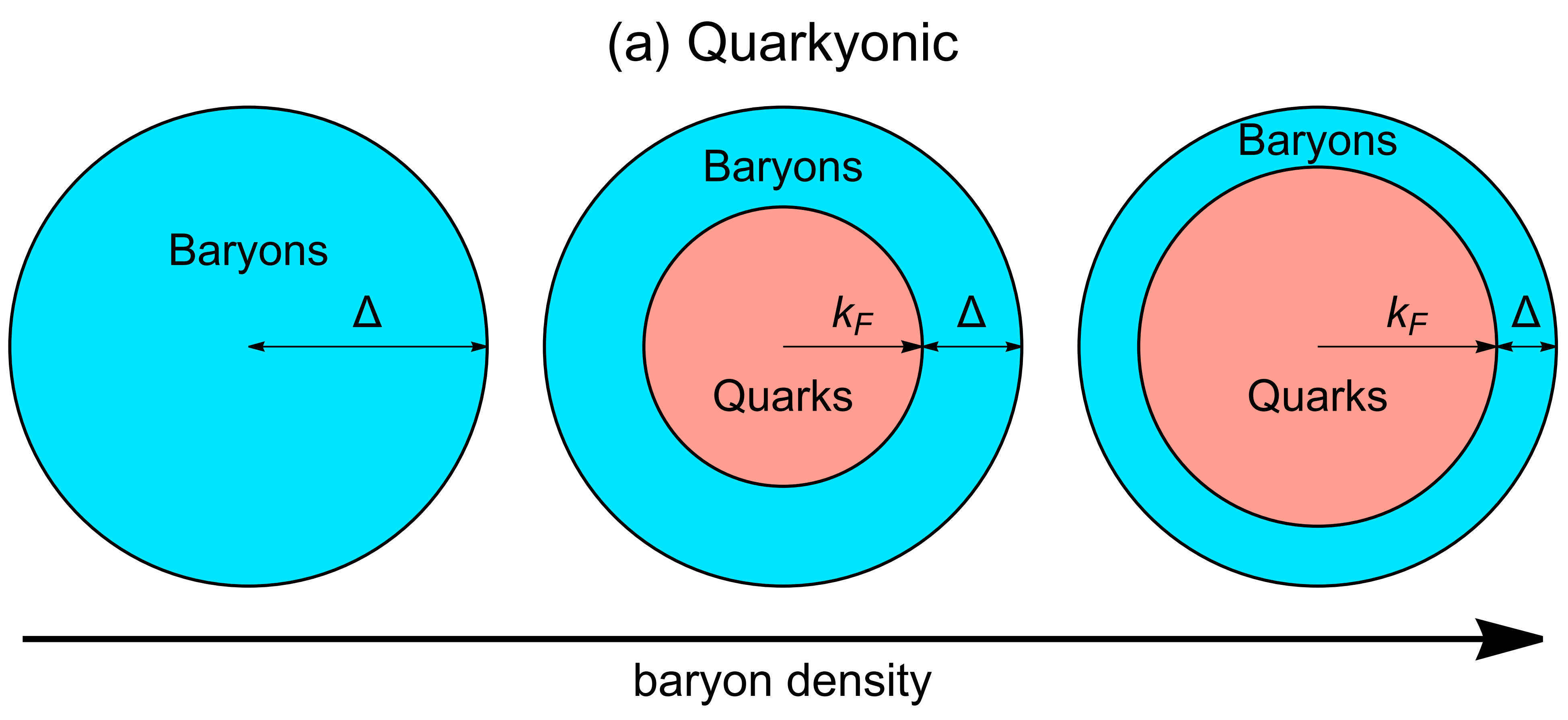
# Quarkyonic or Baryquark Matter or….

- Baryquark Matter
  - energetics
  - properties

- Some observations concerning the recent STAR BESII data

Thanks to: V. Vovchenko, A Bzdak

# Quarkyonic Matter




L.McLerran,R.D.Pisarski, 0706.2191

Construction (K.S.Jeong,L.McLerran,S.Sen,1908.04799):

Introduce repulsive interaction among nucleon only

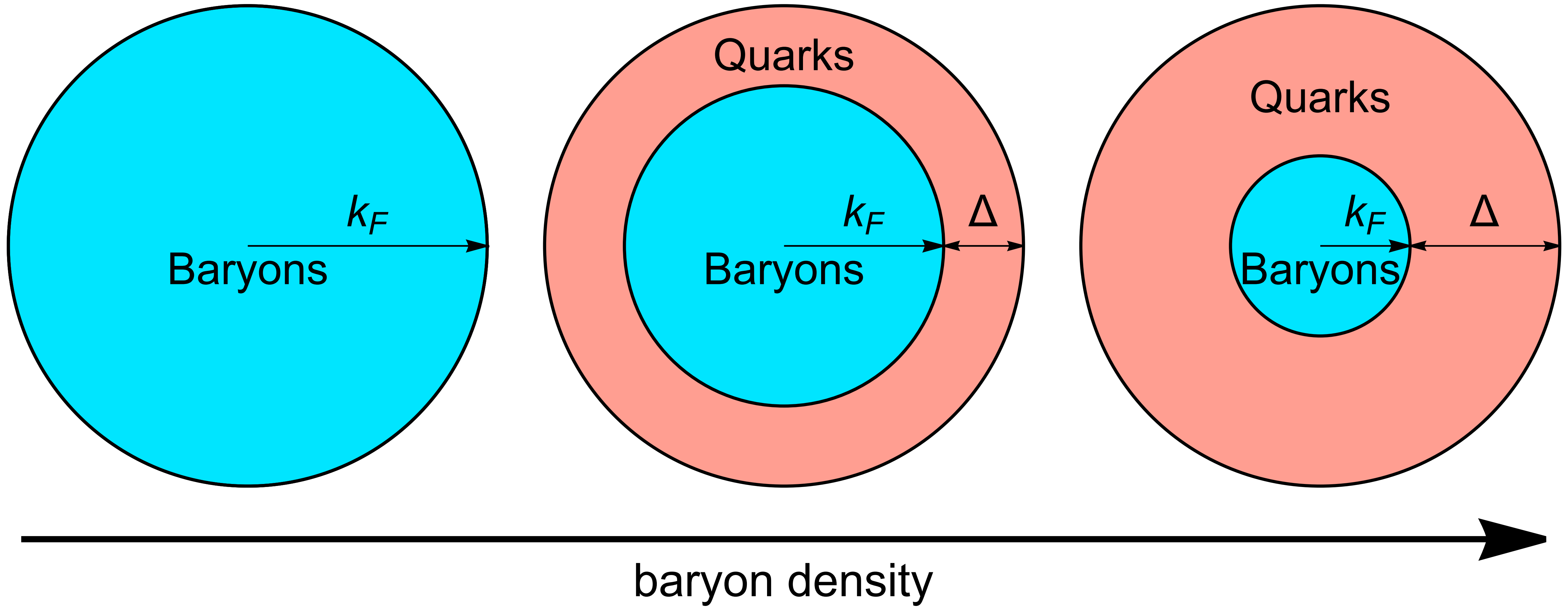
⇒ At a certain density a configuration with quarks at low momentum is favored

Is it?

# Baryquark Matter

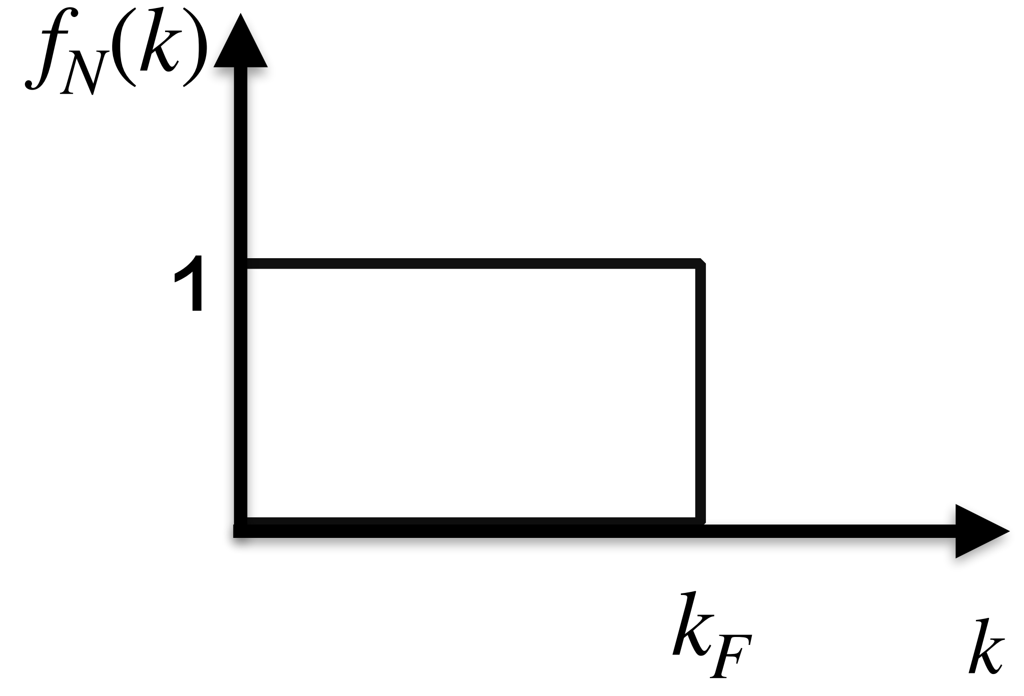
VK, V. Vovchenko, 2211.14674

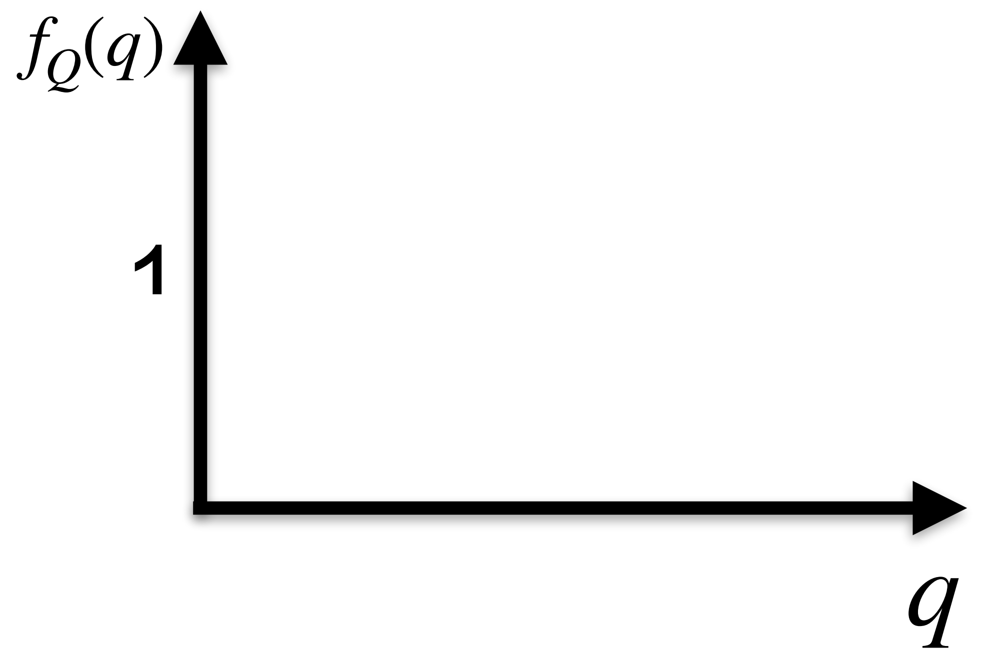
(b) Baryquark



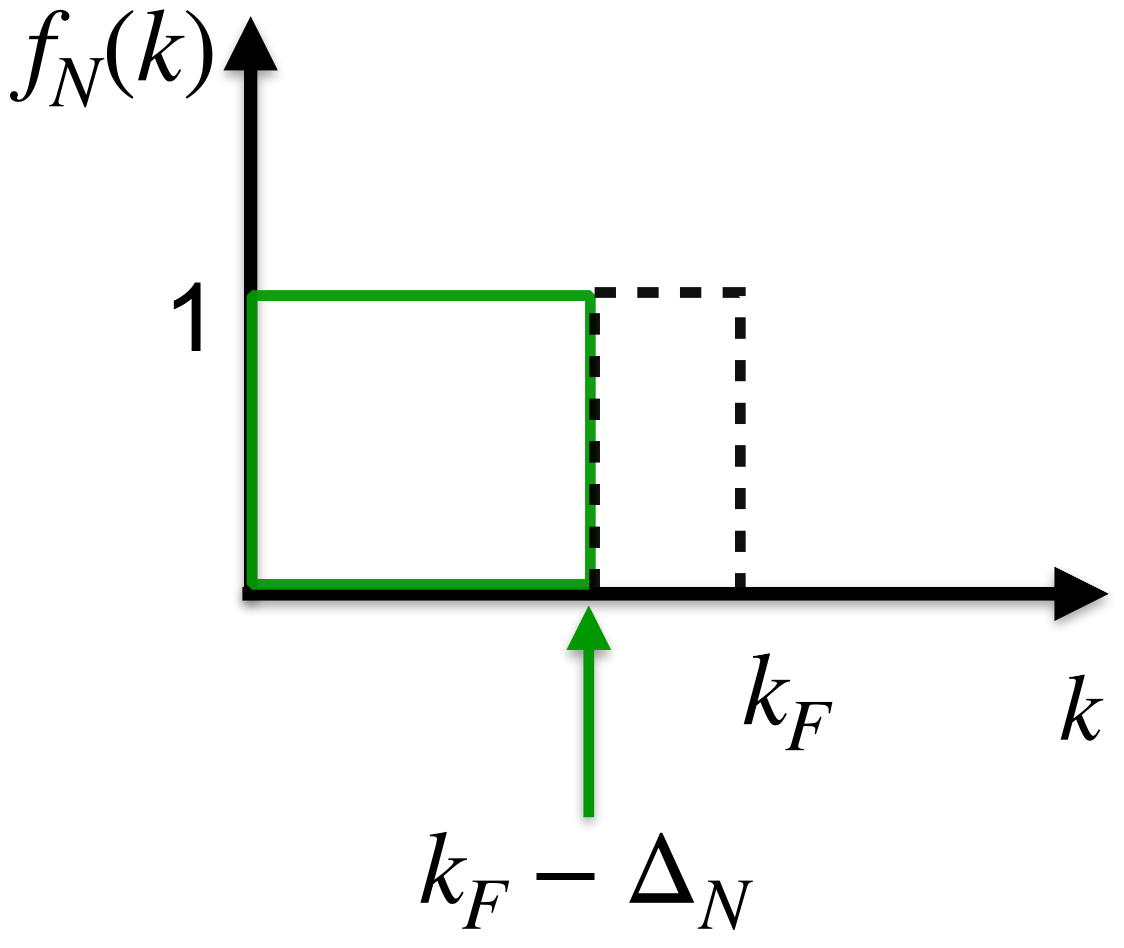
# Energetics for BaryQuark Matter

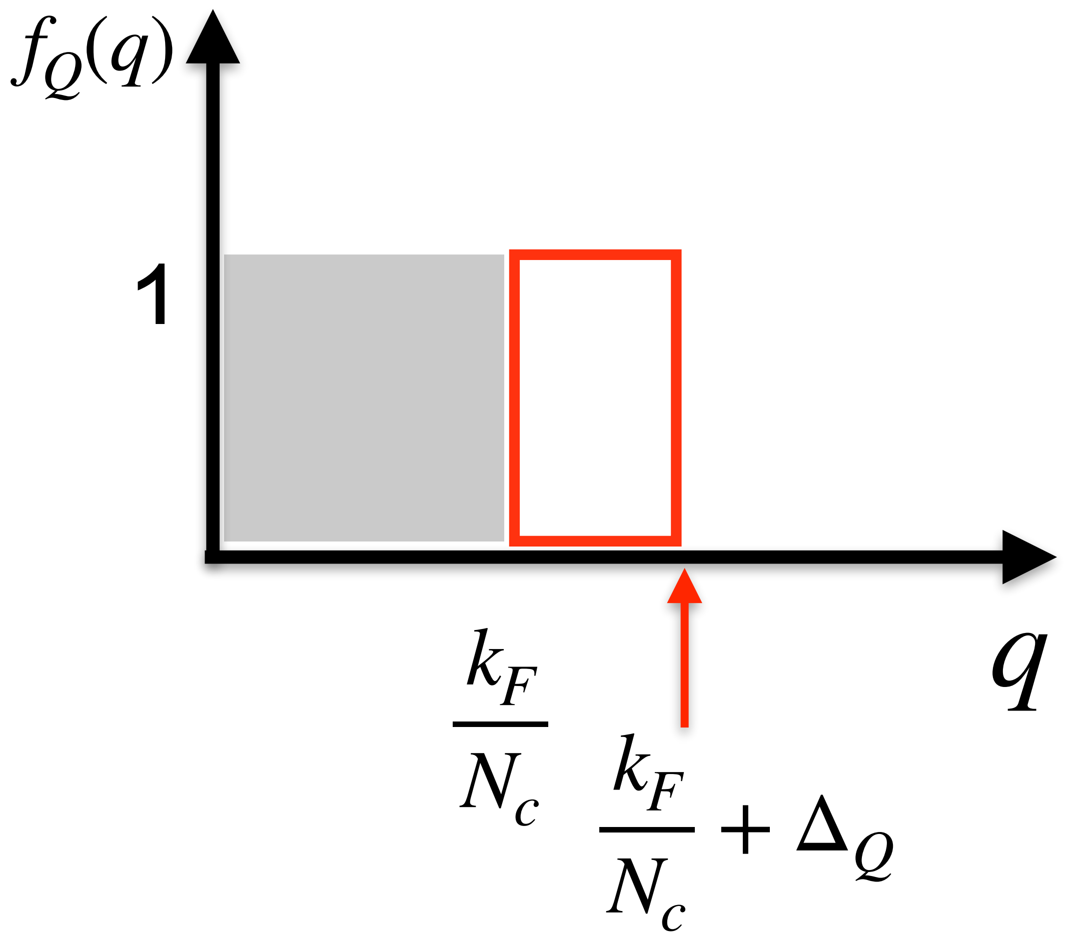
Start with Baryons only




Remove Baryons at Fermi surface and add quarks so that $\rho_B = \rho_N + \rho_Q = const$



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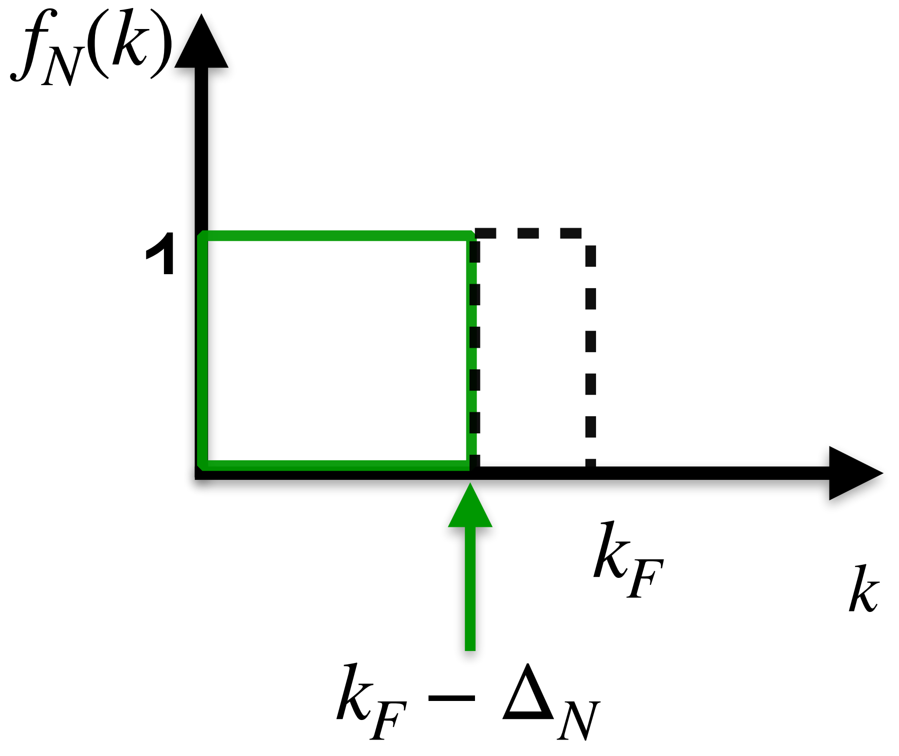
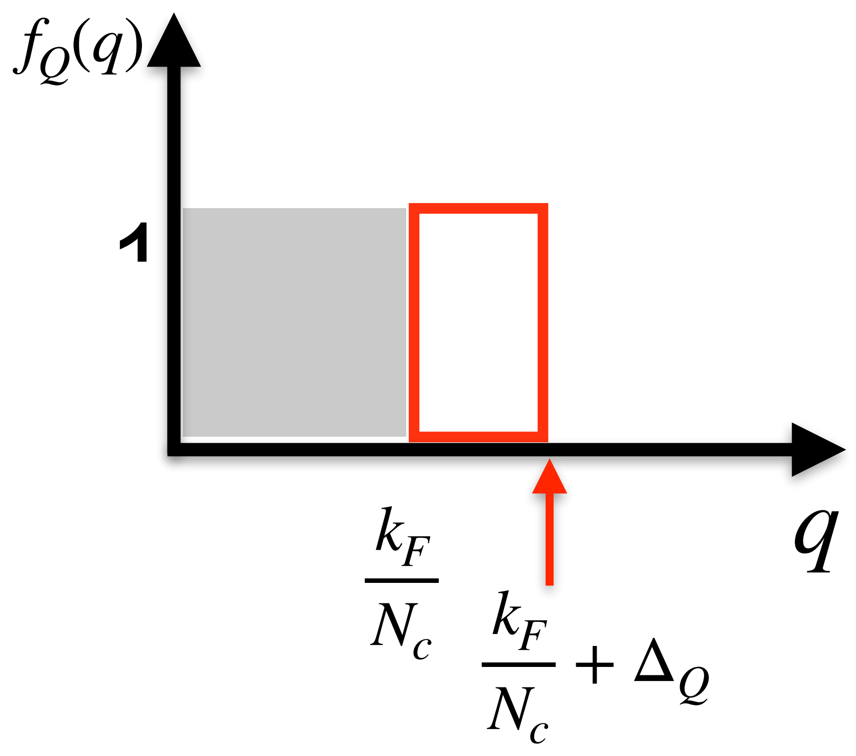



$$\Delta\rho_N = -D\int_{k_F-\Delta_N}^{k_F} k^2 dk \simeq -Dk_F^2\Delta_N$$

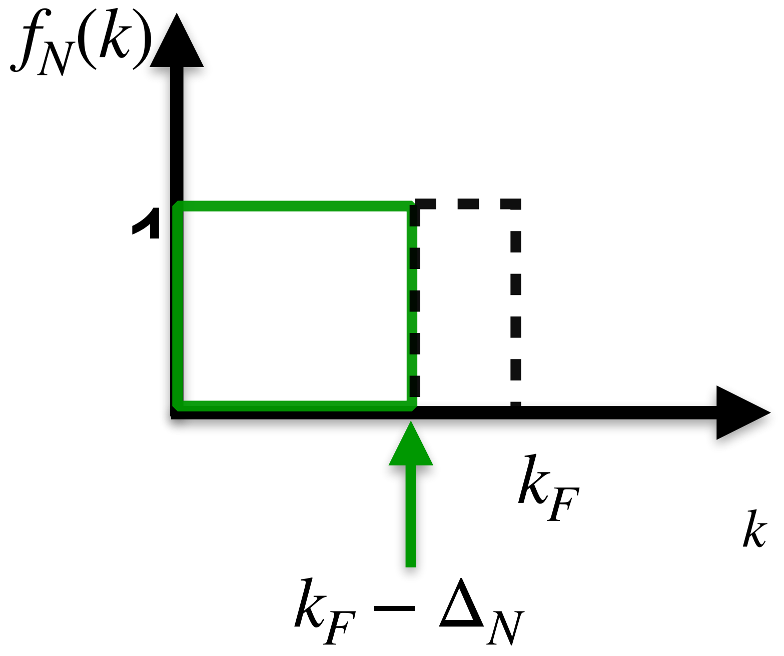
$$\Delta\rho_Q = D\int_{k_F/N_c}^{k_F/N_c+\Delta_Q} k^2 dk \simeq D\left(\frac{k_F}{N_c}\right)^2 \Delta_Q$$

$$\rho_N + \rho_Q = const \ \rightarrow \Delta\rho_N + \Delta\rho_Q = 0$$

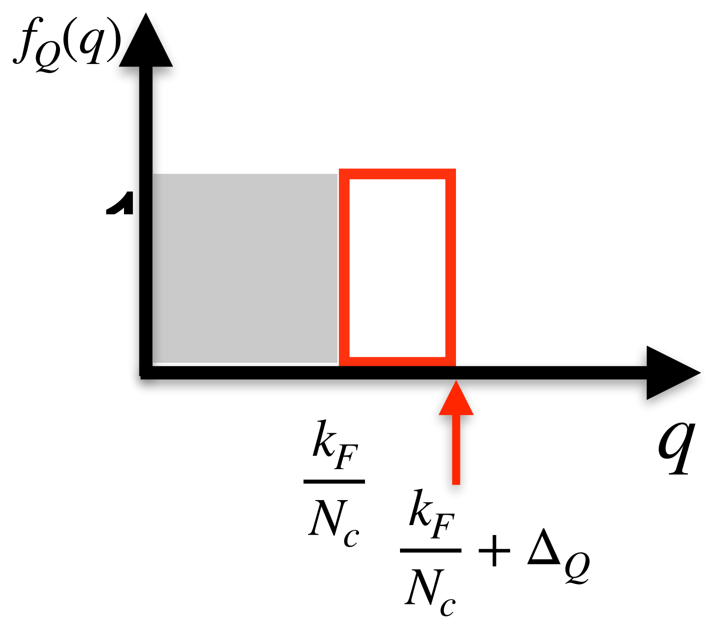
$$\Rightarrow \ \Delta_N = \frac{\Delta_Q}{N_c^2}$$

# Energetics for BaryQuark Matter

Remove Baryons and add quarks so that $\rho_B = \rho_N + \rho_Q = const$




$$\Delta_N = \frac{\Delta_Q}{N_c^2}$$




$$\Delta\epsilon_N \simeq -D\int_{k_F-\Delta_N}^{k_F}\left(\frac{k^2}{2m_N} + m_n\right)k^2dk$$

$$\simeq -\frac{D}{2m_N}k_F^4\Delta_N - m_n\Delta\rho_N$$

$$\Delta\epsilon_Q \simeq -DN_c\int_{k_F/N_c}^{k_F/N_c+\Delta_Q}\left(\frac{k^2}{2m_Q} + m_Q\right)k^2dk$$

$$\simeq N_c^2\frac{D}{2m_N}\left(\frac{k_F}{N_c}\right)^4\Delta_Q + m_n\Delta\rho_Q$$

$$= \frac{D}{2m_N}k_F^4\Delta_N + m_n\,\Delta\rho_N = -\Delta\epsilon_N$$

$\Delta\epsilon_N + \Delta\epsilon_Q = 0$ $\Rightarrow$ Energy unchanged to leading order in $\delta\rho_N$

# Energetics for Quarkyonic Matter

Start with Baryons only


Remove Baryons at low momentum and add quarks so that $\rho_B = \rho_N + \rho_Q = const$

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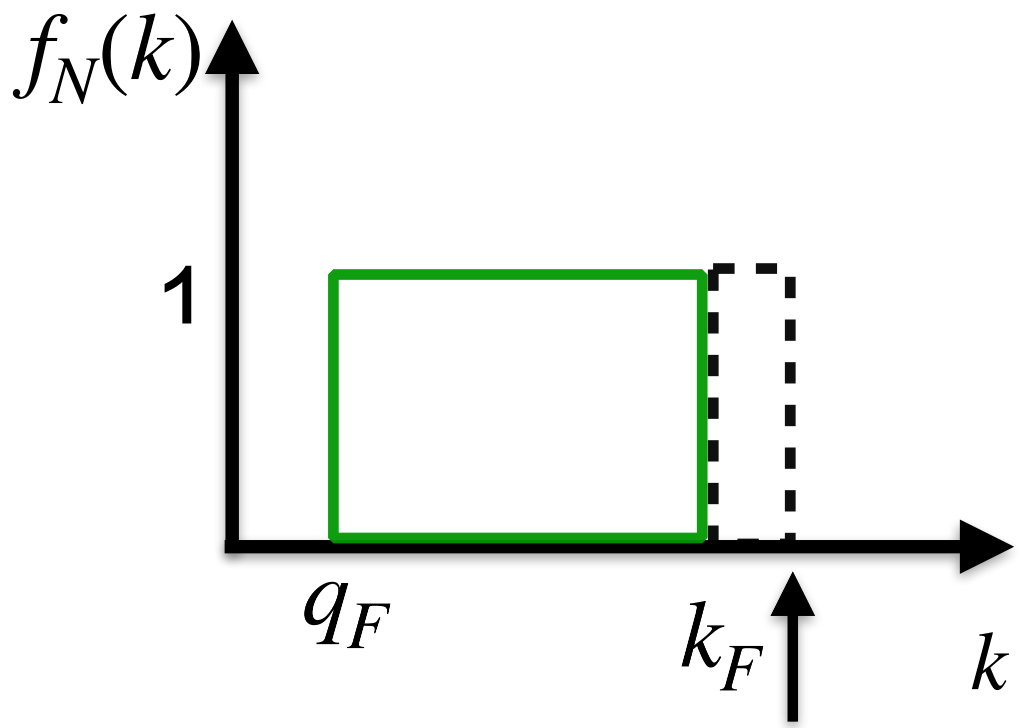
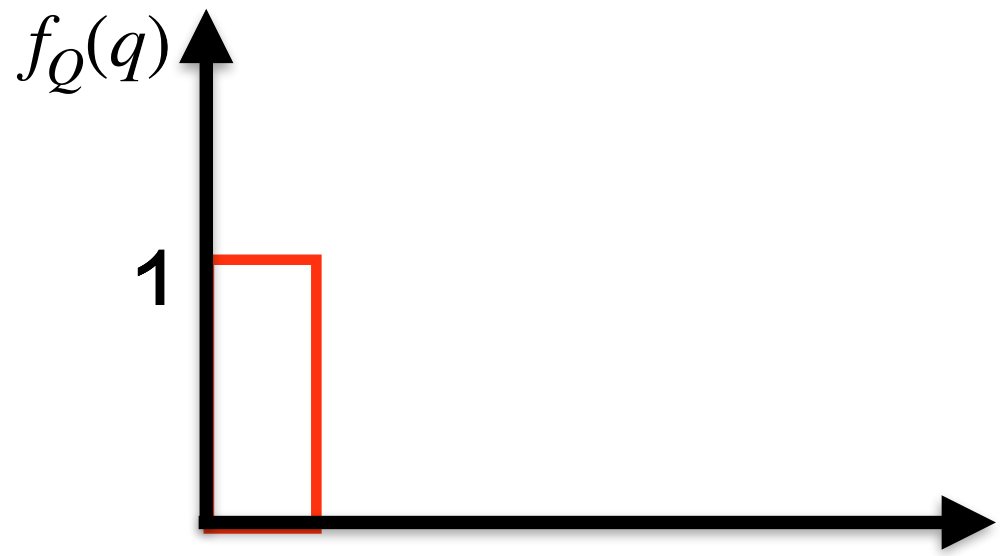



$$\Delta\rho_N = -D\int_0^{q_F} k^2dk + D\int_{k_F} k_F + \Delta N k^2dk$$

$$\simeq -\frac{D}{3}q_F^3 + Dk_F^2\Delta N$$

$$\Delta\rho_Q = D\int_0^{q_F/N_c} k^2dk \simeq \frac{D}{3}\left(\frac{q_F}{N_c}\right)^3$$

$$\rho_N + \rho_Q = const \ \rightarrow \Delta\rho_N + \Delta\rho_Q = 0 \Rightarrow \Delta N > 0$$

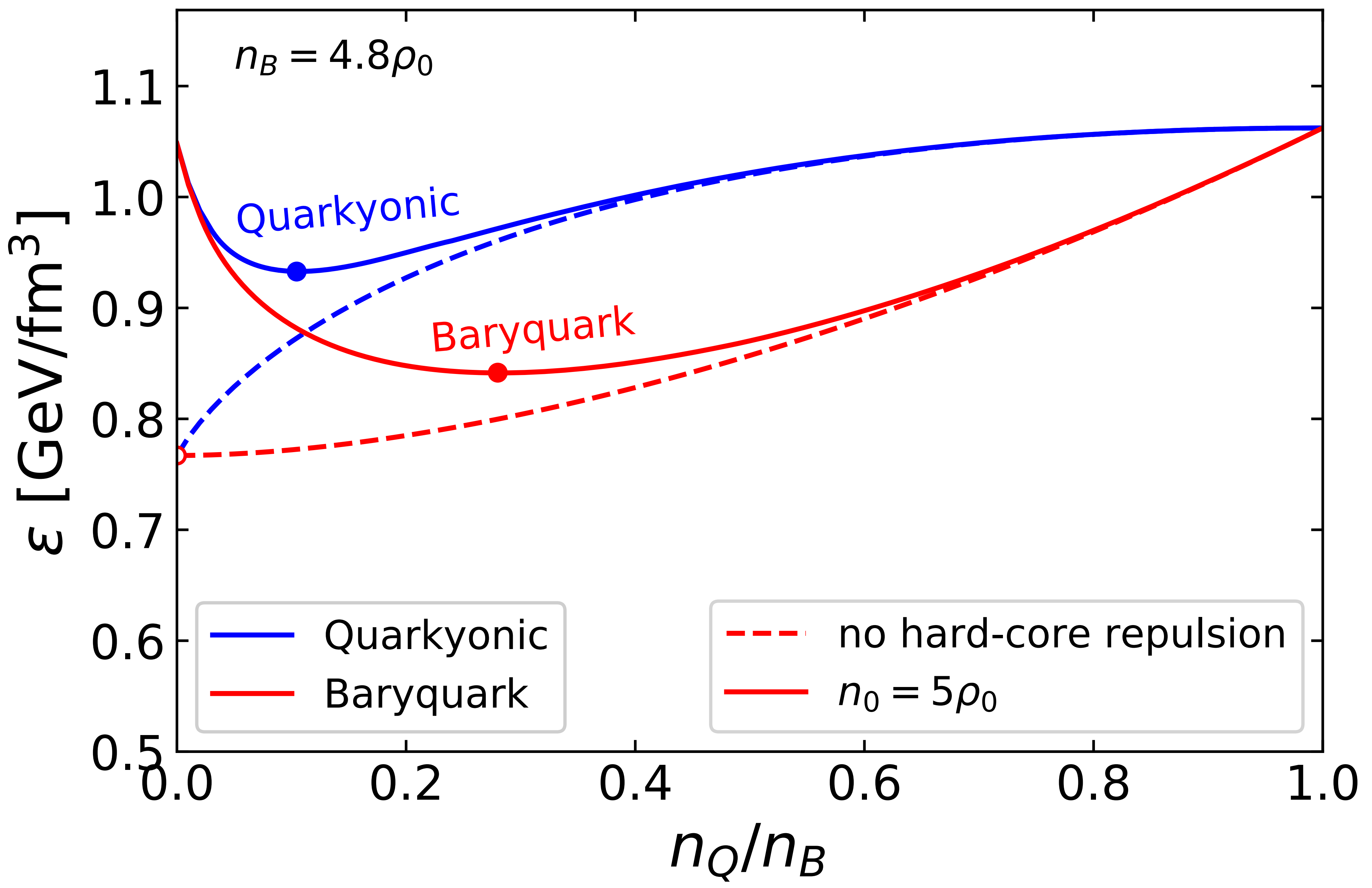
Need to "lift" nucleons from low momentum to top of Fermi energy

$$\Delta E \simeq \frac{D}{3}(N_c^3 - 1)k_F^5\frac{\Delta\rho_Q}{\rho^B} > 0$$

Energetically disfavored compared to BaryQuark Matter

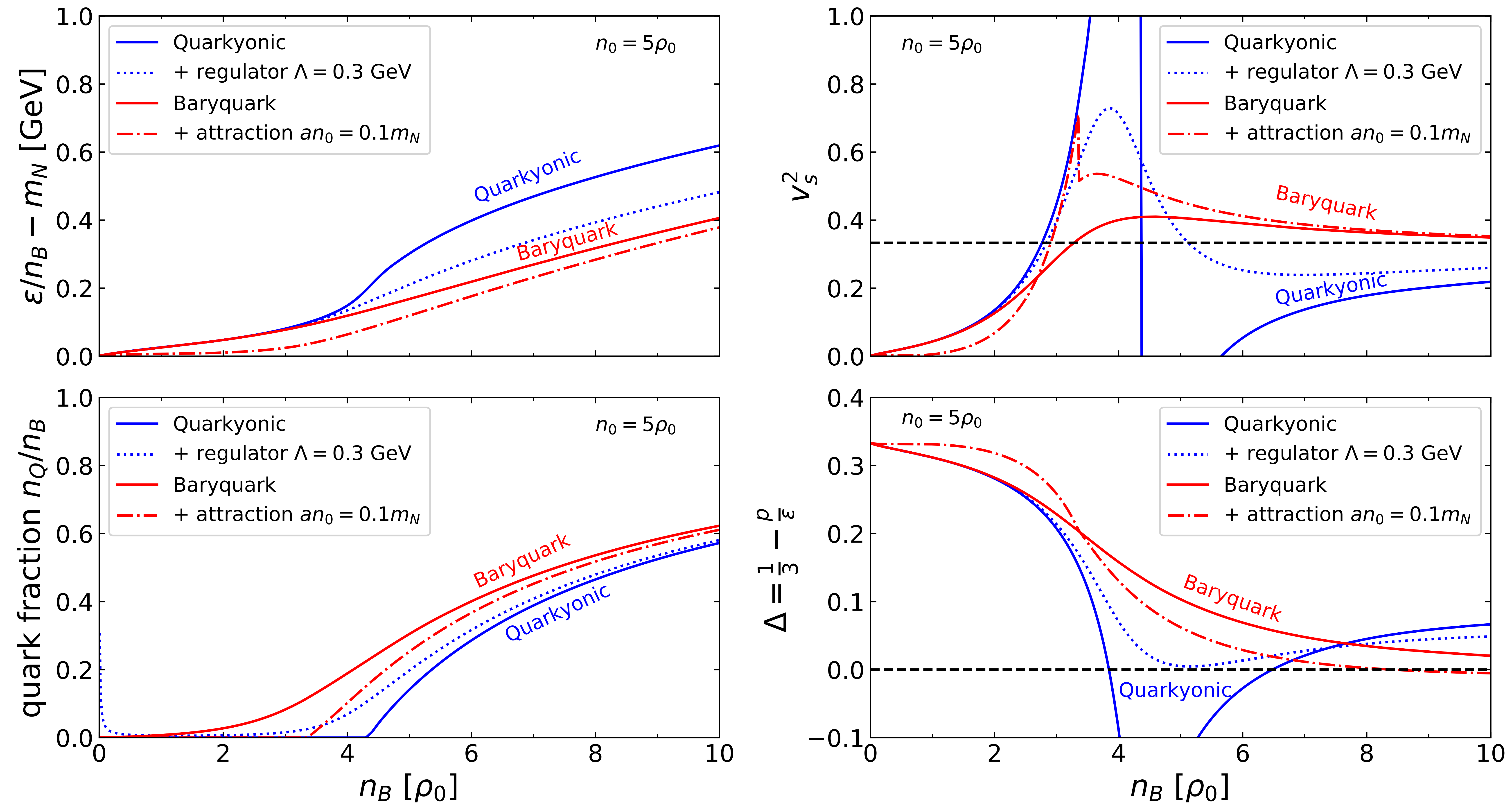
# Energetics

(repulsive) Interaction among nucleons does not help, since nucleon density is the same at given $\frac{\rho_Q}{\rho_B}$




VK, V. Vovchenko, 2211.14674

# Various properties



# End of story?

Consider quark momentum distribution in nucleon with momentum $k$: $\phi_N(q - \frac{k}{N_c})$

(Fujimoto, Kojo, McLerran 2306.04304)

Momentum distribution of quarks inside nucleus:

$$f_Q(q) = \int_{\text{Fermi Sphere}} \phi\left(q - \frac{k}{N_c}\right) d^3k$$

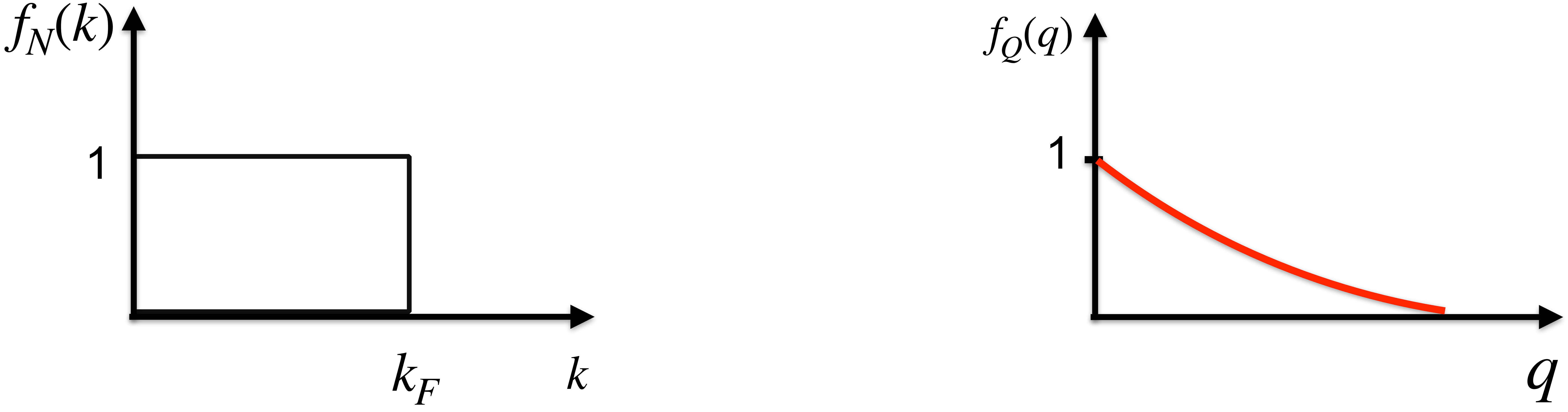
Harmonic Oscillator: $\phi(q) \sim Exp(-\sigma^2 q^2)$ with $\sigma \sim$ RMS of nucleon

Pauli Blocking in the quark sector becomes relevant when $f_Q(q=0) = 1$

For RMS = 1 fm: $f_Q(q=0) = 1$ for $\rho_{crit} = 1.1\rho_0$, for RMS = 0.8 fm: $\rho_{crit} = 2.2\rho_0$

No "asymptotic" densities out of theory land!!!!

# Situation at $\rho_{crit}$




How to add another nucleon?

# Baryquark matter vs quarkyonic matter

- Baryquark energetically favored
- Conceptually not very appealing
- May possibly be fixed with momentum dependent interaction

Much more appealing story

Idyllic Matter

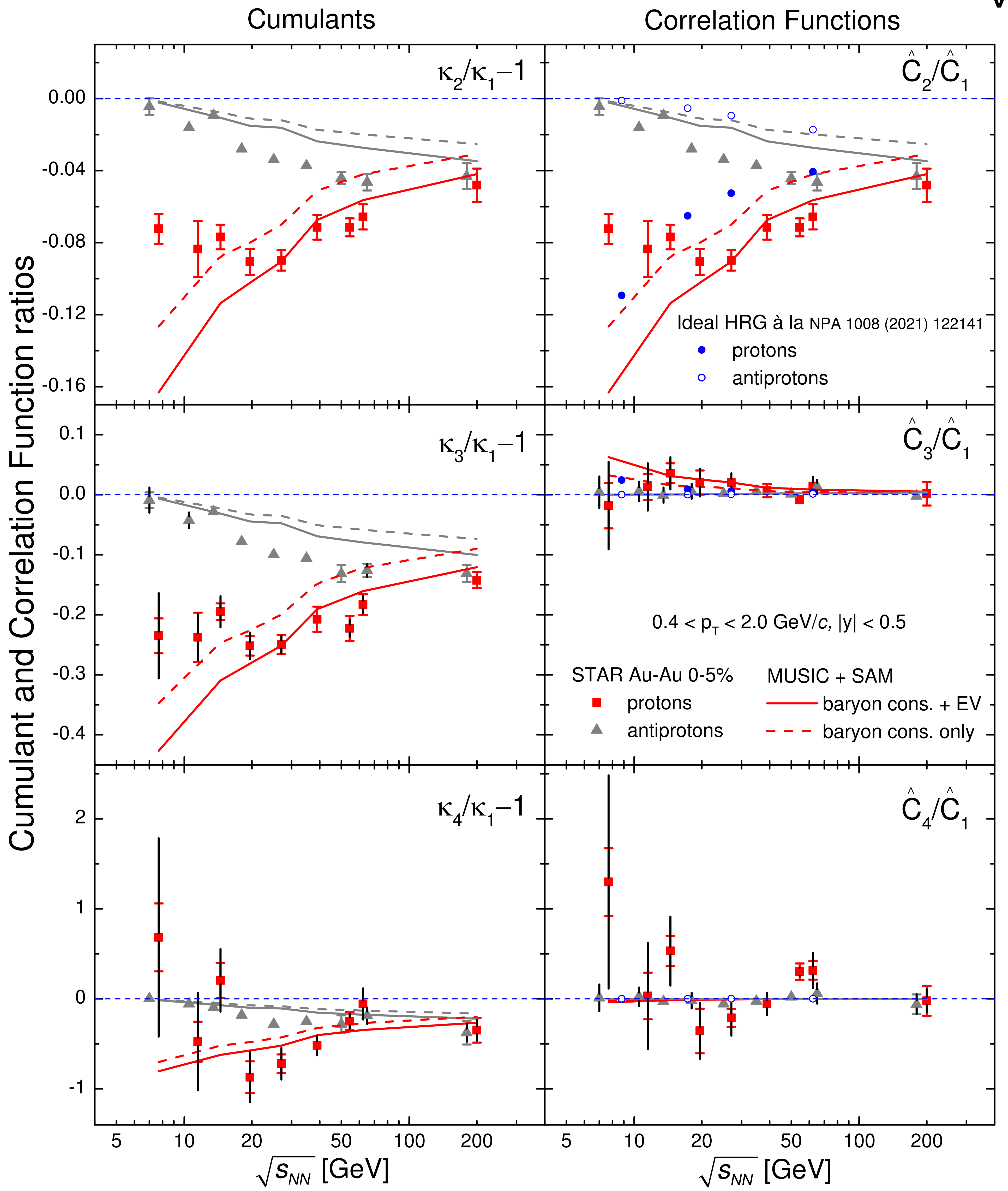
(Fujimoto, Kojo, McLerran 2306.04304)

# Results (Prediction) for proton cumulants

Vovchenko, Shen, VK, 2107.00163

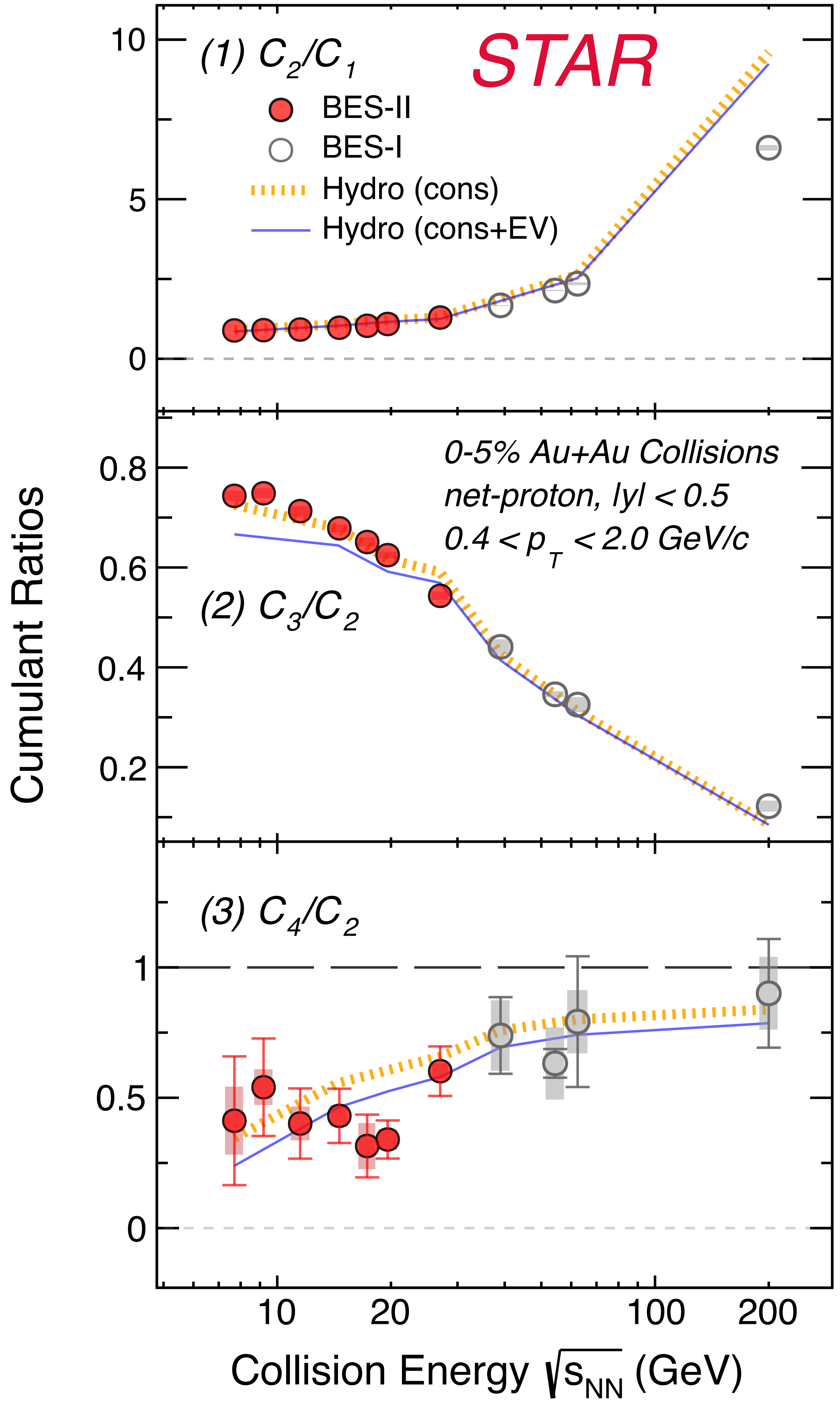
- Viscous hydro
- EOS tuned to LQCD
- Correct for global charge conservation
- Protons NOT baryons

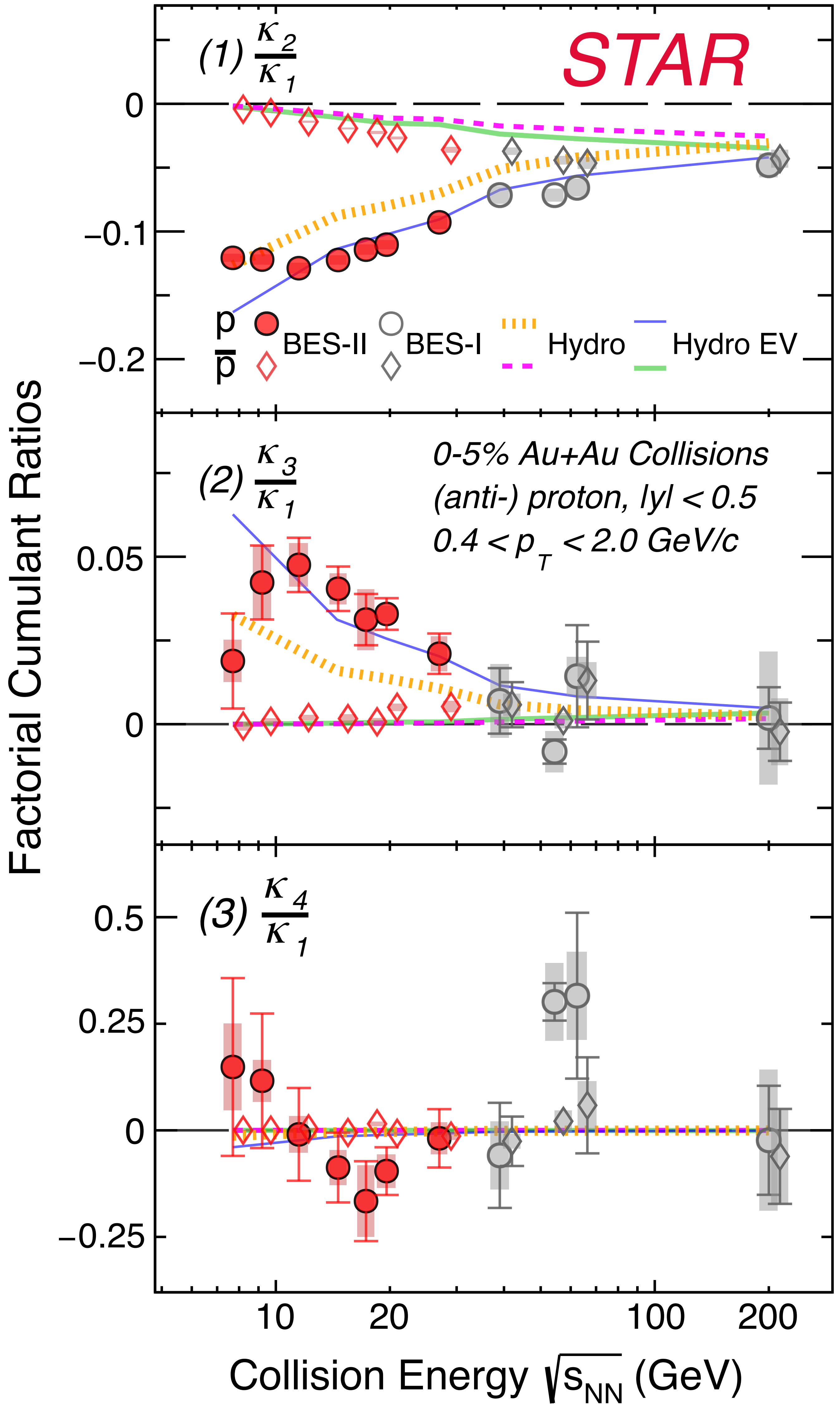
- Baseline!
  No critical point or phase transition



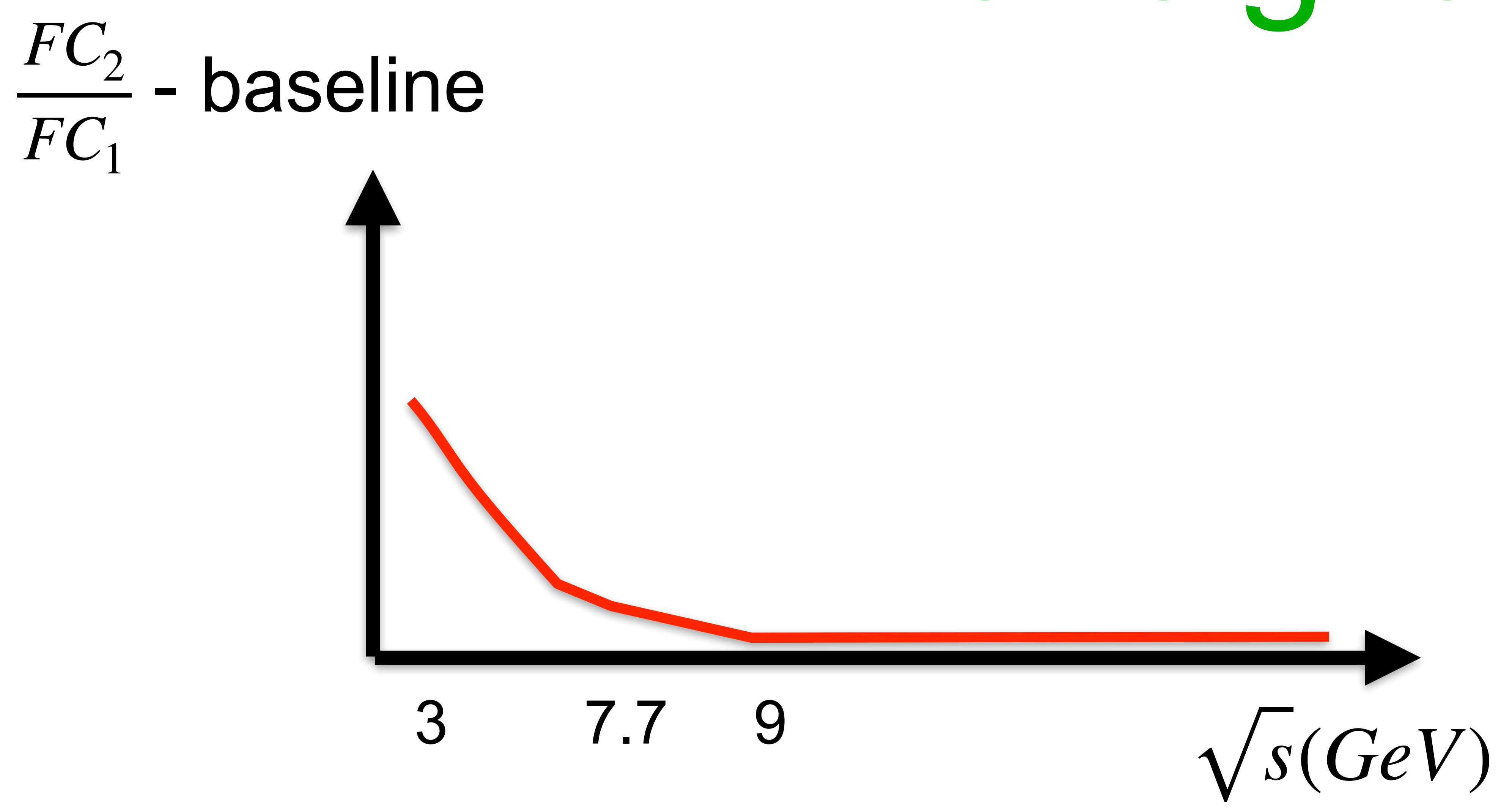

See also: Braun-Munzinger et al, NPA 1008 (2021) 122141

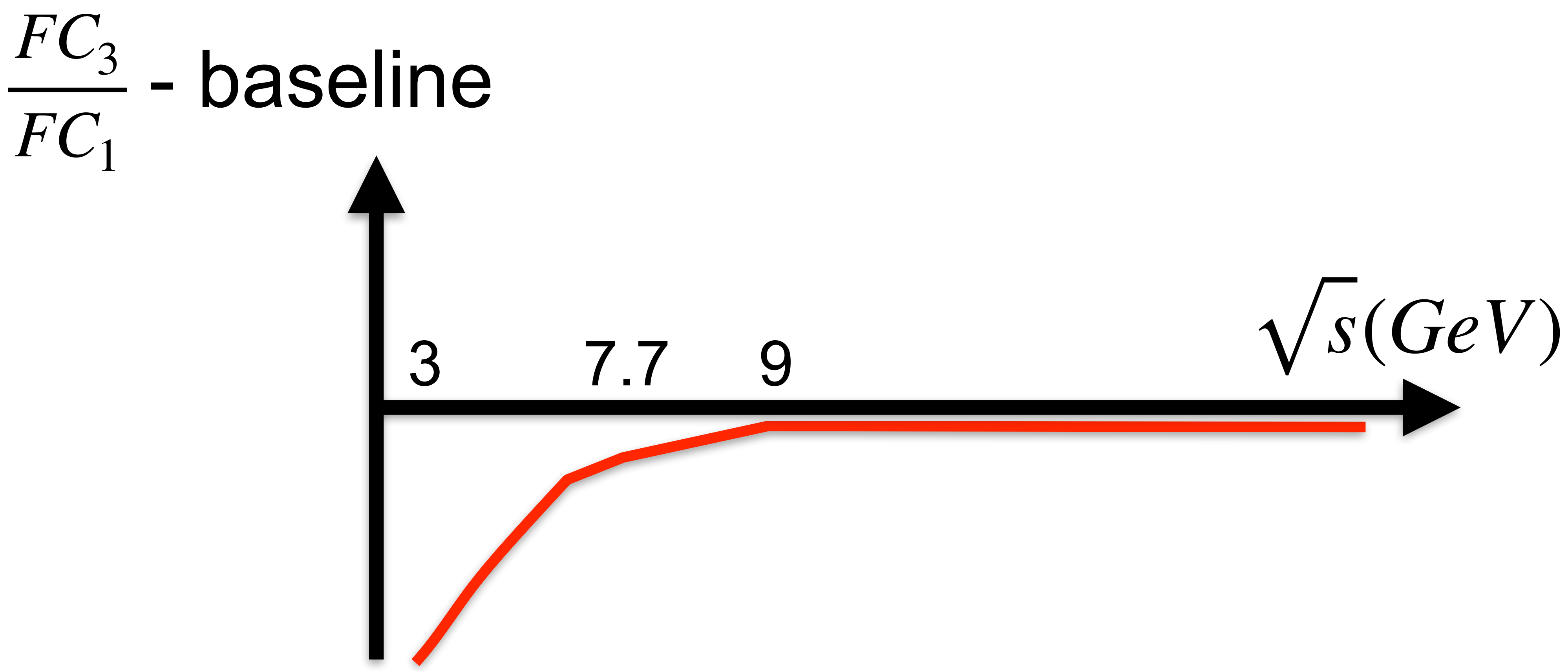
# New STAR data (BESII)

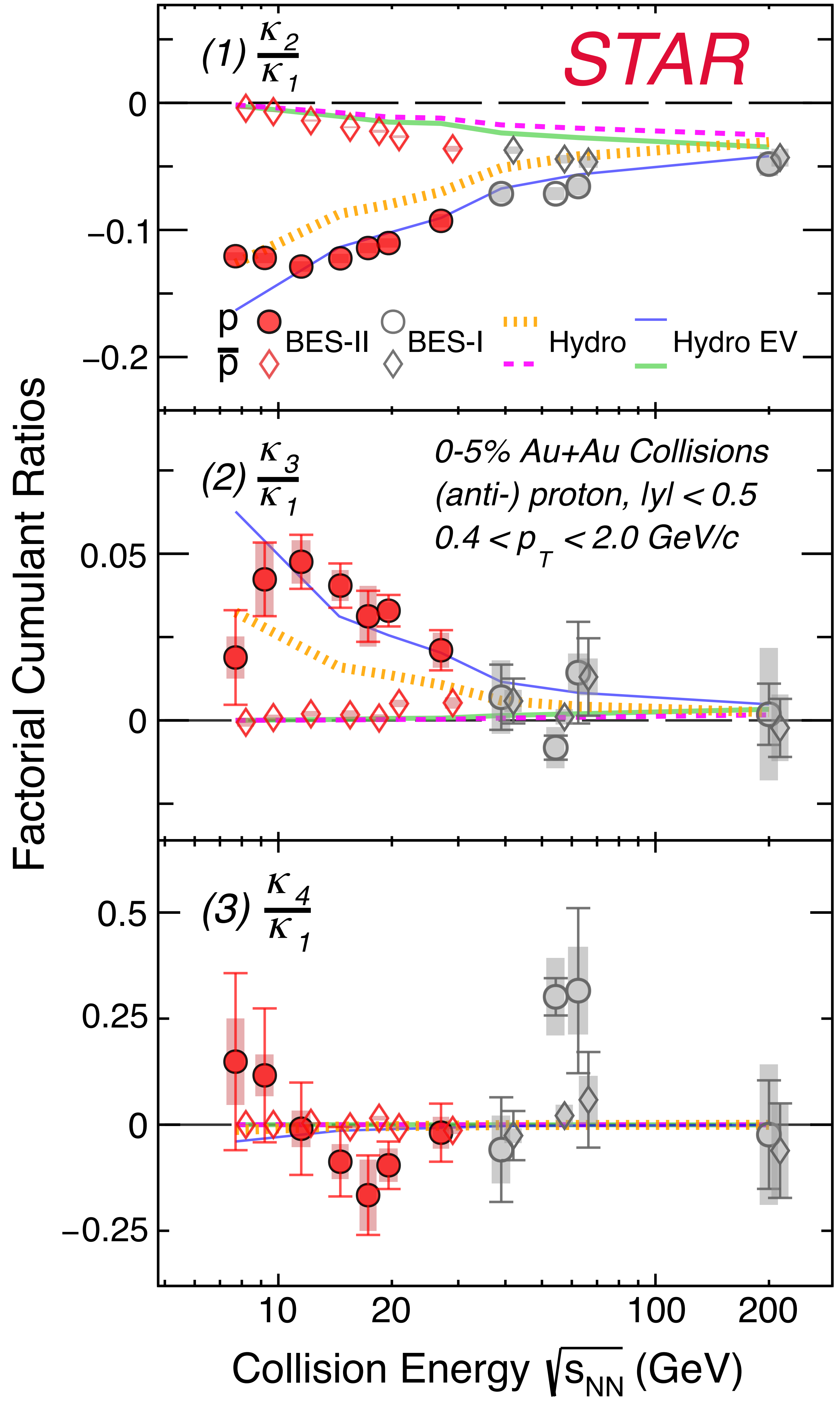


# The “signal” (relative to baseline)

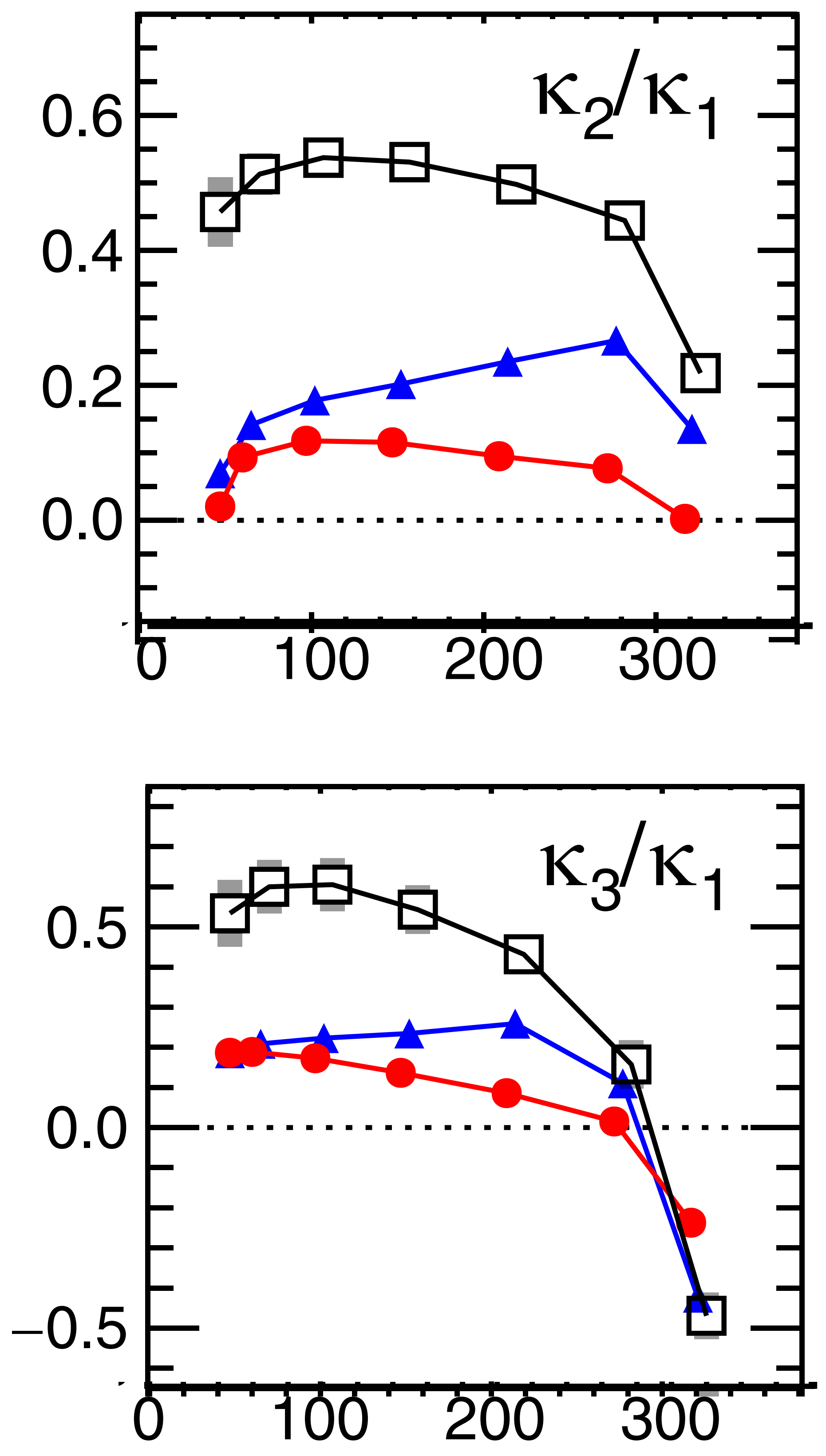


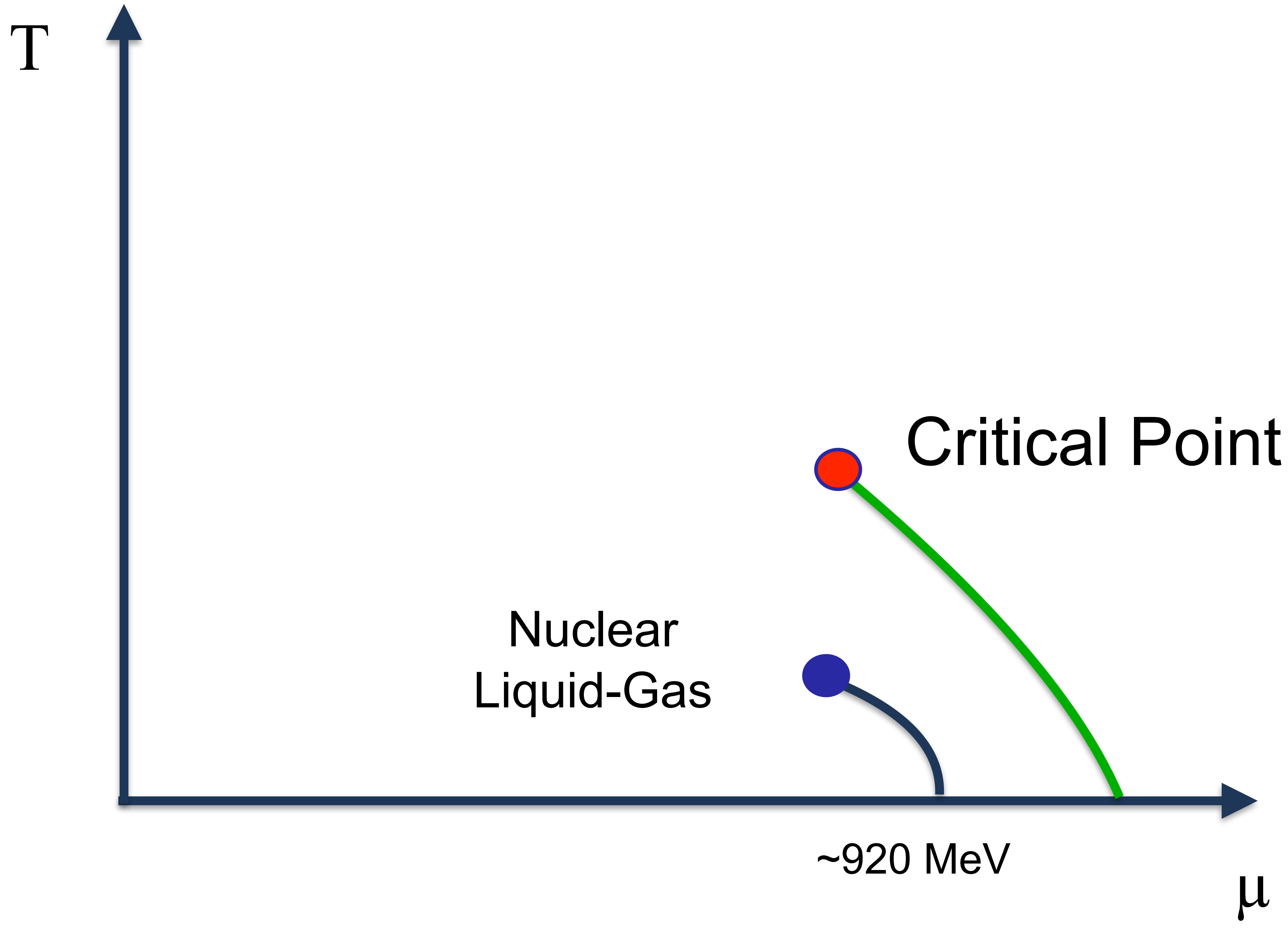



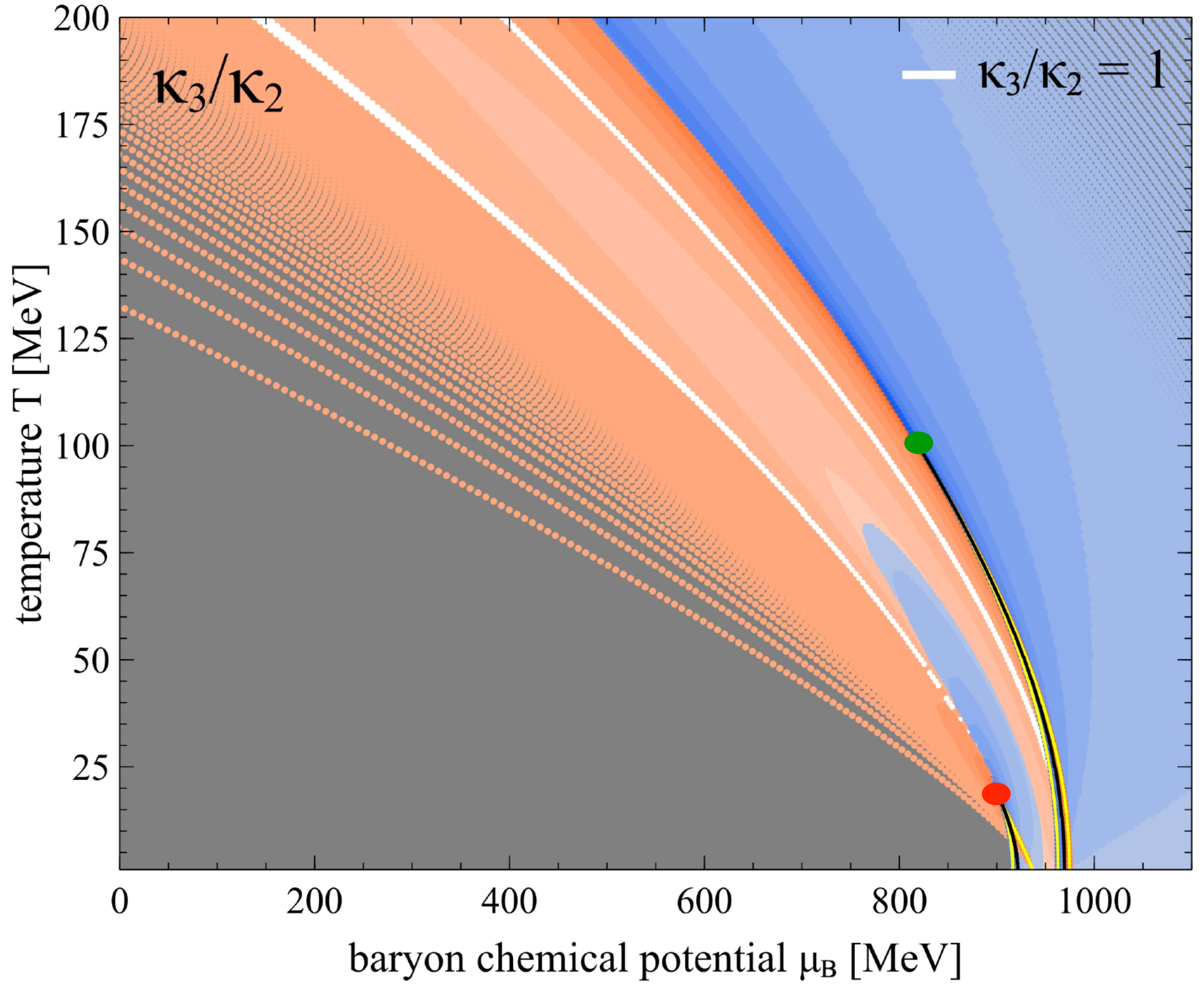

STAR 2209.11940

$\sqrt{s} = 3GeV$



# Liquid Gas?




Model calculation by Agnieszka Sorensen
arXiv:2011.06635

# Test for Baseline

A. Bzdak, V. Vovchenko and V.K. in preparation

Both global charge conservation and volume fluctuations are long range correlations

Factorial cumulant: $$C_n = \int_{\Delta_Y} dy_1 \cdots \int_{\Delta_Y} dy_n C(y_1, \cdots, y_n)$$

$C(y_1, \cdots, y_n)$: n-particle correlations function

Long range correlations: $C(y_1, \cdots, y_n) = const$ within $\Delta Y$

$$\Rightarrow C_n \sim (\Delta Y)^n$$

$$\Rightarrow \frac{C_n}{{C_1}^n} = const \text{ as function of } \Delta Y$$

# Baryon number conservation

Within acceptance:

$$P(n,\bar{n}) = \sum_{N,\bar{N}} B(n,N;\alpha)B(\bar{n},\bar{N},\bar{\alpha})P(N,\bar{N})$$

$$\alpha = \frac{\langle N\rangle_{\Delta Y}}{\langle N+\bar{N}\rangle_{4\pi}} \qquad \bar{\alpha} = \frac{\langle \bar{N}\rangle_{\Delta Y}}{\langle N+\bar{N}\rangle_{4\pi}}$$

$B(n,N,\alpha)$ Binomial distribution with Bernoulli prob $\alpha$

Factorial cumulants:

$$C_k(n;\Delta Y) = \alpha^n C_k(N,4\pi)$$

$$C_k(\bar{n};\Delta Y) = \bar{\alpha}^n C_k(\bar{N},4\pi)$$

analogous to “efficiency” corrections

$\Rightarrow \dfrac{C_k}{C_1{}^k} = const$ as function of $\Delta Y$ for both protons and anti protons

# Include volume fluctuations

Holzmann et al. 2403.03598

$$C_1[N] = \langle N_w \rangle C_1[n] = \langle N_w \rangle \langle n \rangle = \langle N \rangle \,,$$

$$C_2[N] = \bar{C}_2[N] + \langle N \rangle^2 \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} \,,$$

$$C_3[N] = \bar{C}_3[N] + 3 \langle N \rangle \bar{C}_2[N] \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} + \langle N \rangle^3 \frac{\kappa_3[N_w]}{\langle N_w \rangle^3} \,,$$

$$C_4[N] = \bar{C}_4[N] + 4 \langle N \rangle \bar{C}_3[N] \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} + 3\bar{C}_2^2[N] \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} + 6 \langle N \rangle^2 \bar{C}_2[N] \frac{\kappa_3[N_w]}{\langle N_w \rangle^3} + \langle N \rangle^4 \frac{\kappa_4[N_w]}{\langle N_w \rangle^4} \,.$$

Since $\bar{C}_n \sim \alpha^n \Rightarrow C_n \sim \alpha^n$

$\bar{C}_n$ : Factorial cumulant WITHOUT volume fluctuations

$C_n$ : Factorial cumulant WITH volume fluctuations

If $\dfrac{C_k}{C_1{}^k} \neq const$ as function of $\Delta Y$: Some other (short range) physics is at play as well (Example: excluded volume)

# Summary

- Baryquark matter is energetically favored over Quarkyonic matter
- Pauli blocking of the quark sector sets in at $\rho \simeq 1 - 2\rho_0$
- Consequences: Larry's talk

- STAR has deliver on the BESII data
  - cannot hide behind errorbars anymore
- Interpretation requires some care
  - we won't get better data in this energy regime anytime soon
- possible test of a baseline involving baryon number conservation and volume fluctuations

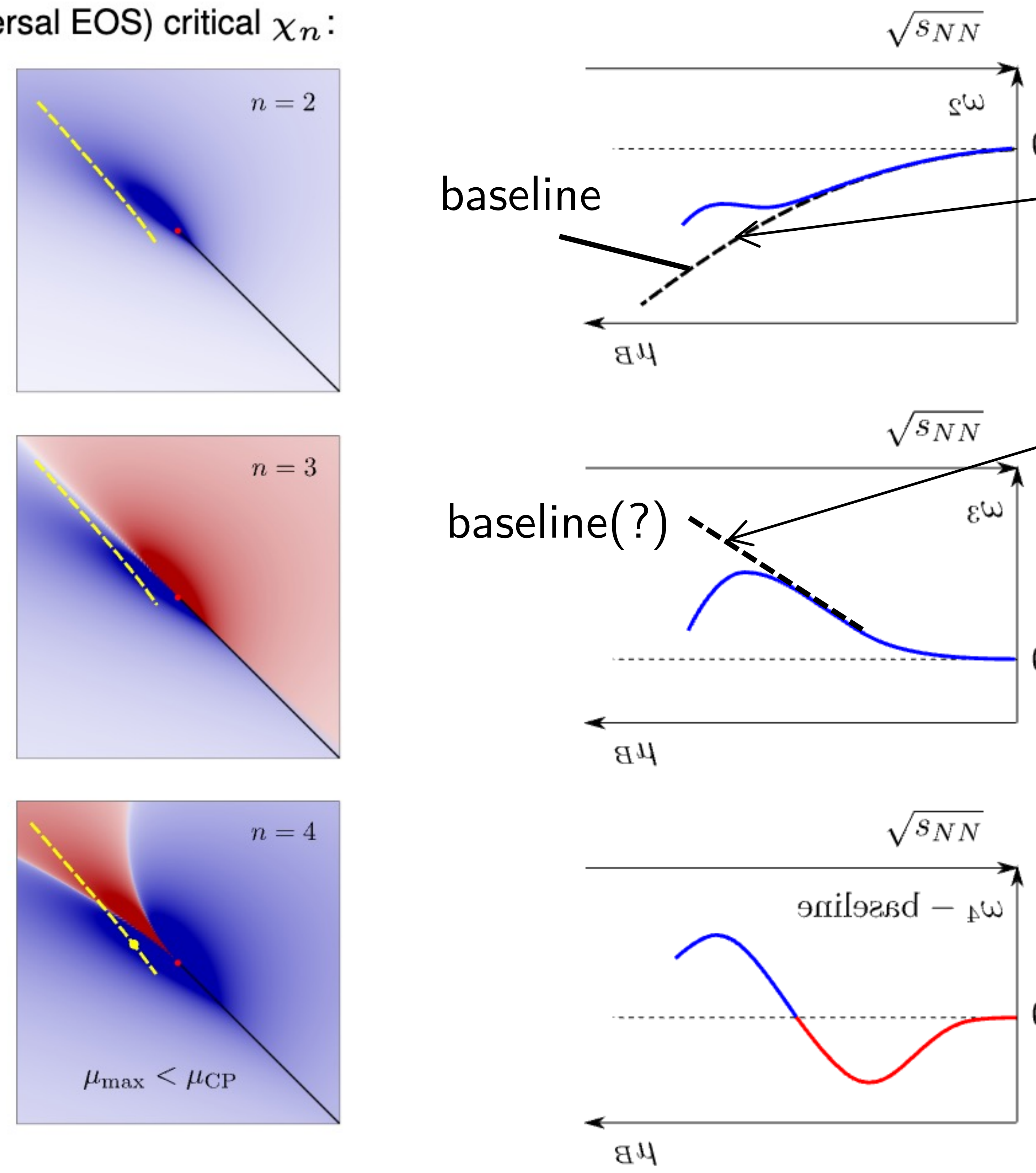
# Backup

# Factorial cumulants from RHIC-BES-II

From M. Stephanov (SQM2024):

$\omega_n = \hat{C}_n / \hat{C}_1$

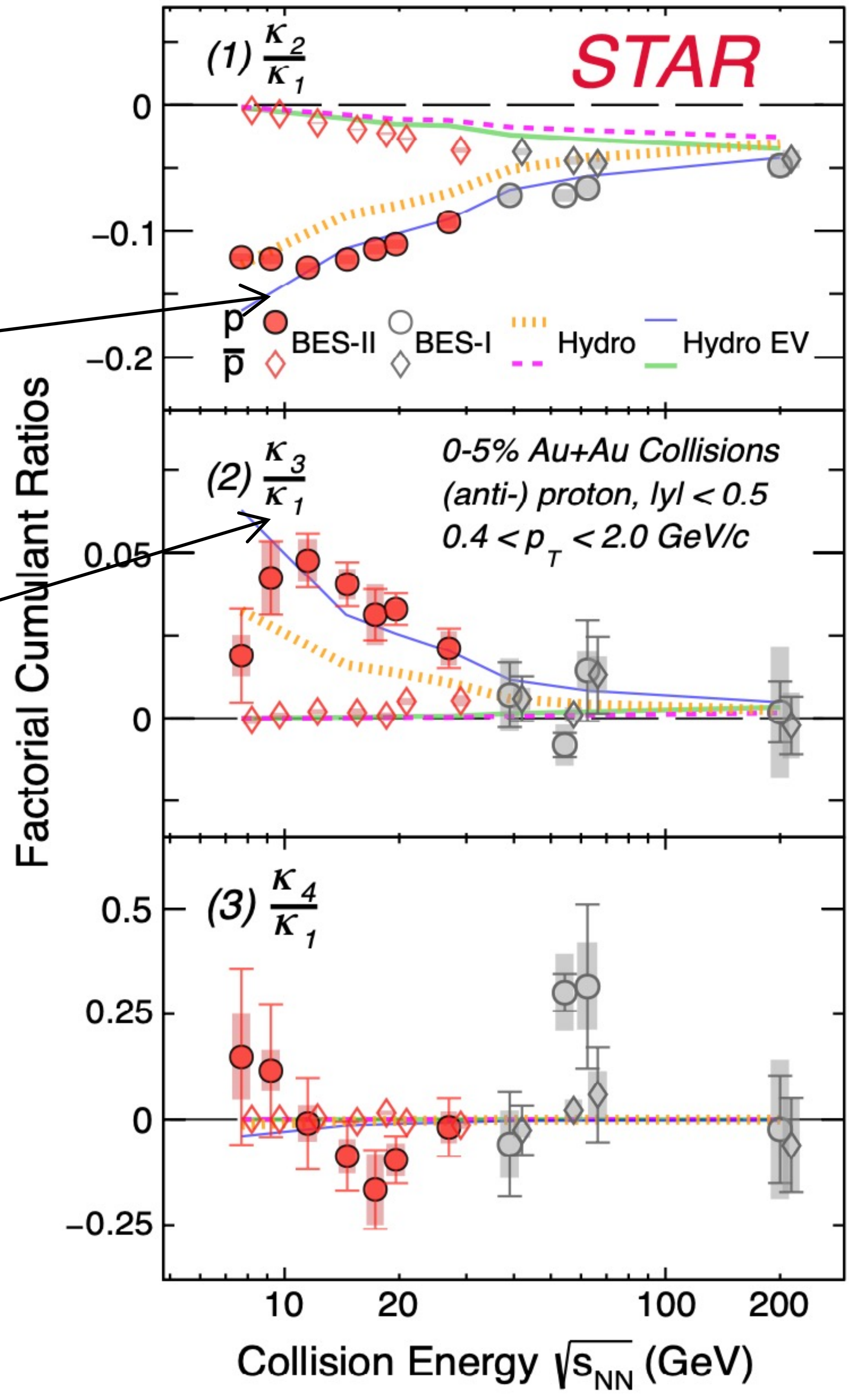
(universal EOS) critical $\chi_n$:




Bzdak et al review 1906.00936

Expected signatures: bump in $\omega_2$ and $\omega_3$, dip then bump in $\omega_4$ for CP at $\mu_B > 420$ MeV

STAR data:




A. Pandav, CPOD2024

baseline (hydro):

VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

- describes right side of the peak in $\hat{C}_3$
- implies
  - *positive* $\hat{C}_2$ − baseline $> 0$
  - *negative* $\hat{C}_3$ − baseline $< 0$

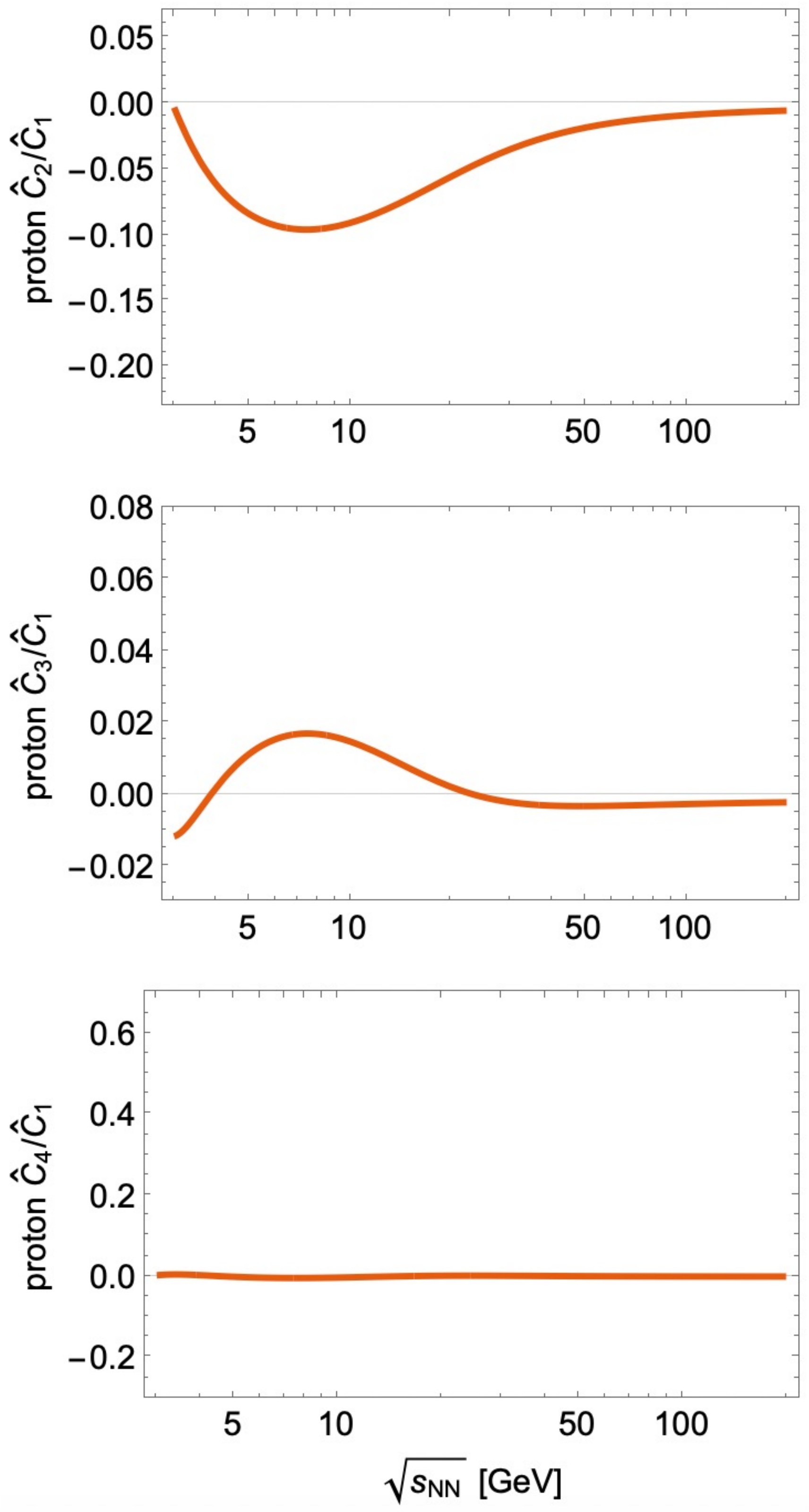
Vovchenko, RHIC-AGS Users meeting, June 2024

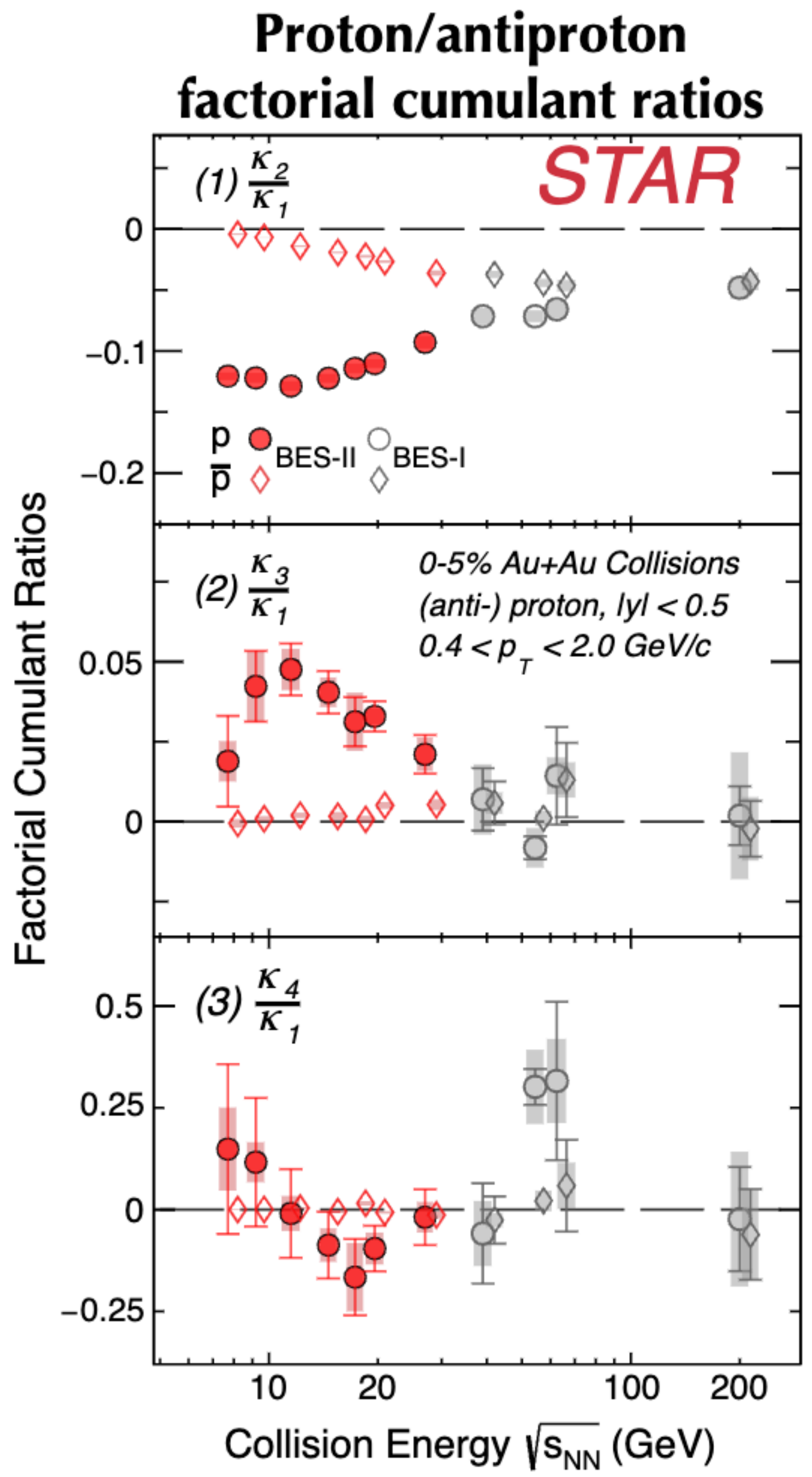
# Factorial cumulants and nuclear liquid-gas transition

UNIVERSITY OF HOUSTON

Calculation in a van der Waals-like HRG model along the freeze-out curve*

VV, Gorenstein, Stoecker, EPJA 54, 16 (2018)




NB: The calculation is grand-canonical

*Poberezhnyuk et al., PRC 100, 054904 (2019)

Vovchenko, RHIC-AGS Users meeting, June 2024

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