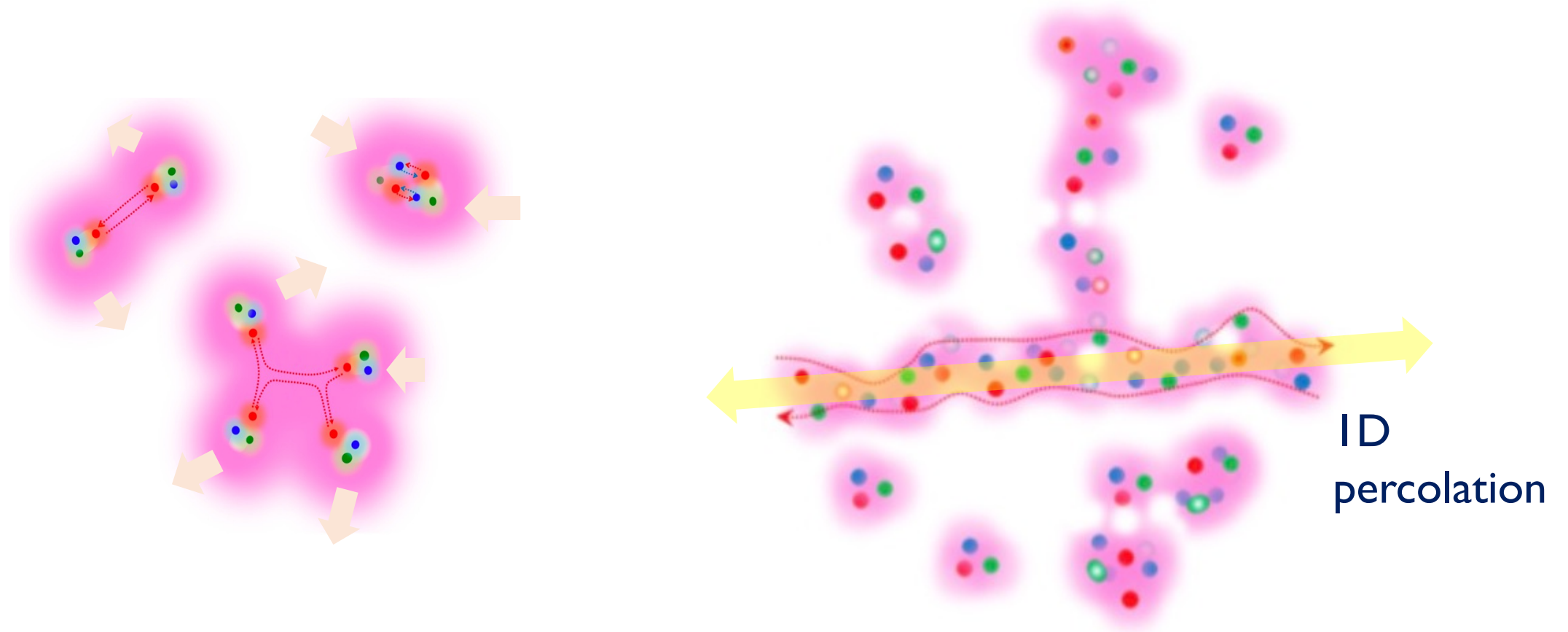
at ~ 0.15 - 0.18 GeV

"Soft" & "Hard" scales in a nucleon



Soft Deconfinement

relating "multi-quark exchanges" to "delocalization of quark w.f."



how can this transition be described?

Strategy (in dilute regime)

Separate **fast** quark dynamics from **slow** baryon dynamics

=> *Born-Oppenheimer* descriptions

1, The velocity : $k_B/E_B \sim 1/Nc \ll k_q/E_q \sim 1$ $(k_B \sim k_q \sim n_B^{1/3})$
 $n_B = n_q^R = n_q^G = n_q^B$

2, Find **quark eigenstates** for **a given** baryon configuration

3, Take the "**time** average" \rightarrow "**ensemble** average" of baryons

A model of **quantum** percolation

[Kirkpatrick-Eggarter '72,...]

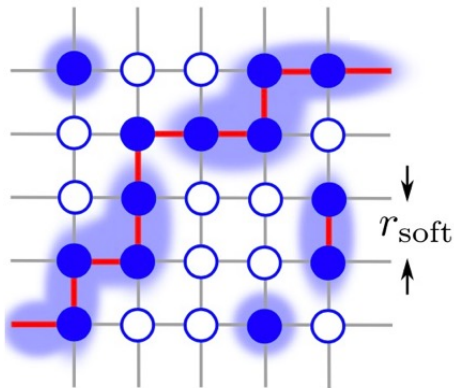
tight-binding
Hamiltonian

$$H = \sum_n |n\rangle \varepsilon_n \langle n| + \sum_{n \neq m} |n\rangle V_{nm} \langle m|$$

$|n\rangle$: a quark state
exists at a site n

$$V_{nm} = -V \quad (V > 0)$$

nearest-neighbor hopping



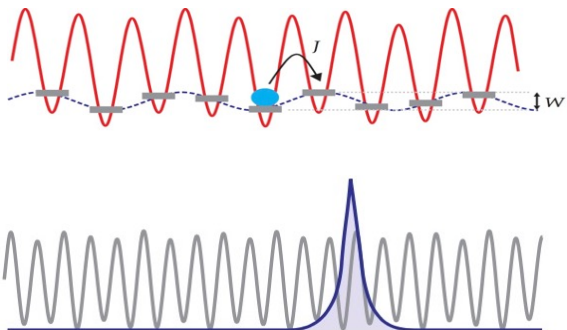
- ε, V depends on a given baryon distribution

- $\varepsilon \rightarrow \infty$ when quarks are out of baryons

- quarks hop **only within** connected clusters (setup)

→ geometrical (classic) percolation must occur first

"dirty" potentials



- **interference** may kill amplitudes (*Anderson localization*)

connected path does not necessarily lead to delocalization



mode-by-mode percolation

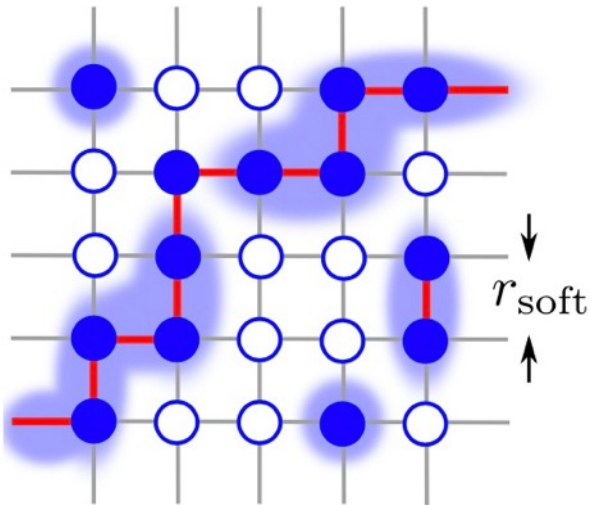
(some modes delocalize earlier, the others later)

Delineating quark wavefunctions

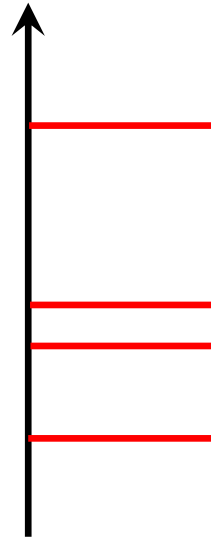
procedures

(e.g. 3D lattice model)

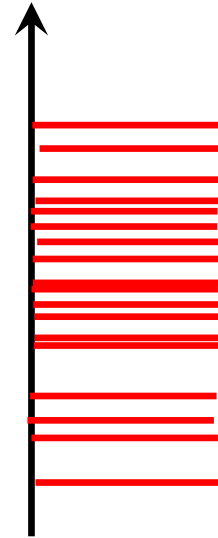
solve a **single** particle problem
for a **given geometry**



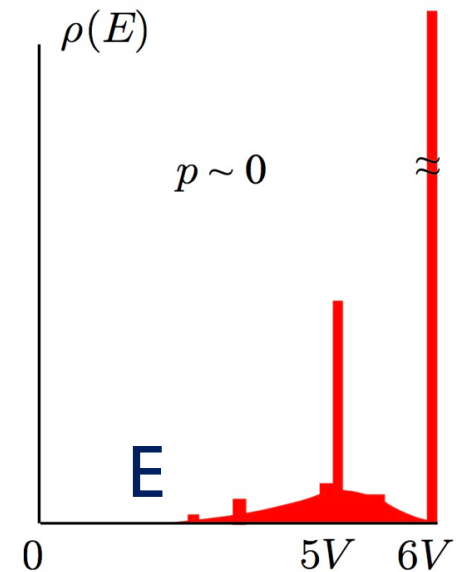
prepare single
particle levels



sum geometries
(& normalize)



make histogram
(take inf. vol. limit)

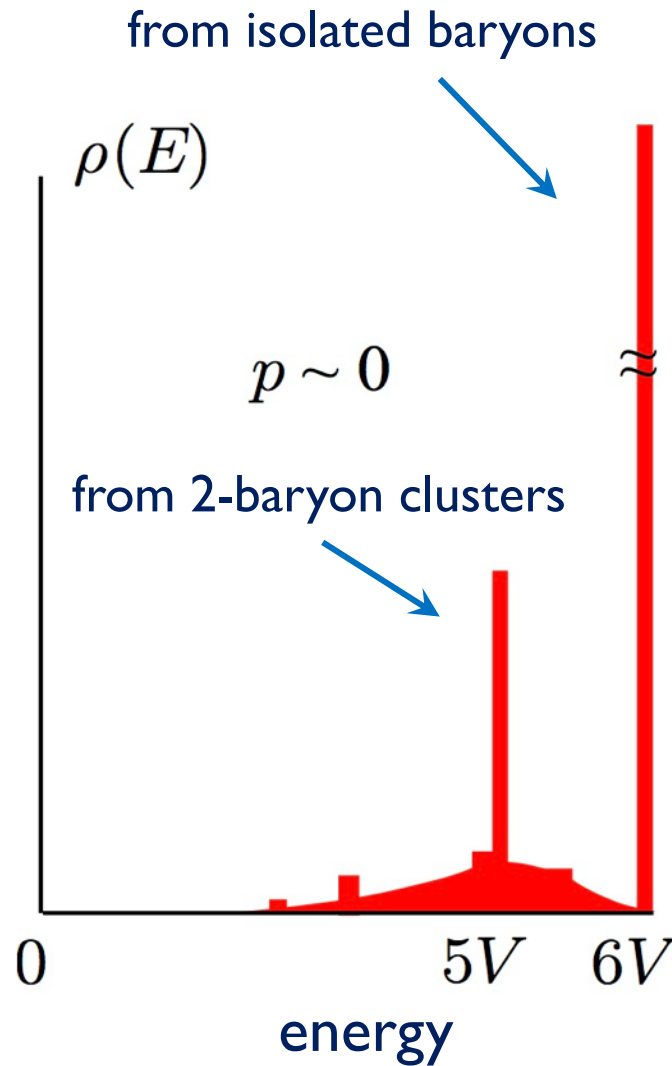


\Rightarrow we diagnose the **quark contents** of given baryon configurations

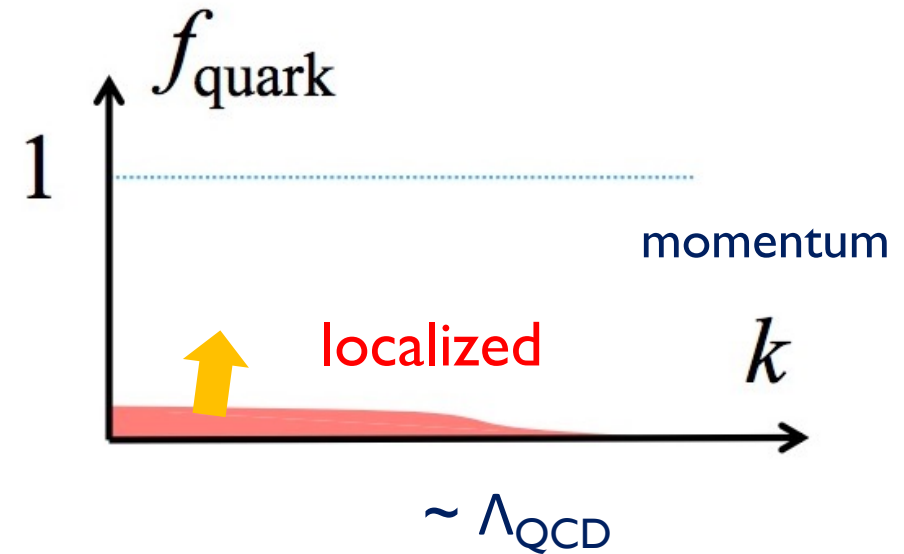
quark Fermi sea & **mode-by-mode** percolation

$$\int dE \rho(E) = 1$$

localized quark
 \rightarrow energy $\sim R_B^{-1}$



occupation probability

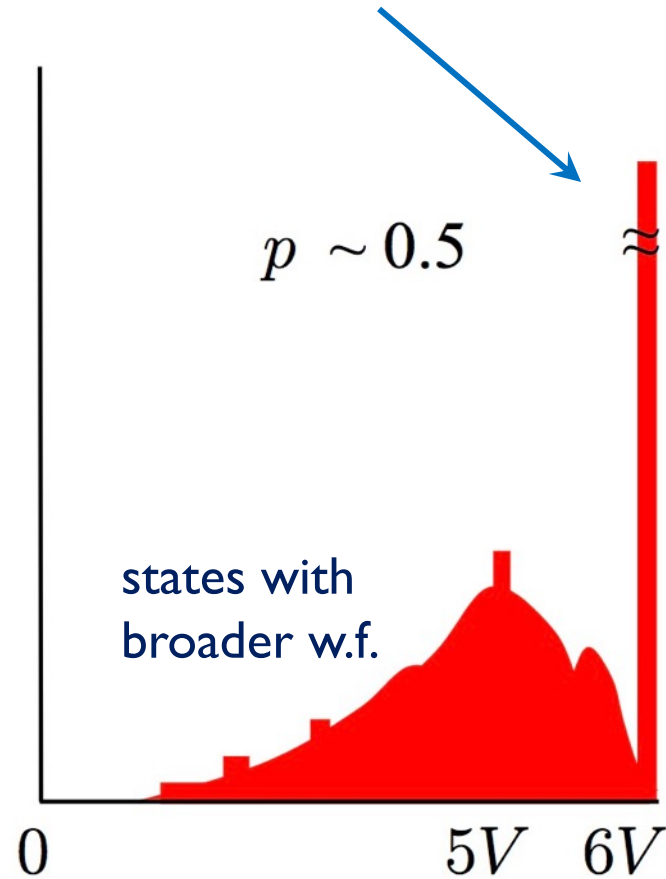
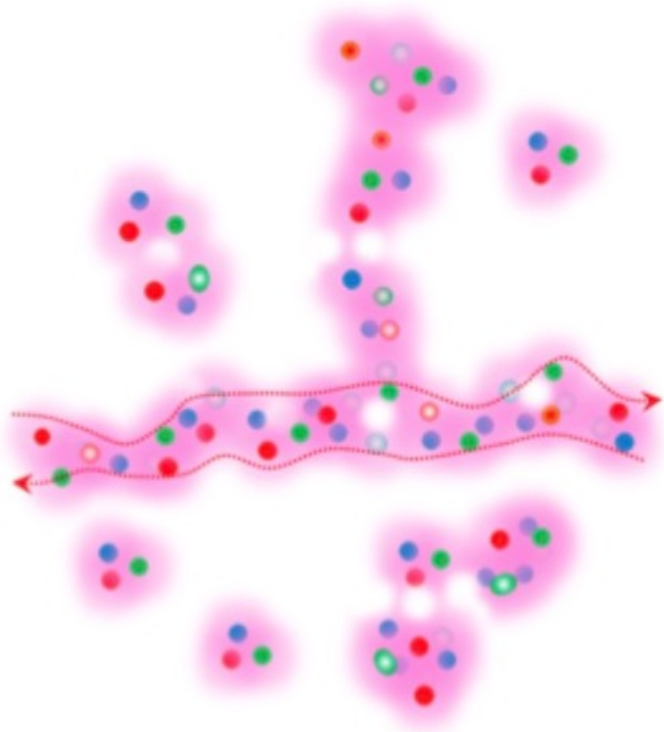


"nuclear"

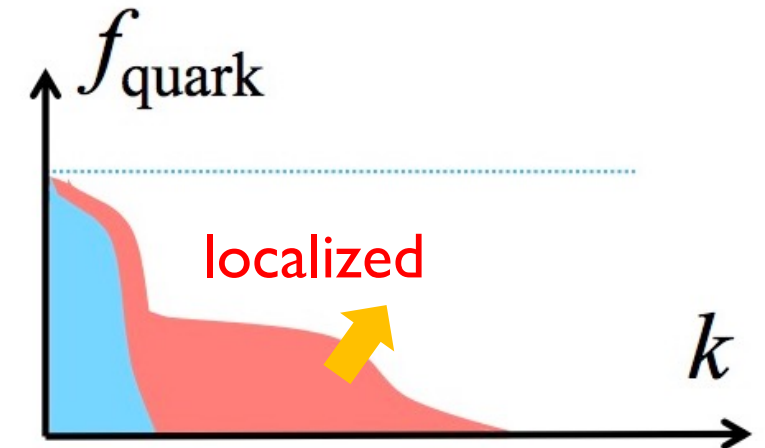
quark Fermi sea & mode-by-mode percolation

$$\int dE \rho(E) = 1$$

isolated baryons + sub-clusters



occupation probability



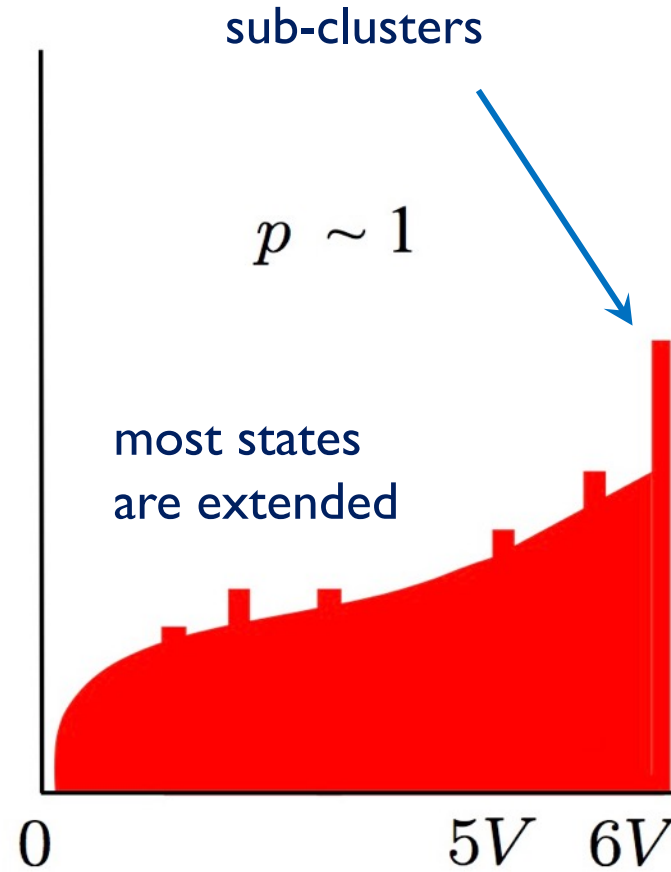
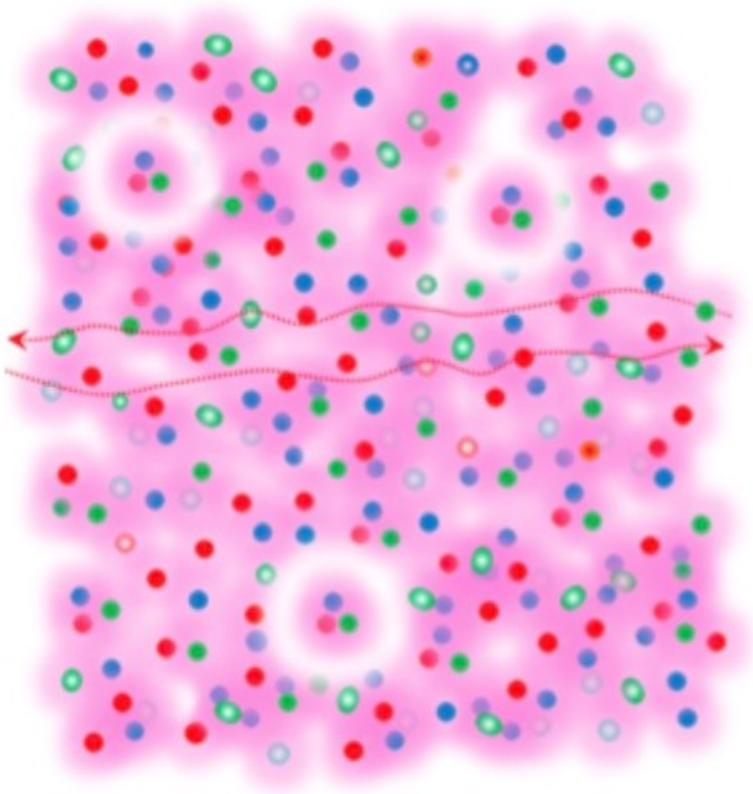
extended $\sim \Lambda_{\text{QCD}}$

quark bases reasonable

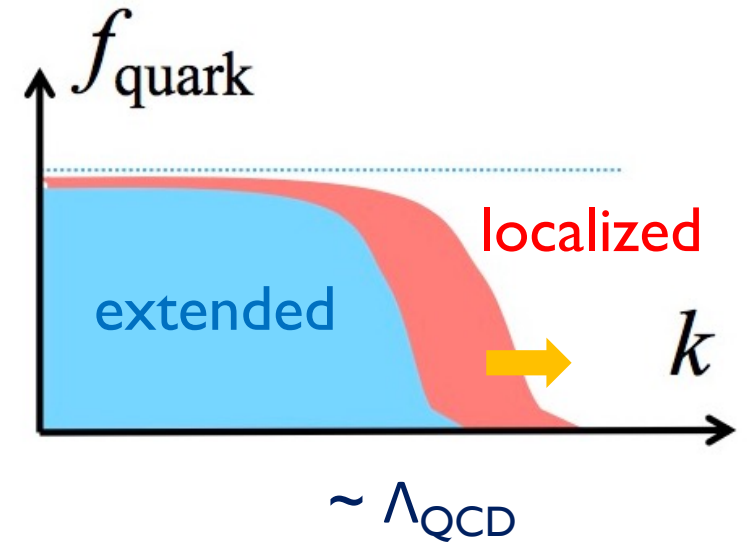
"quark-hadron continuity"

quark Fermi sea & **mode-by-mode** percolation

$$\int dE \rho(E) = 1$$



occupation probability

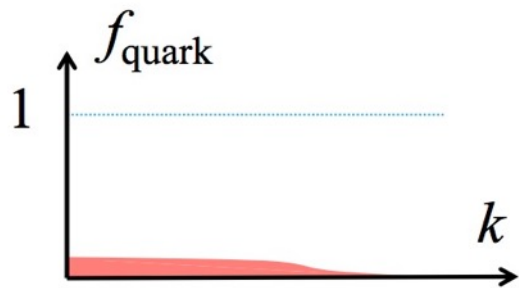


"quarkyonic"

[McLerran-Pisarski '06,...]

a cartoon

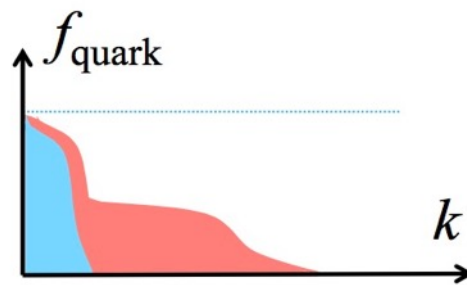
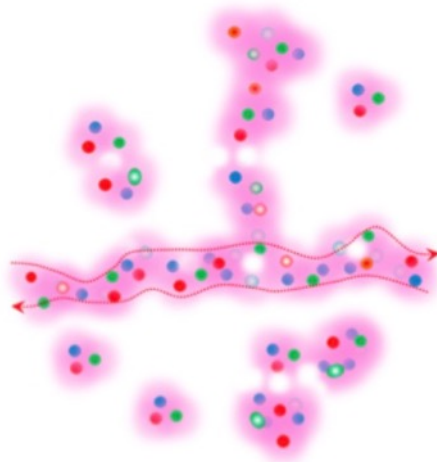
nuclear



$\sim 2n_0$

"Soft" Deconf.

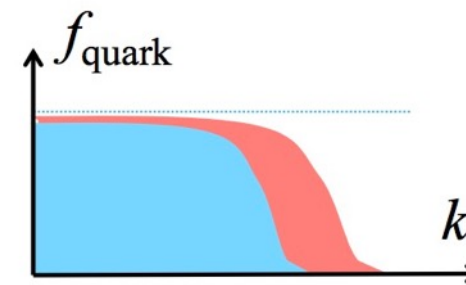
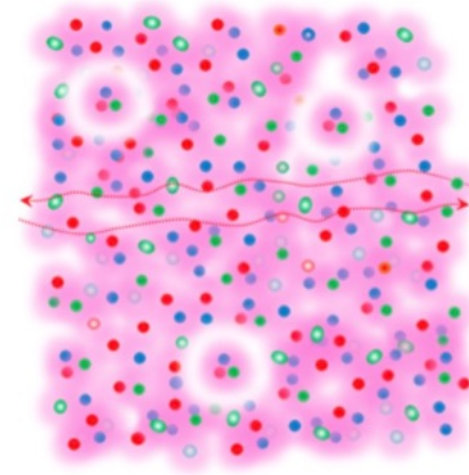
(mode-by-mode percolation)



Hints from NSs

"Hard" Deconf.

(core dominance)



$\sim 4-7n_0$

$\sim 50n_0$

partonic

(pQCD)

[Freedman-McLerran,
Kurkela+,...]



n_B