EMMI Workshop
at the
University of Wrocław

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Aspects of Criticality II

Bottomonium spectral functions in thermal QCD

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FASTSUM Collaboration

COSMIC WISPERS

EuroHPC Joint Undertaking

cost EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

LA VA lattice virtual academy

STRONG 2020

FASTSUM Collaborators for Bottomonium

Gert Aarts, Chris Allton, Naeem Anwar,
Ryan Bignell,
Tim Burns,
Rachel Horsham d'Arcy,
Ben Jäger, Seyong Kim, MpL, Benjamin Page, Sinead Ryan, Jon-Ivar Skullerud,
Antonio Smecca,
Tom Spriggs

Review with references:
Prog.Part.Nucl.Phys. 133 (2023) 104070

+ *Unpublished work*

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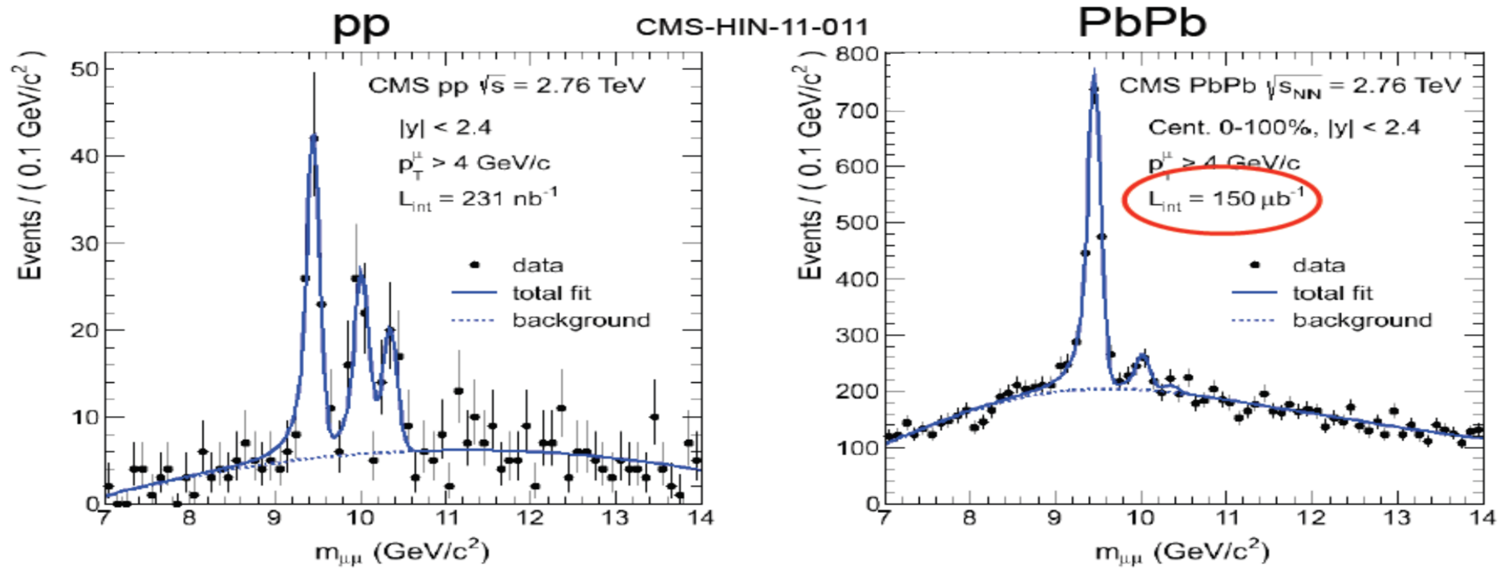
Bottomonium as a probe of QGP

A long history



$N_{\Upsilon(2S)}/N_{\Upsilon(1S)}|_{pp} = 0.56 \pm 0.13 \pm 0.01$

$N_{\Upsilon(3S)}/N_{\Upsilon(1S)}|_{pp} = 0.21 \pm 0.11 \pm 0.02$

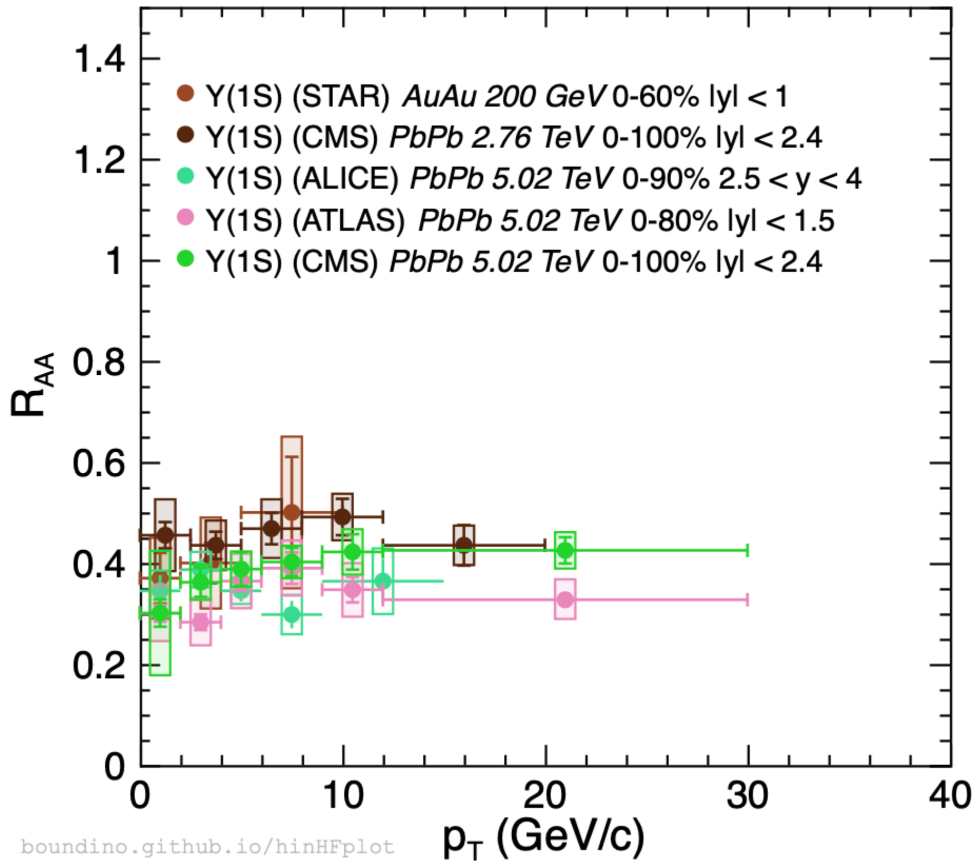
$N_{\Upsilon(2S)}/N_{\Upsilon(1S)}|_{PbPb} = 0.12 \pm 0.03 \pm 0.01$

$N_{\Upsilon(3S)}/N_{\Upsilon(1S)}|_{PbPb} < 0.07$

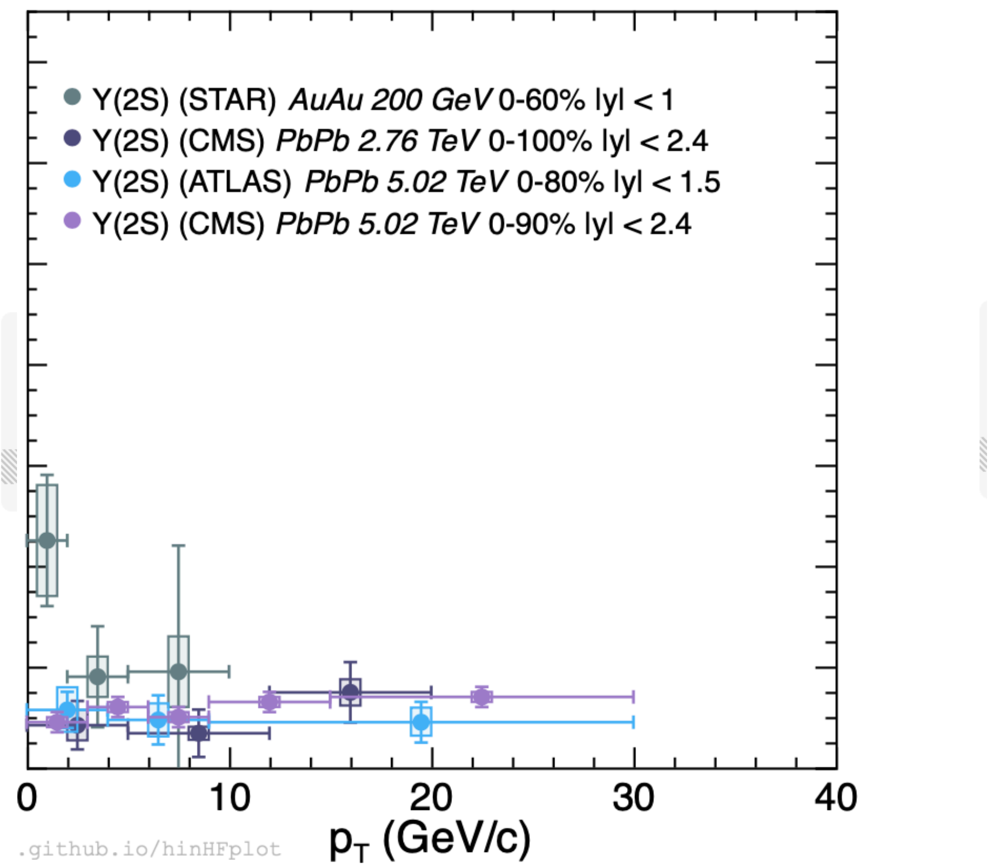
CMS
Eur.Phys.J. C76 (2016) no.3, 107

..still continuing ..

Plots from Boundino



→ PRL 130 (2023) 112301 → PLB 770 (2017) 357
→ PLB 822 (2021) 136579 → PRC 107 (2023) 054912
→ PLB 790 (2019) 270



→ PRL 130 (2023) 112301 → PLB 770 (2017) 357
→ PRC 107 (2023) 054912 → arXiv:2303.17026

Bottomonium — starting from the summary:

Exp/Pheno:

Beauty sector: good overall consistency of the following facts:

- Similar production of Υ(1S) from RHIC -> LHC
- Higher states strongly suppressed
- Washing out of the spectral function (but the Υ(1S) which survive up to T = 0.45 GeV)

Not paying too much attention at CNM effects:

Paul Gossieaux @ SQM2024

Lattice   Bottomonium spectral functions:

Methods based on inverse Laplace : no clear winner

Alexander Rothkopf 2211.10680
Ch. Allton in *Prog.Part.Nucl.Phys.* 133 (2023) 104070

Rationale :

Laplace transform inversion works on continuum models

Fitting models help recover missing information

A good fitting model is a necessary requirement

Salvatore Cuomo in *Prog.Part.Nucl.Phys.* 133 (2023) 104070

Plan

Two paths to spectral functions: inversion and analytic continuation

Overview of bottomonium results from inversion methods

Living in Euclidean space : sum rules and ‘moments’

The spectral function is defined as:

$$\rho(\omega, \vec{p}) = -\frac{1}{\pi} \mathrm{Im}\, D^R(\omega, \vec{p}).$$

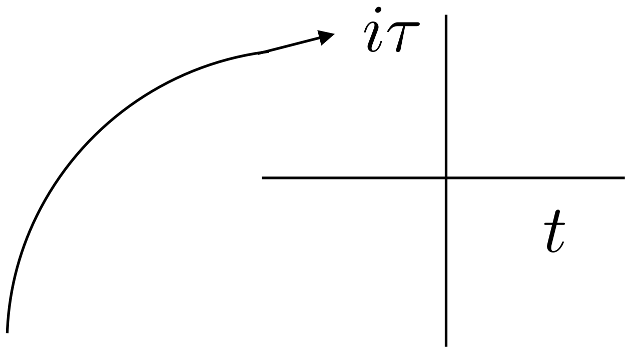
$$D^E(p_0) = \int_{-\infty}^{\infty} d\tau D^E(\tau) \exp(ip_0\tau),$$

$$D^E(p_0) = \frac{1}{TN} \sum_{n=-N/2}^{N/2-1} D^E\left(\frac{n}{NT}\right) \exp\left(ip_0 \frac{n}{NT}\right) \quad p_0 = 2n\pi T$$

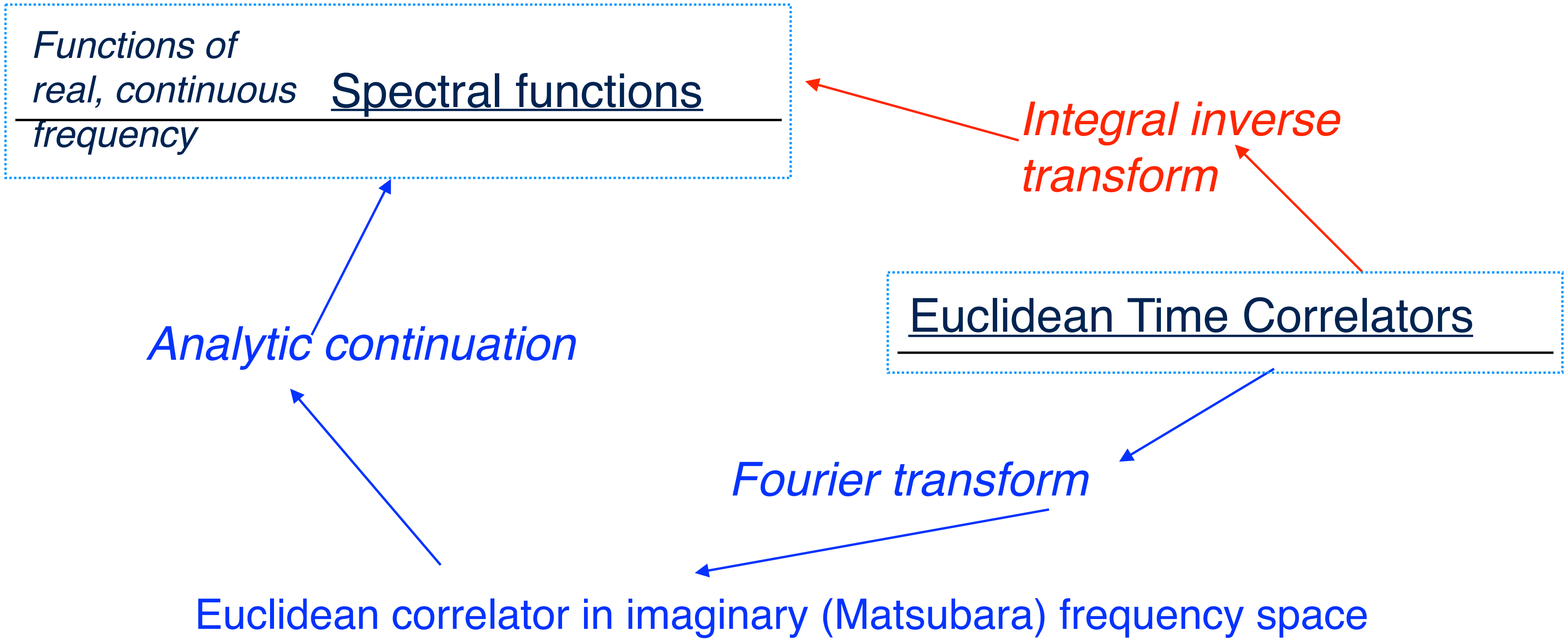
$$D^R(\omega, \vec{p}) = -D^E(p_0 \to \mathrm{i}\omega - \epsilon, \vec{p}),$$

omega real time energy

From Correlators to Spectral functions



Computed on the lattice: Euclidean (imaginary) Time Correlators



The 'red' path simplified:

Relativistic

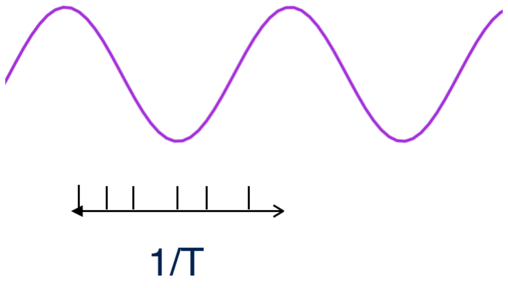
$$D(\tau) = \int_0^{\infty} \frac{e^{-\tau\omega} + e^{-(\beta-\tau)\omega}}{1 - e^{-\beta\omega}} S(\omega) d\omega$$

Non-relativistic

$$D(\tau) = \int_{-M_0}^{\infty} e^{-\tau\omega} S(\omega) d\omega$$

Relativistic propagators:

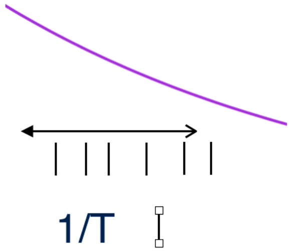
$$G(\tau) = G(-\tau + 1/T)$$



Inverse Laplace: makes life easier..

Non-relativistic propagators : only forward

$$G(\tau) \neq G(-\tau + 1/T)$$



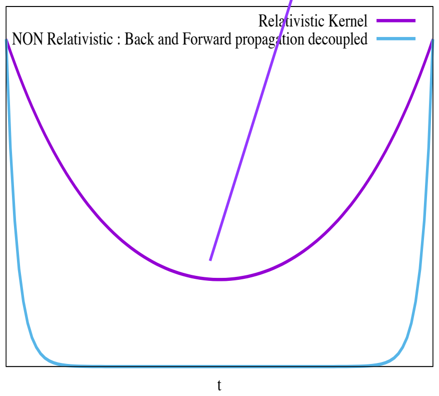
Relativistic and non-relativistic kernels

$$K(\tau,\omega) = \frac{\left(e^{-\omega\tau} + e^{-\omega(1/T-\tau)}\right)}{1 - e^{-\omega/T}}.$$

$$K(\tau,\omega) \simeq \left(e^{-\omega\tau} + e^{-\omega(1/T-\tau)}\right):$$

QCD

K



1/T

NRQCD

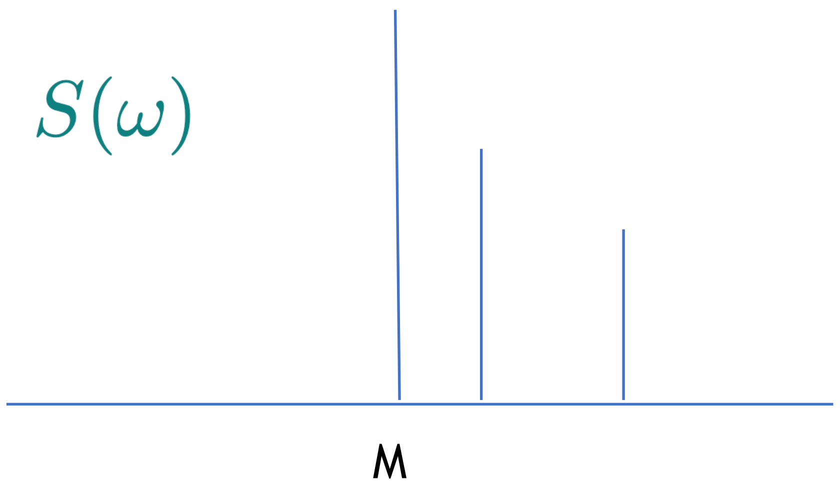
In practice one retains only n=0

$$
\begin{aligned}
D(\tau) &= \sum_{n=0}^{\infty}\int_{-\infty}^{\infty} e^{(-\beta\omega)^n} e^{-\tau\omega} S(\omega)d\omega \\
&= \sum_{n=0}^{\infty}\int_{-\infty}^{\infty} e^{-\omega(\tau+n\beta)} S(\omega)d\omega
\end{aligned}
$$

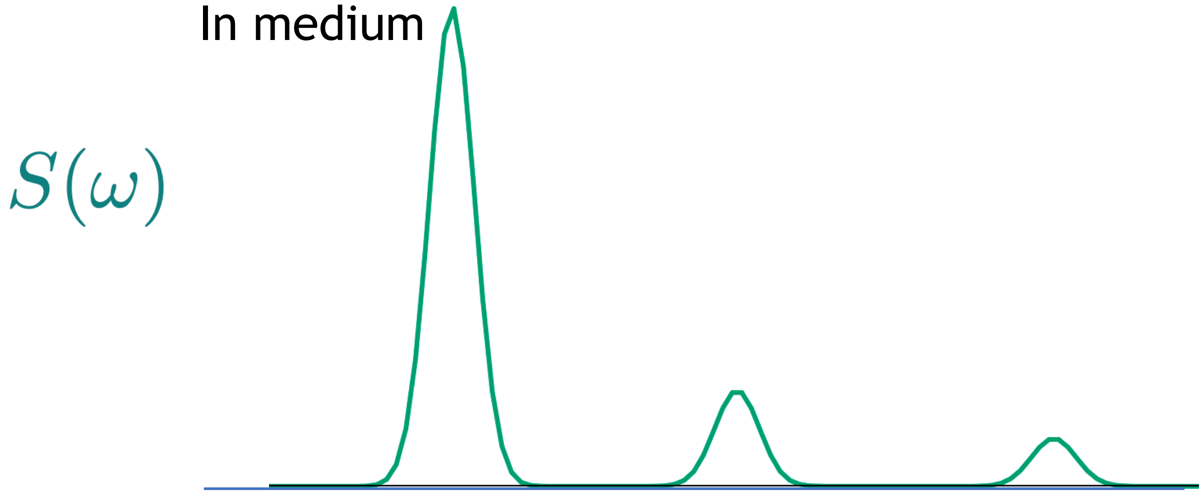
and periodicity is lost

Spectral functions and two point functions : a challenge for LFT

In vacuum



In medium



$$G(t) = \int \delta(M-\omega)e^{-\omega t} \propto e^{-Mt}$$

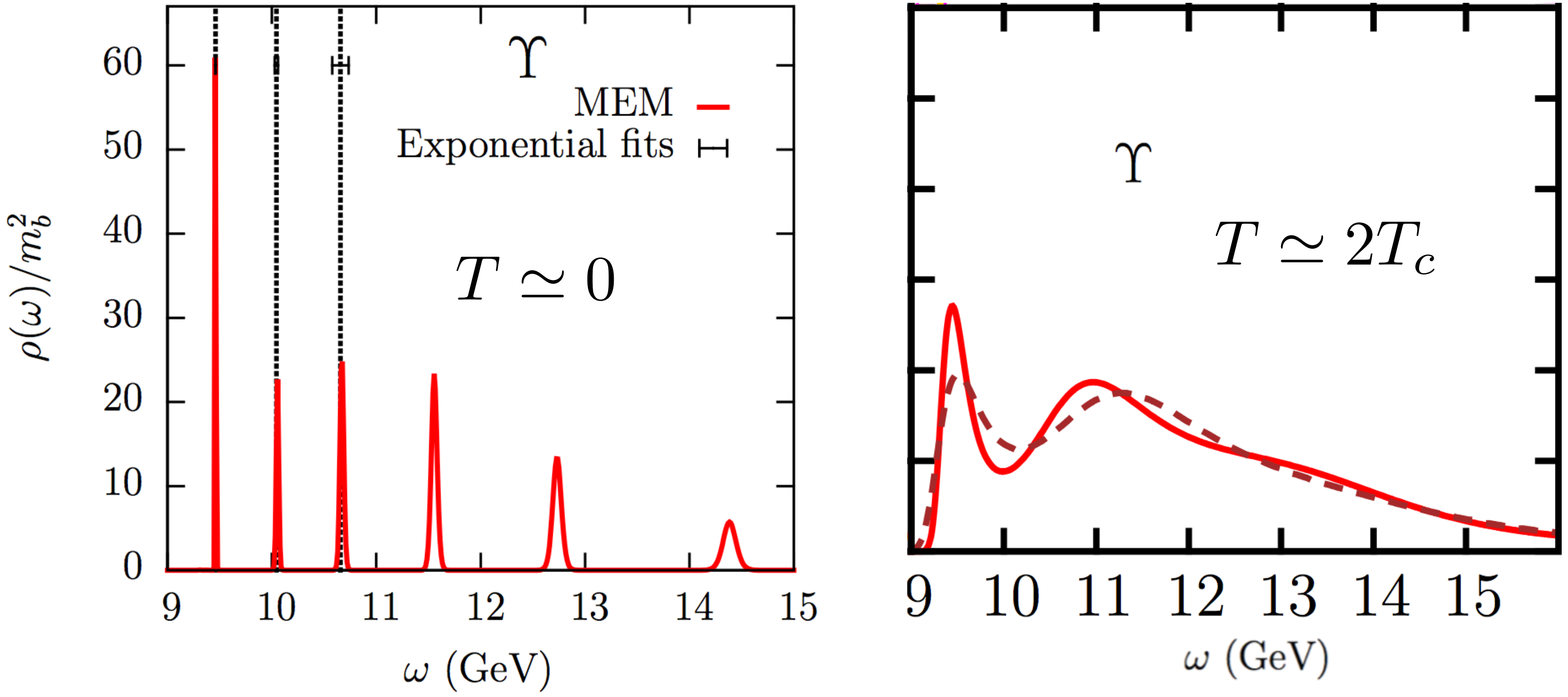
$$G(t) = \int S(\omega)e^{-\omega t}$$

Bottomonium via NRQCD

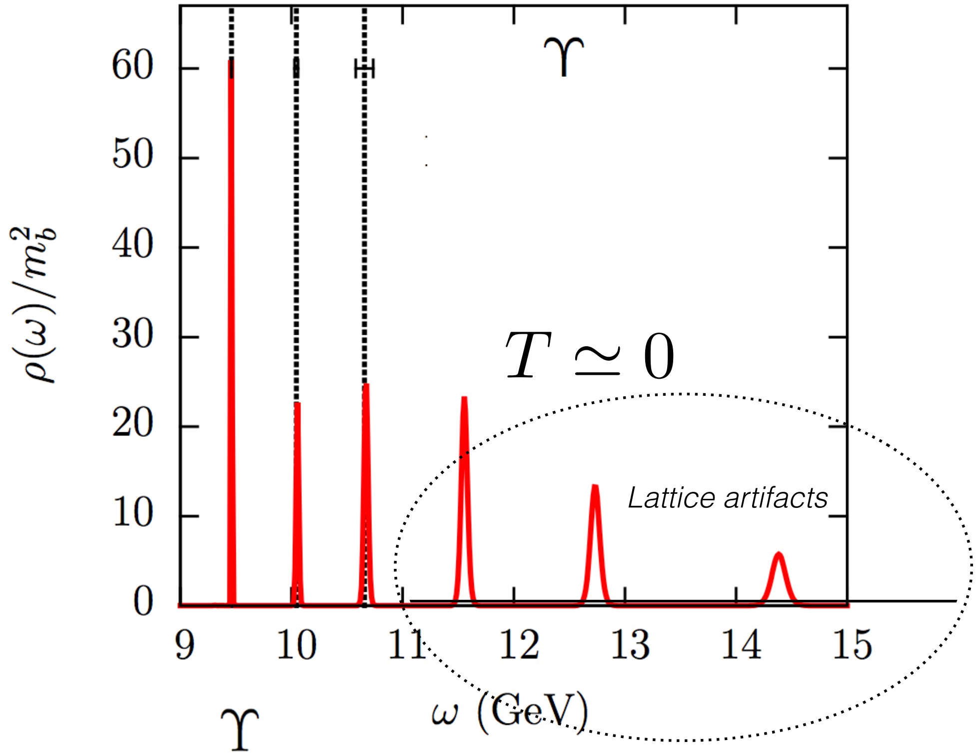
Zero temperature NRQCD works beautifully for the spectrum

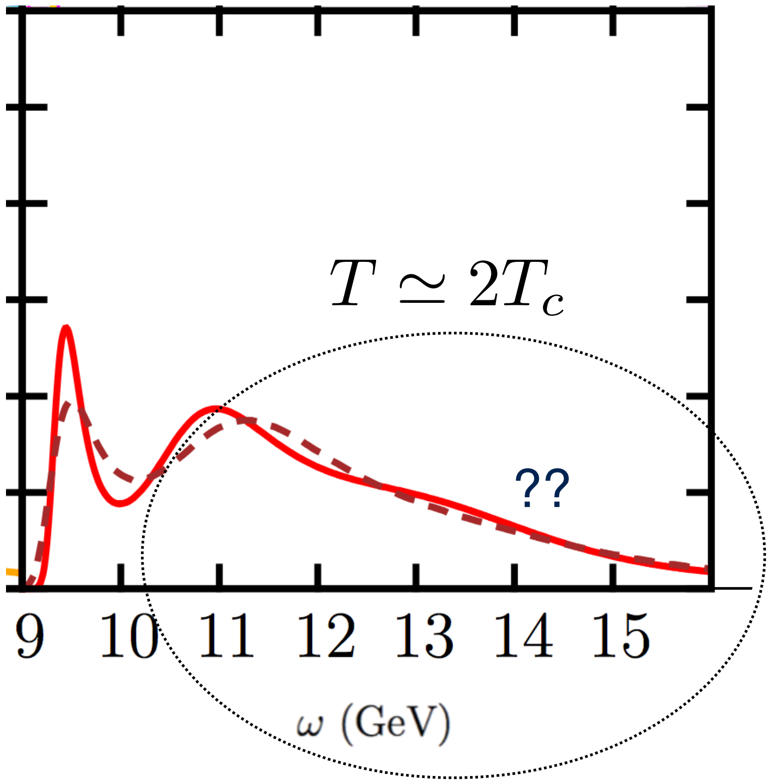
$n^{S+1}L_J$	State	$a_\tau M$	$E_0 + M$ (MeV)	M_{expt} (MeV)
$1^1\mathrm{S}_0$	η_b	0.20549(4)	9409(12)	9398.0(3.2)
$2^1\mathrm{S}_0$	η_b'	0.311(3)	10004(21)	9999(4)
$1^3\mathrm{S}_1$	Υ	0.21460(5)	9460*	9460.30(26)
$2^3\mathrm{S}_1$	Υ'	0.318(3)	10043(22)	10023.26(31)
$1^1\mathrm{P}_1$	h_b	0.2963(4)	9920(15)	9899.3(1.0)
$1^3\mathrm{P}_0$	χ_{b0}	0.2921(4)	9896(15)	9859.44(52)
$1^3\mathrm{P}_1$	χ_{b1}	0.2964(4)	9921(15)	9892.78(40)
$1^3\mathrm{P}_2$	χ_{b2}	0.2978(4)	9928(15)	9912.21(40)

Bottomonium spectral functions from the lattice



Bottomonium spectral functions from the lattice





Issues:

Control the systematics!

FASTSUM setup

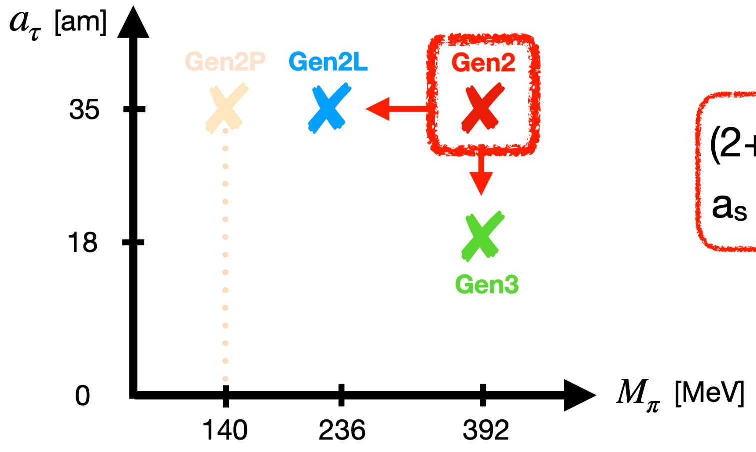
Review by C.Allton:

https://www.ggi.infn.it/talkfiles/slides/slides5843.pdf

Study of Numerical Methods

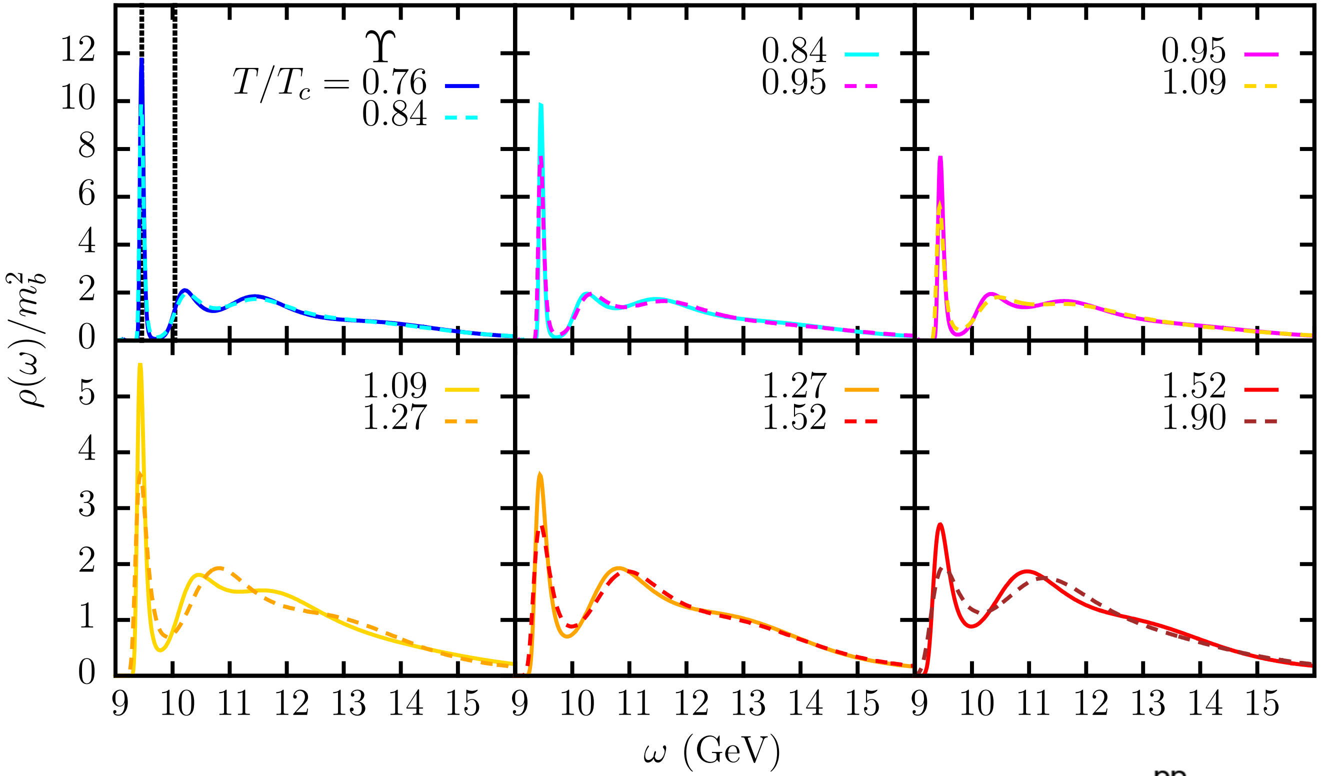
1. Exponential (Conventional δ f'ns)
2. Gaussian Ground State (+ δ f'n excited)
} Maximum Likelihood (Minimise χ^2)
3. Moments of Correlation F'ns — Direct Method - "no" fit
4. BR Method
5. Maximum Entropy Method
} Bayesian Approaches
6. Kernel Ridge Regression — Machine Learning
7. Backus Gilbert — from Geophysics

Lattice Parameters



(2+1) flavour
a_s ~ 0.12 fm

Upsilon's spectral functions from MEM (NRQCD)

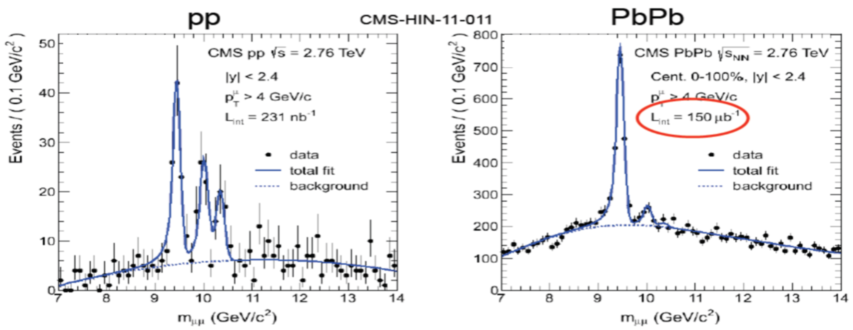


Persistence of the ground state at all temperatures

Melting of excited states

Modifications of the ground state

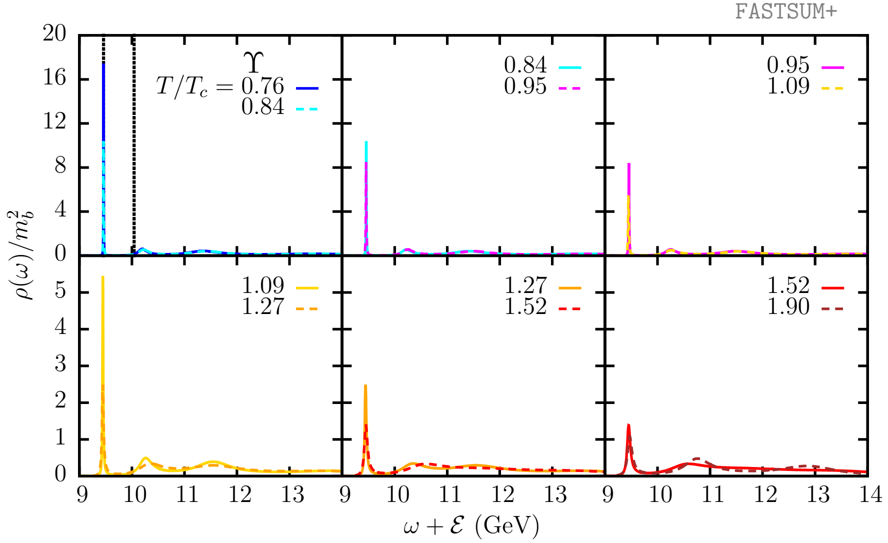
Pattern reminiscent of experimental observations



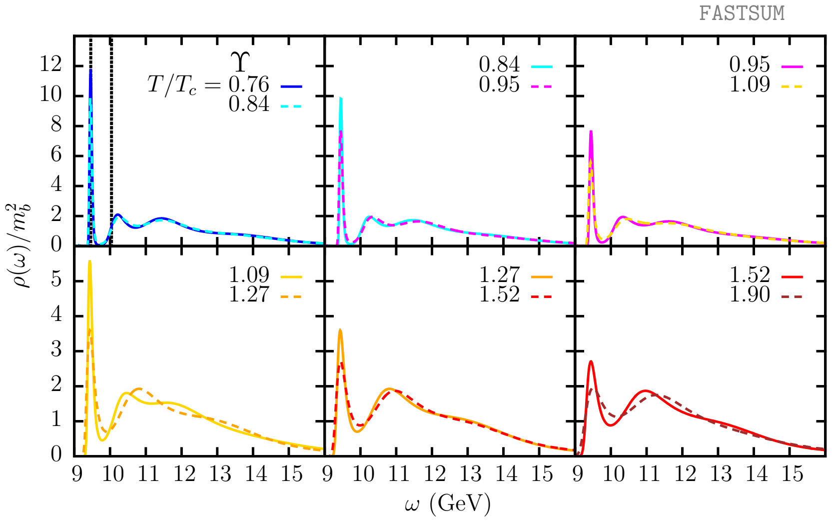
MEM vs BR

Y. Burnier, A. Rothkopf 2013

BR



MEM



FASTSUM +
Y. Burnier and
A. Rothkopf 2015

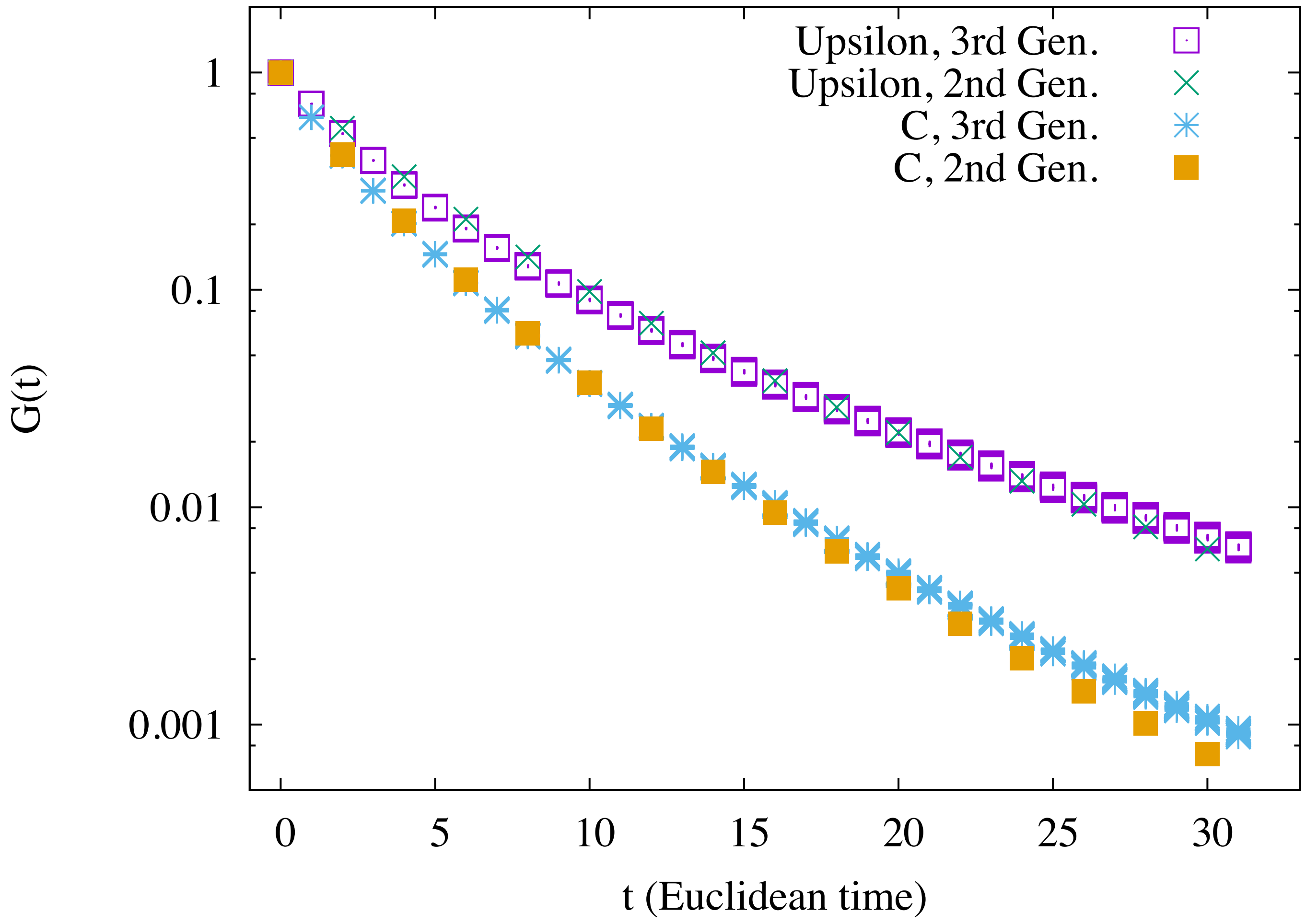
The same set of correlators has been analized by standard MEM and by the Burnier-Rothkopf method

Can we trust the width?

Y. Burnier, A. Rothkopf 2013

Going to
a finer lattice:

a τ from 35 to 18 am   T = 1.9Tc Nt=32



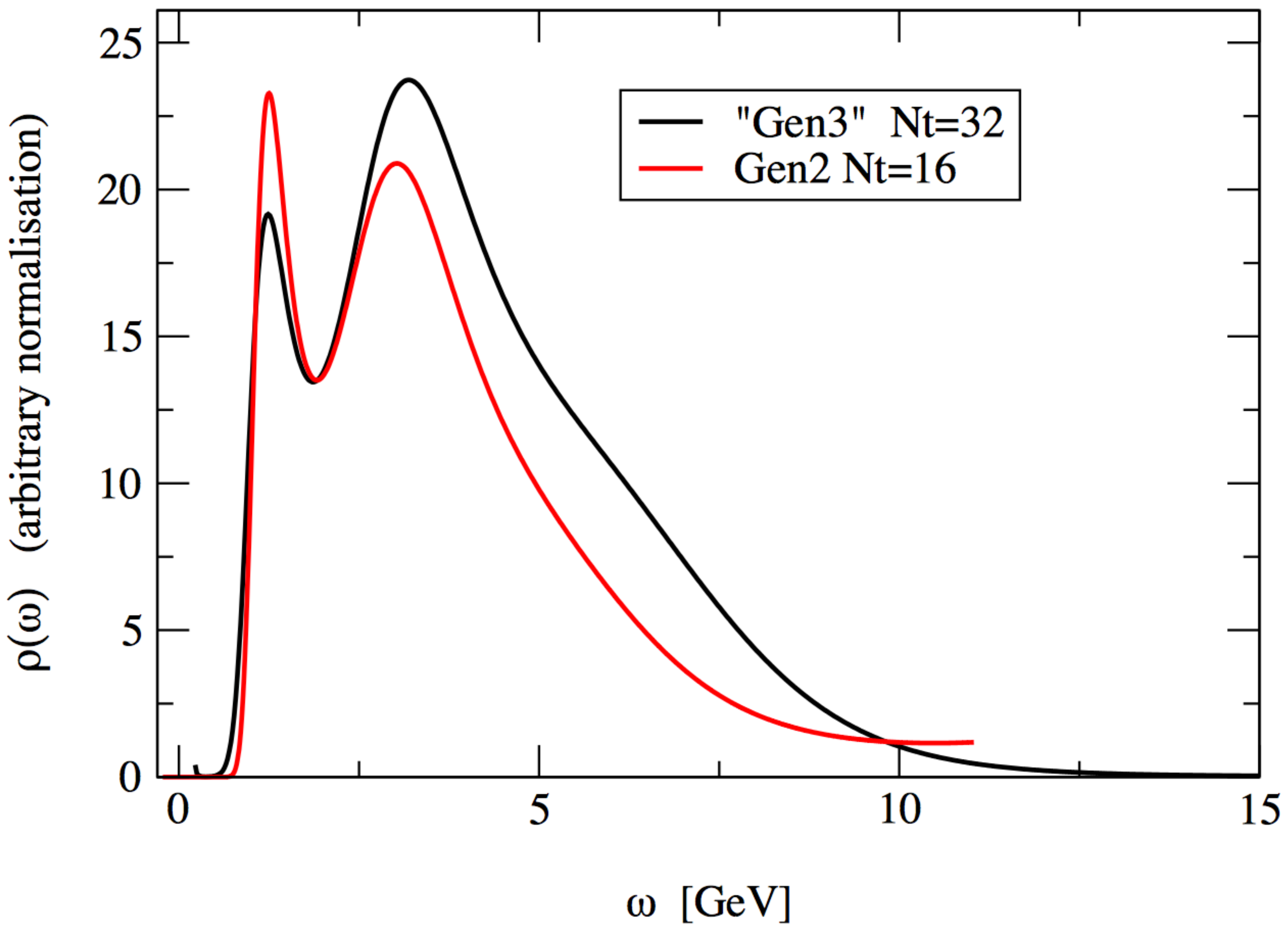
Good agreement for
Υ and χ
rescaled propagators.

Caveat:
Small discrepancies
may hide important
differences

Thanks to NPQCD
for parameters tuning

Υ

Spectral function

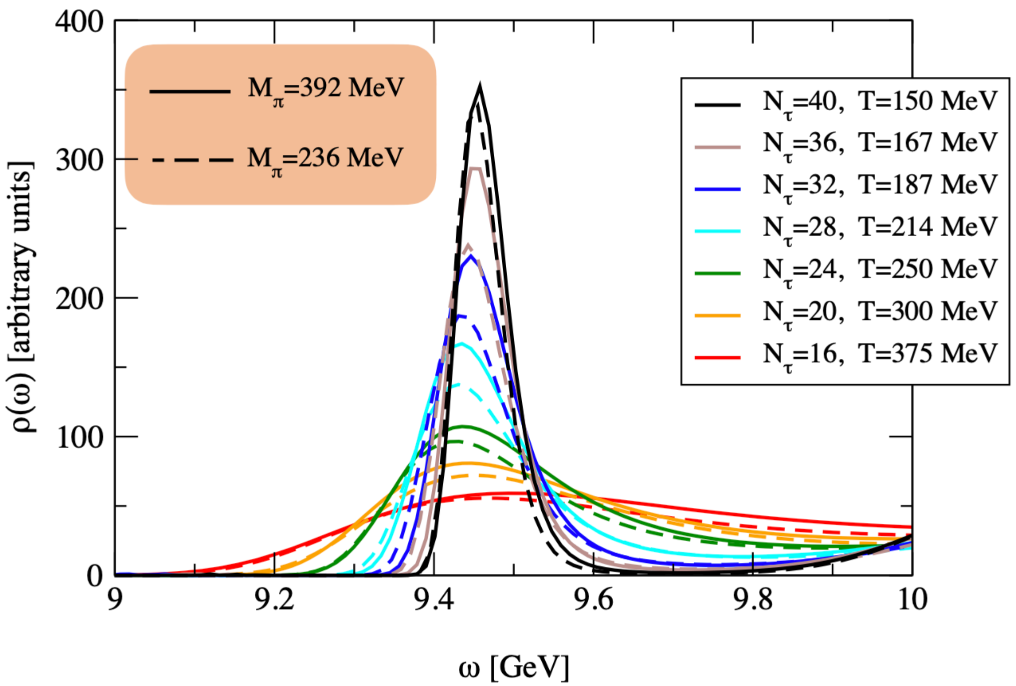


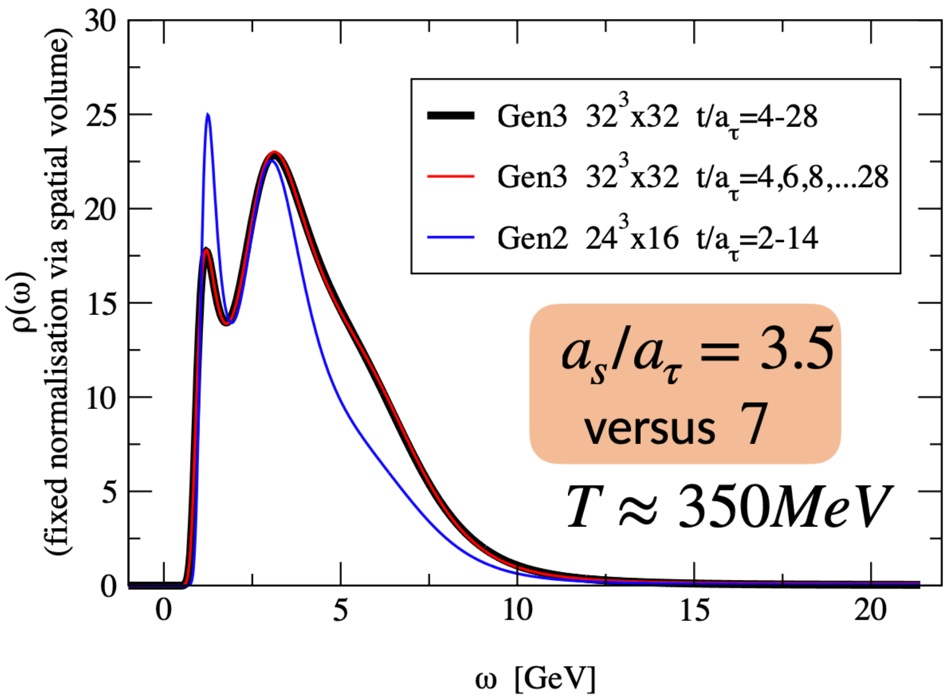
Lattice systematics - are "small"

Slide by C. Allton

Going lighter $m_q \searrow$

Going finer $a_\tau \searrow$

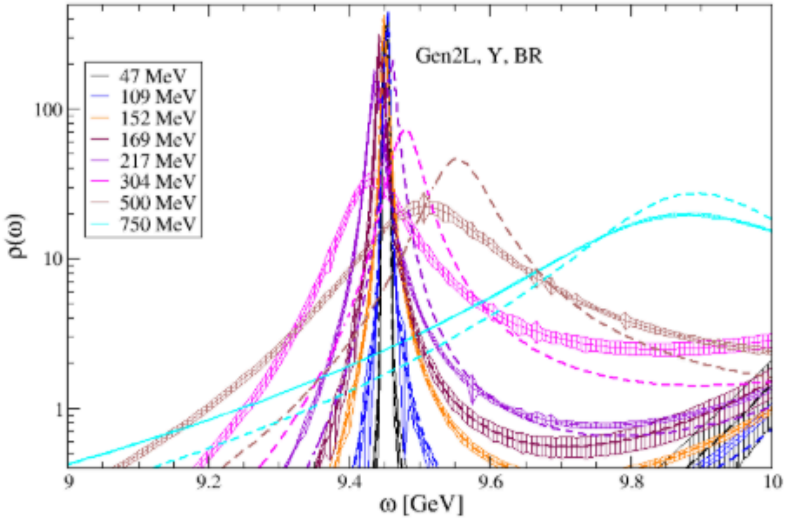




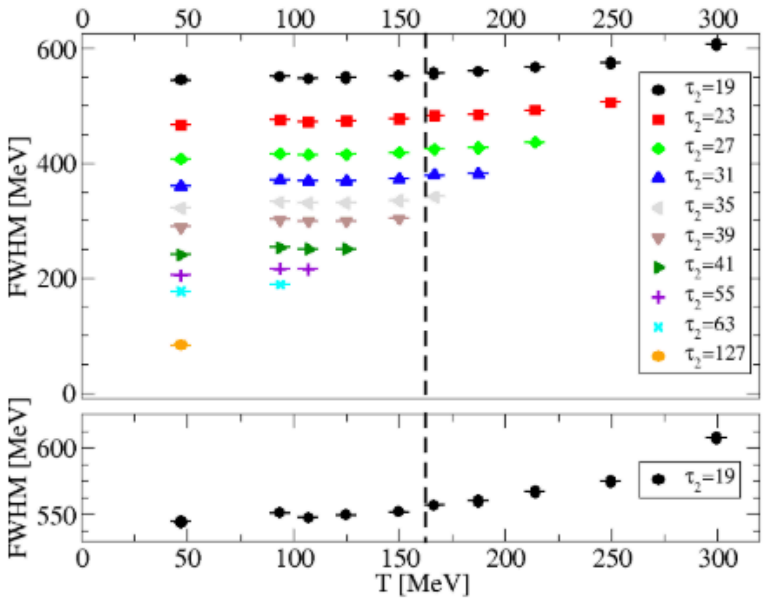
w/o NRQCD additive constant

Understanding and quantifying the systematics

FASTSUM *EPJ Web Conf*. 274 (2022) 05011

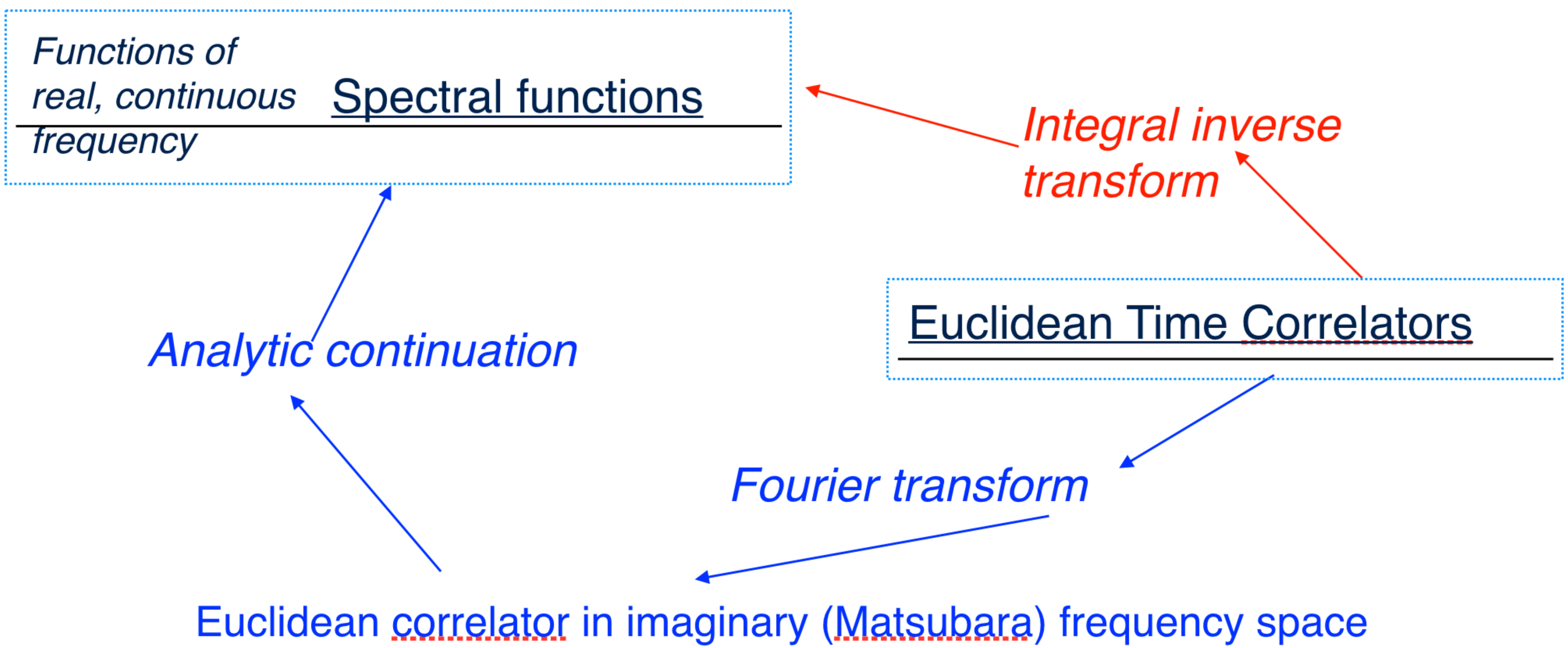


Sensitivity to the details of the implementation of the BR method



Sensitivity to the details of a Gaussian ansatz

Interlude

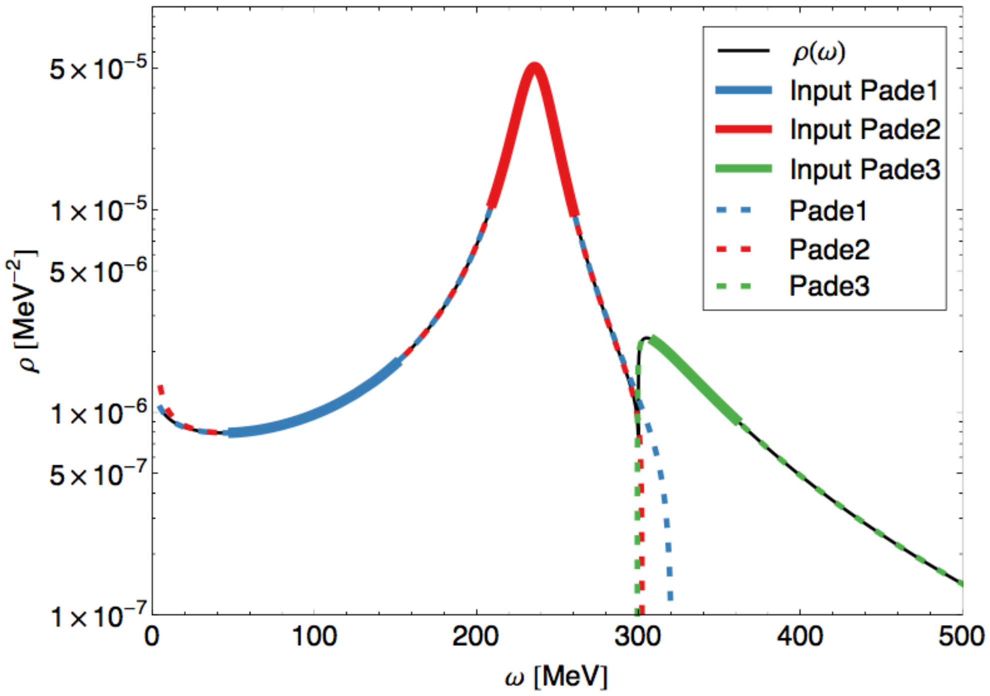


Functions of
real, continuous
frequency
Spectral functions
Integral inverse
transform
Euclidean Time Correlators
Analytic continuation
Fourier transform
Euclidean correlator in imaginary (Matsubara) frequency space

Crucial step : Analytic continuation

Try RVP - a variant of Pade' approximants which has proven very performing

Tripolt, Haritan, Wambach,Moiseyev



Model spectral function

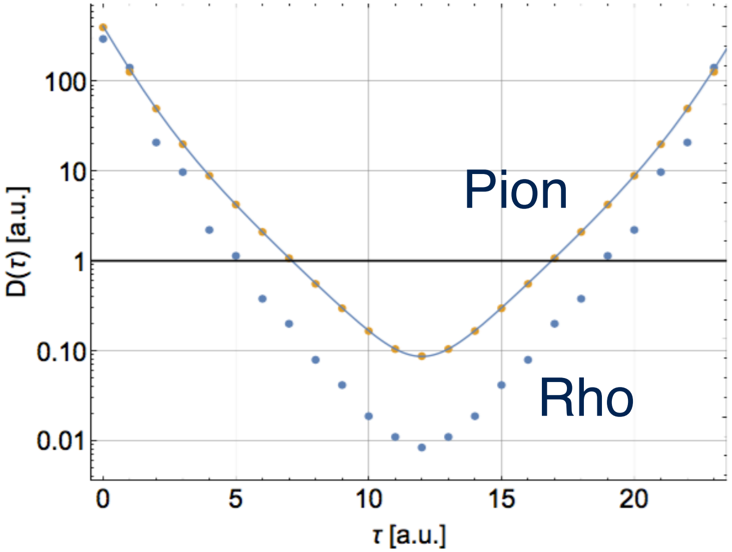
Use Pade' in form:

$$C_M(\eta) = \cfrac{F(\eta_1)}{1 + \cfrac{z_1(\eta-\eta_1)}{1+\cfrac{z_2(\eta-\eta_2)}{\vdots\, z_M(\eta-\eta_M)}}},$$

$$C_M(\eta_i) = F(\eta_i), \;\; i = 1, 2, \ldots, M.$$

..partial and preliminary..
to get a feeling on the possibilities ..

Tripolt, Rothkopf, MpL



Propagators in the conformal phase of QCD (Nf=12)

NB: pion is not a pseudoGoldstone

Under discussion: behaviour of propagators in the conformal window?

$$D(\tau) = \sum a_i e^{-m_i \tau}$$

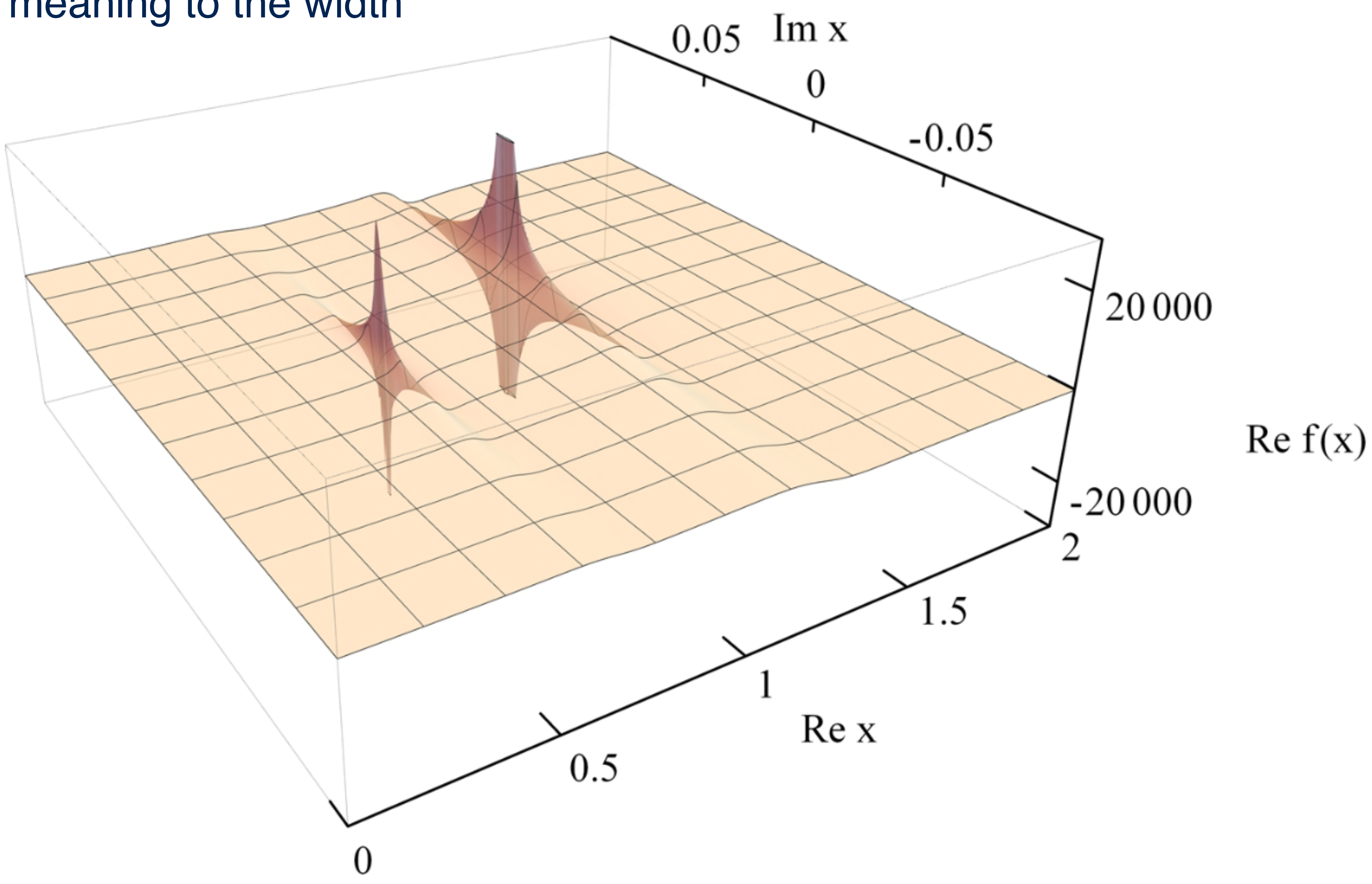
or

$$D(\tau) = \frac{e^{-m_i \tau}}{\tau^{\alpha(y_h)}}$$

Iwasaki et al.

Location of the poles : a plus of this approach - gives meaning to the width

Tripolt, Rothkopf, MpL



Living in the Euclidean

Observation:

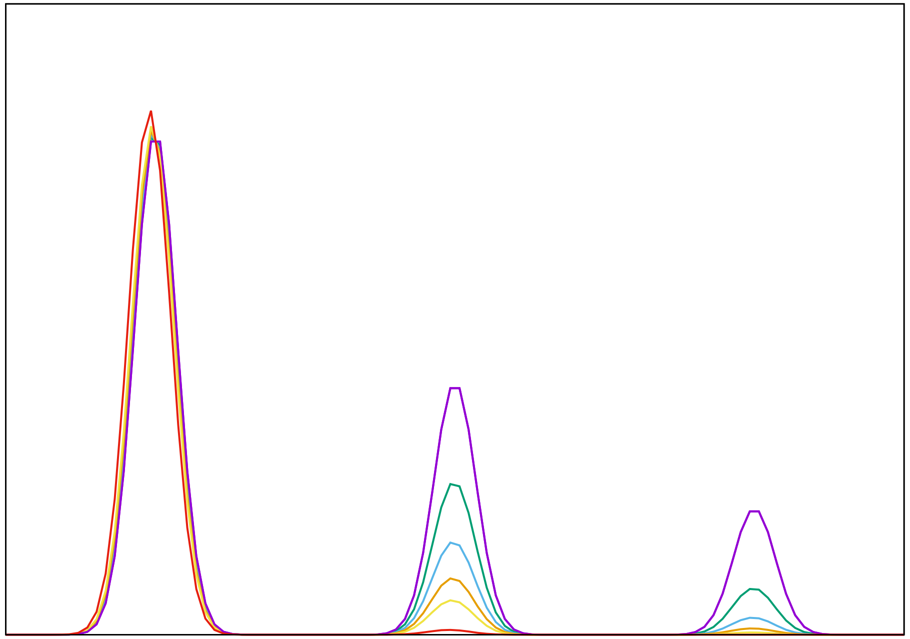
Post’s inversion formula

$$f(t) = \mathcal{L}^{-1}\{F\}(t) = \lim_{k\to\infty} \frac{(-1)^k}{k!} \left(\frac{k}{t}\right)^{k+1} F^{(k)}\left(\frac{k}{t}\right)$$

Calls attention on the derivatives

Another observation:

$$WS(\omega, k) = e^{(-k\omega)}S(\omega)$$



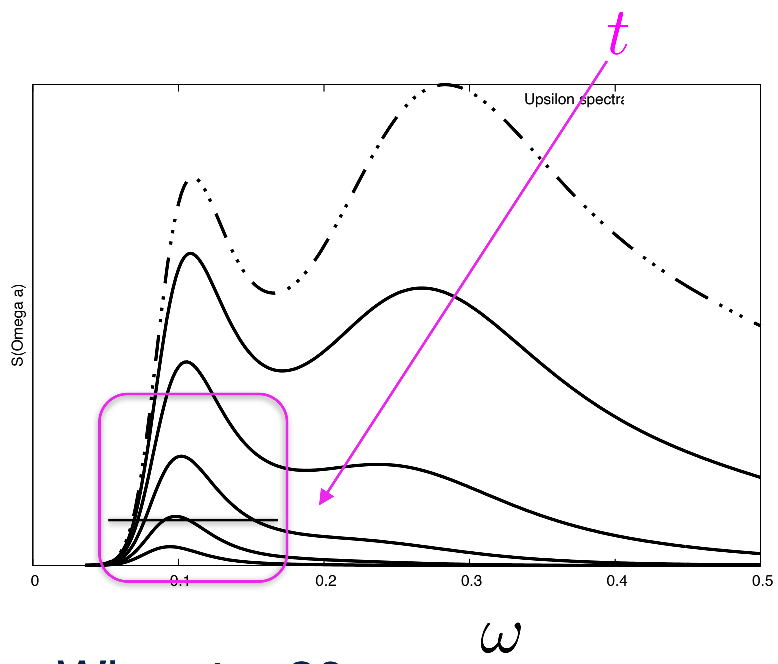
As k increases, the secondary peaks disappear.

If the fundamental peak is Gaussian, it remains so.

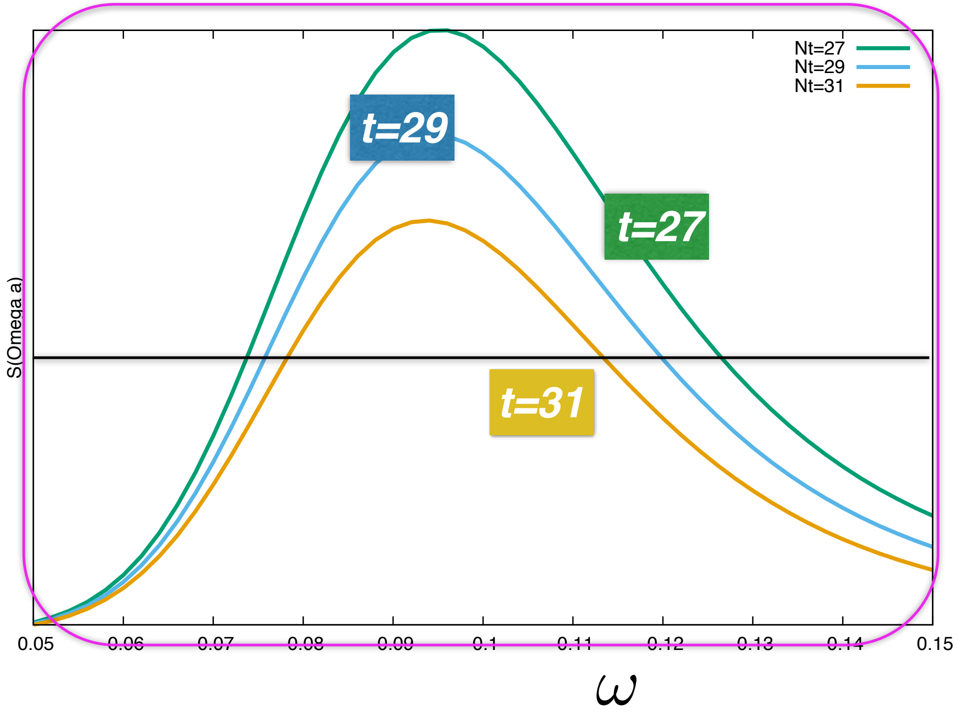
Weighted Spectral Functions

$$e^{-\omega t} S(\omega)$$

using the Upsilon spectral function



When t > 20
only the fundamental
peak is discernable



Sum rules

$$-\frac{1}{G(t)}\frac{dG(t)}{dt} = m_{eff}(t) = \frac{\int \omega e^{-\omega t} S(\omega) \frac{d\omega}{2\pi}}{G(t)} =< \omega >_{e^{-\omega t} S(\omega)}$$

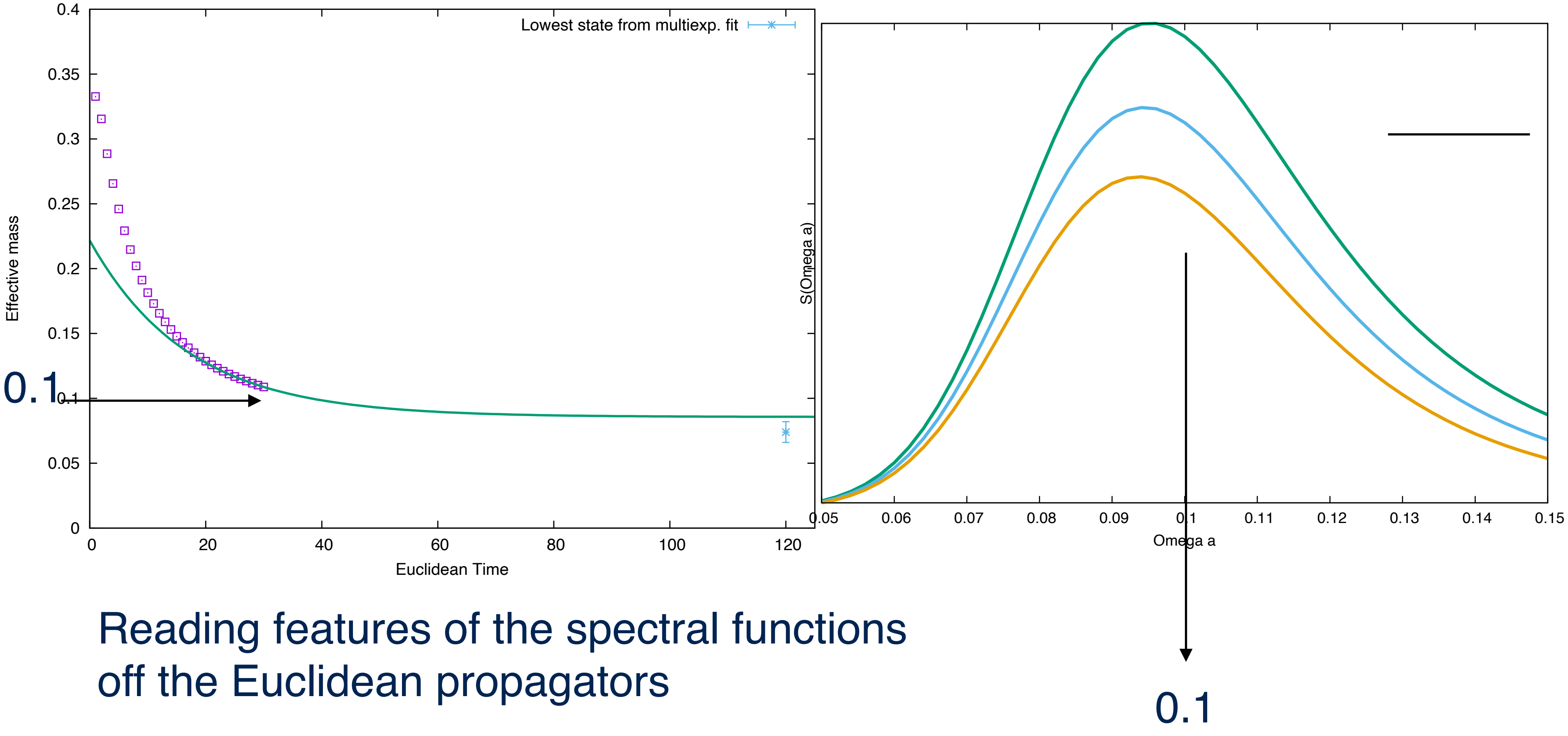
$$\frac{dm_{eff}(t)}{dt} =< (\omega - < \omega >_t)^2 >_t$$

$$e^{-\omega t} S(\omega)$$

P. Petreczky et al: observation that a decreasing
Effective mass implies a width

Results from the Upsilon propagator

$$-\frac{1}{G(t)}\frac{dG(t)}{dt} = m_{eff}(t) = \frac{\int \omega e^{-\omega t} S(\omega)\frac{d\omega}{2\pi}}{G(t)} = <\omega>_{e^{-\omega t}S(\omega)}$$

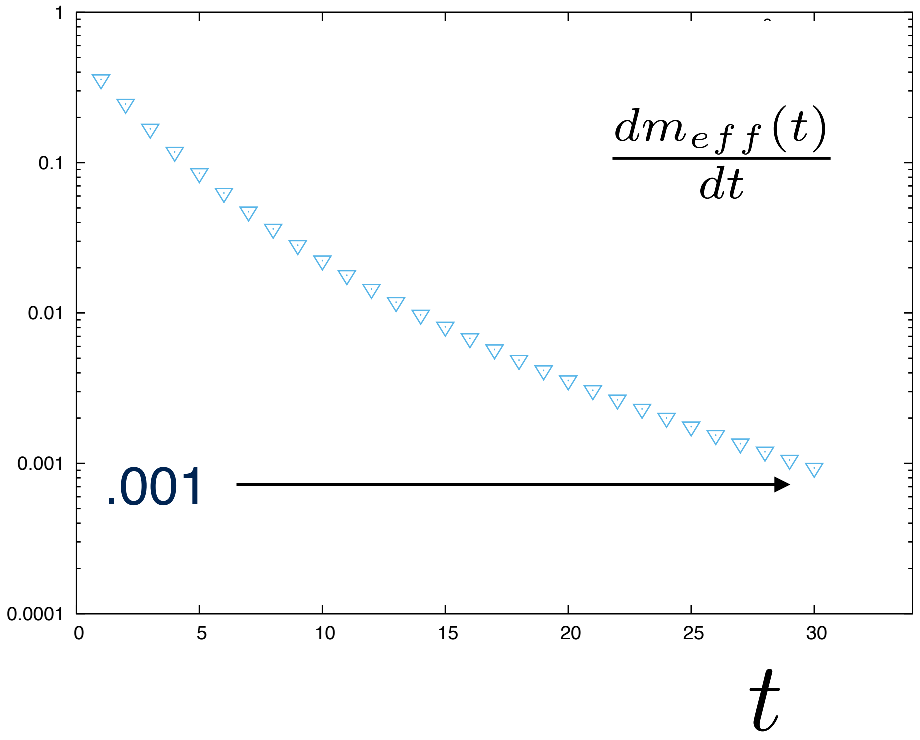


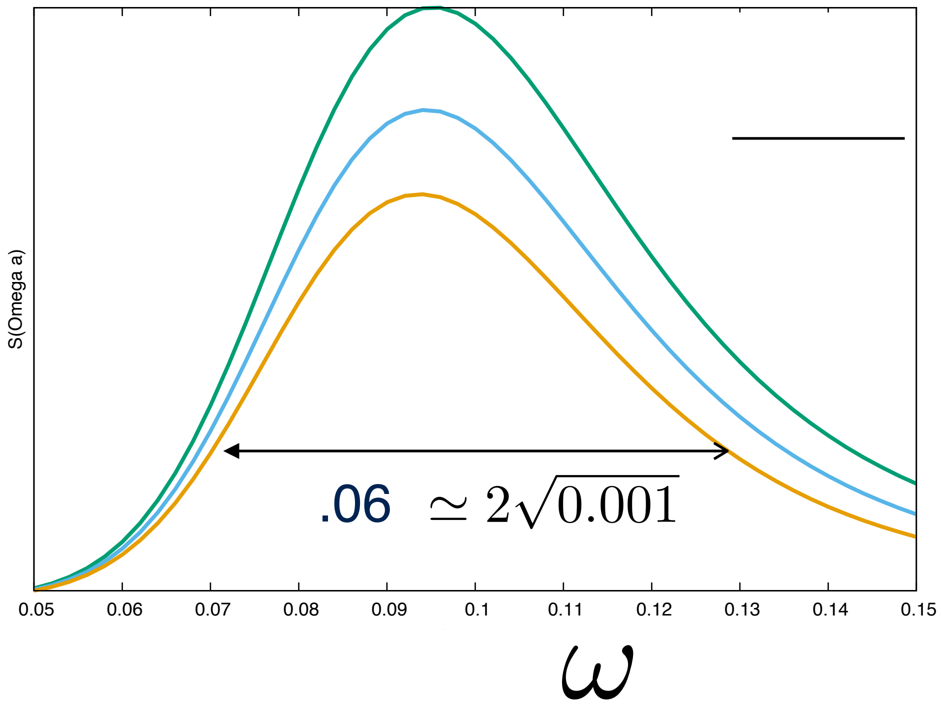
Reading features of the spectral functions
off the Euclidean propagators

The derivative of the effective mass at time t

is the width of the *weighted spectral functions*

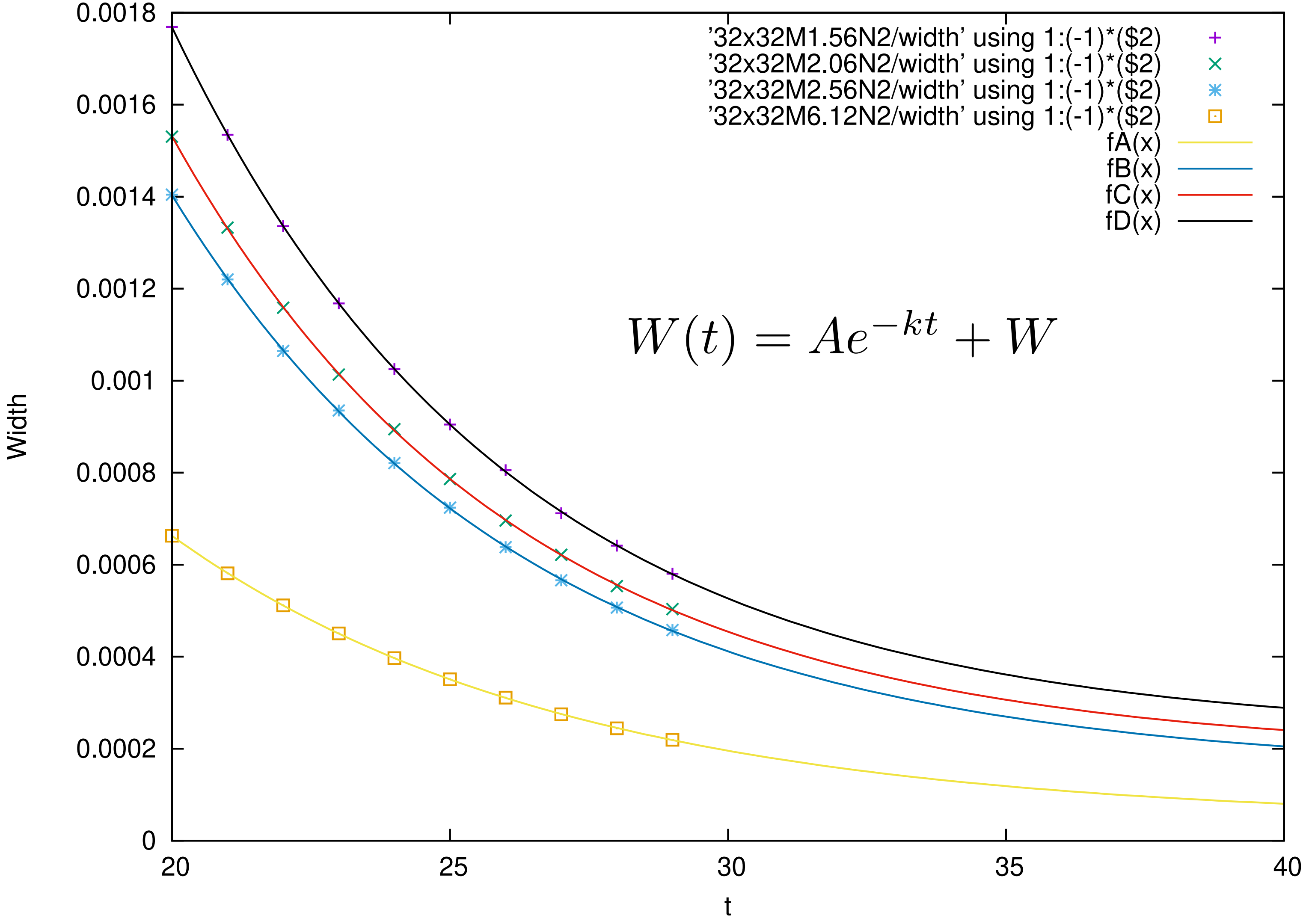
$$\frac{dm_{eff}(t)}{dt} =< (\omega - <\omega>_t)^2 >_t$$



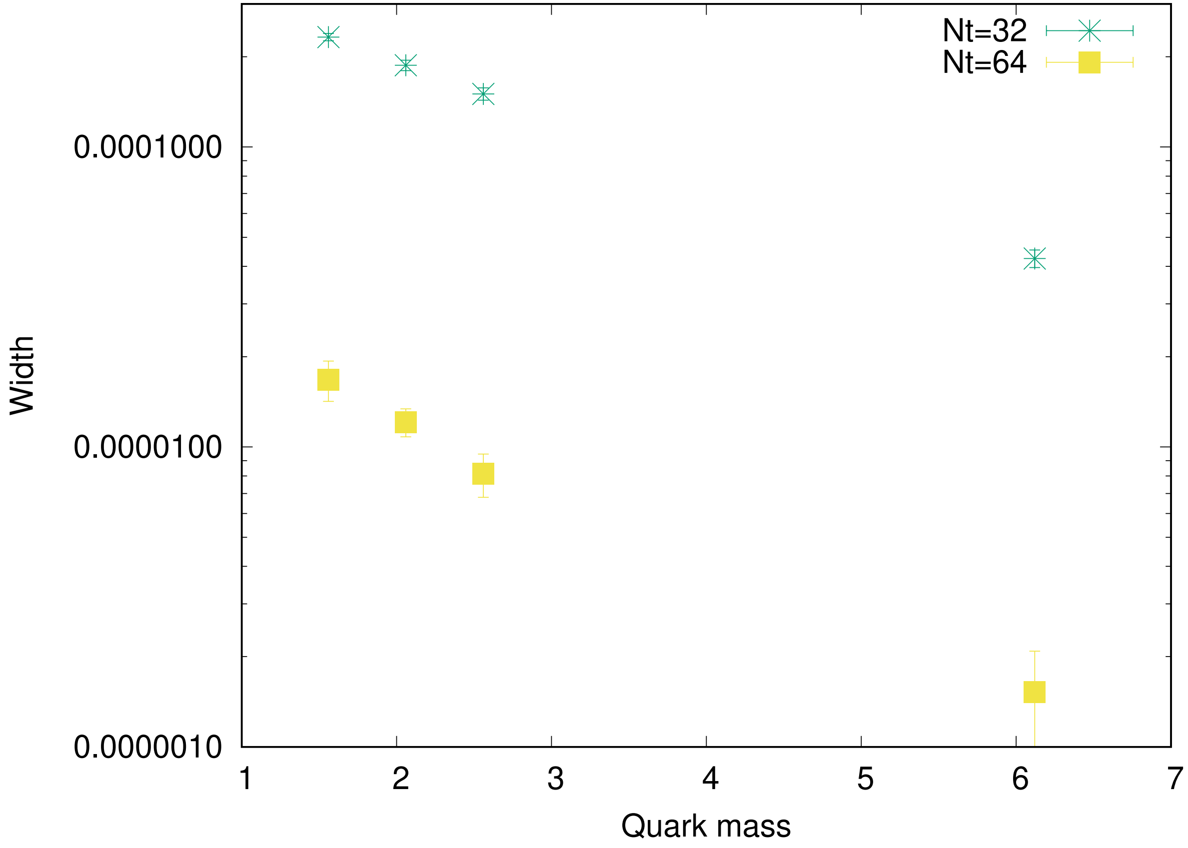


Reading features of the spectral functions
off the Euclidean propagators

Experimenting with variable masses



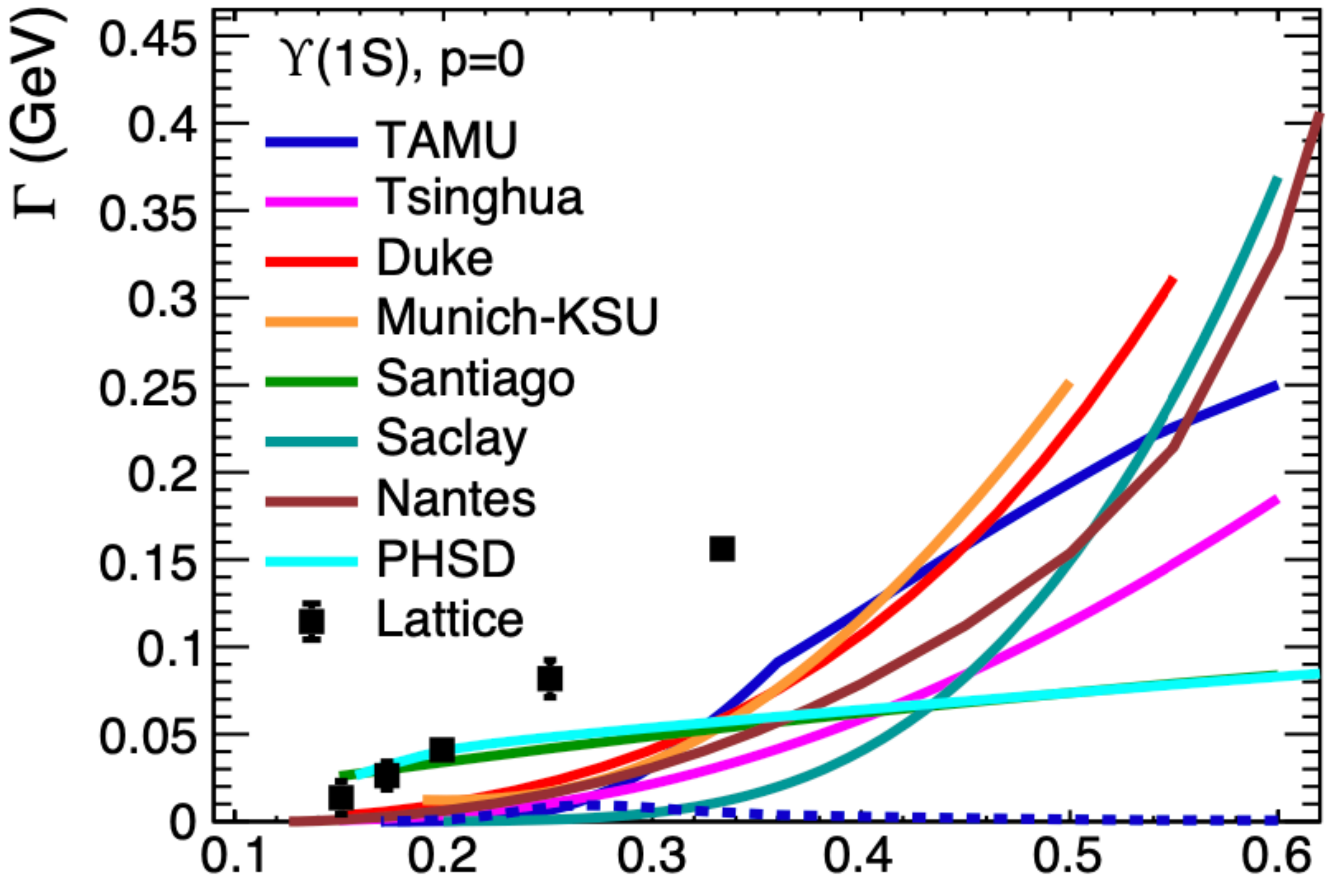
“Width” as a function of the quark mass for different temperatures



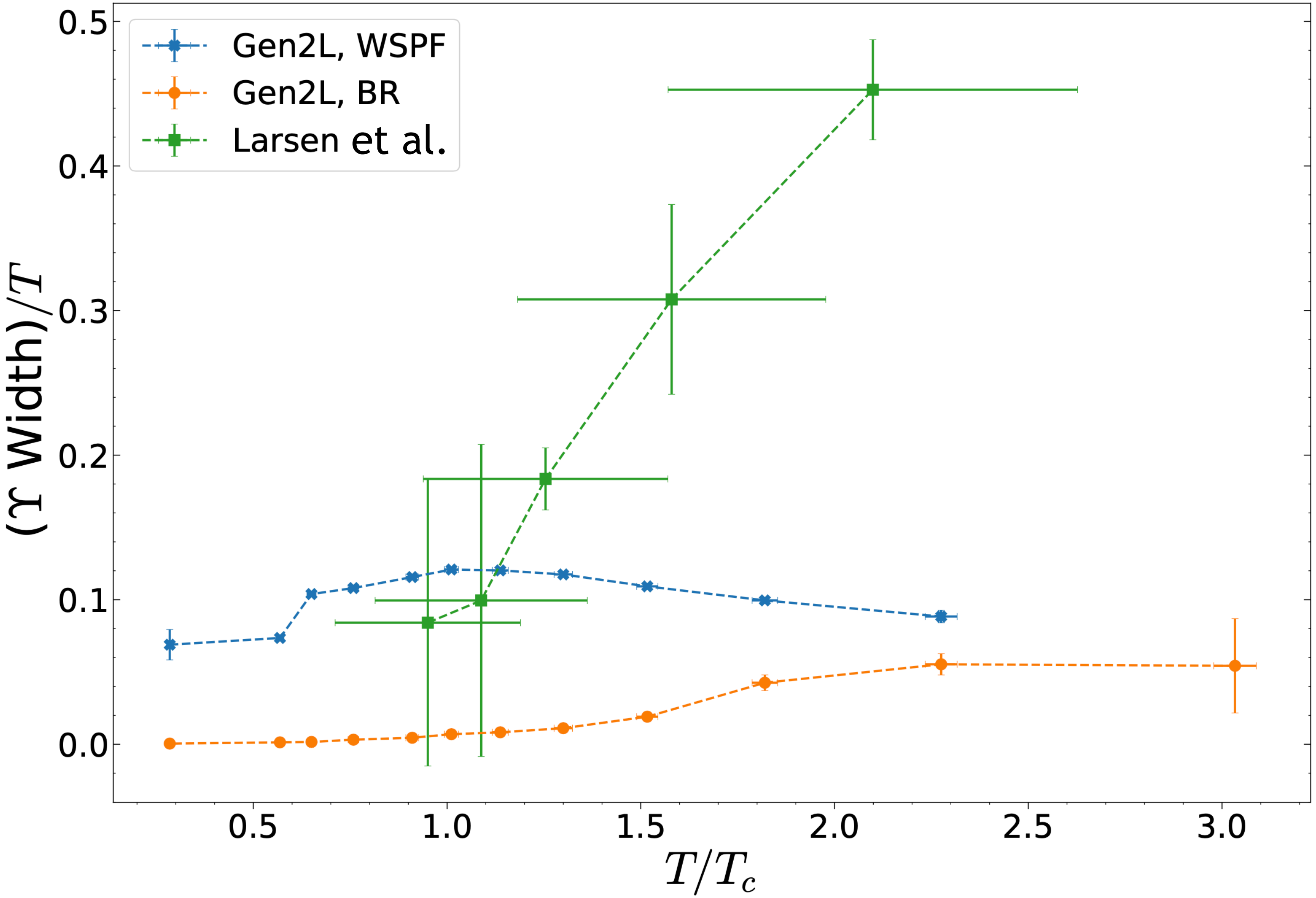
Comparative Study of Quarkonium Transport in Hot QCD Matter

A. Andronic*[a], P.B. Gossiaux*[b], P. Petreczky*[c], R. Rapp*[d], M. Strickland*[e], J.P. Blaizot[f], N. Brambilla[g], P. Braun-Munzinger[h,i], B. Chen[j], S. Delorme[k], X. Du[l], M. A. Escobedo[m,l], E. G. Ferreiro[l], A. Jaiswal[n], A. Rothkopf[o], T. Song[h], J. Stachel[i], P. Vander Griend[p], R. Vogt[q], B. Wu[d], J. Zhao[b], and X. Yao[r]

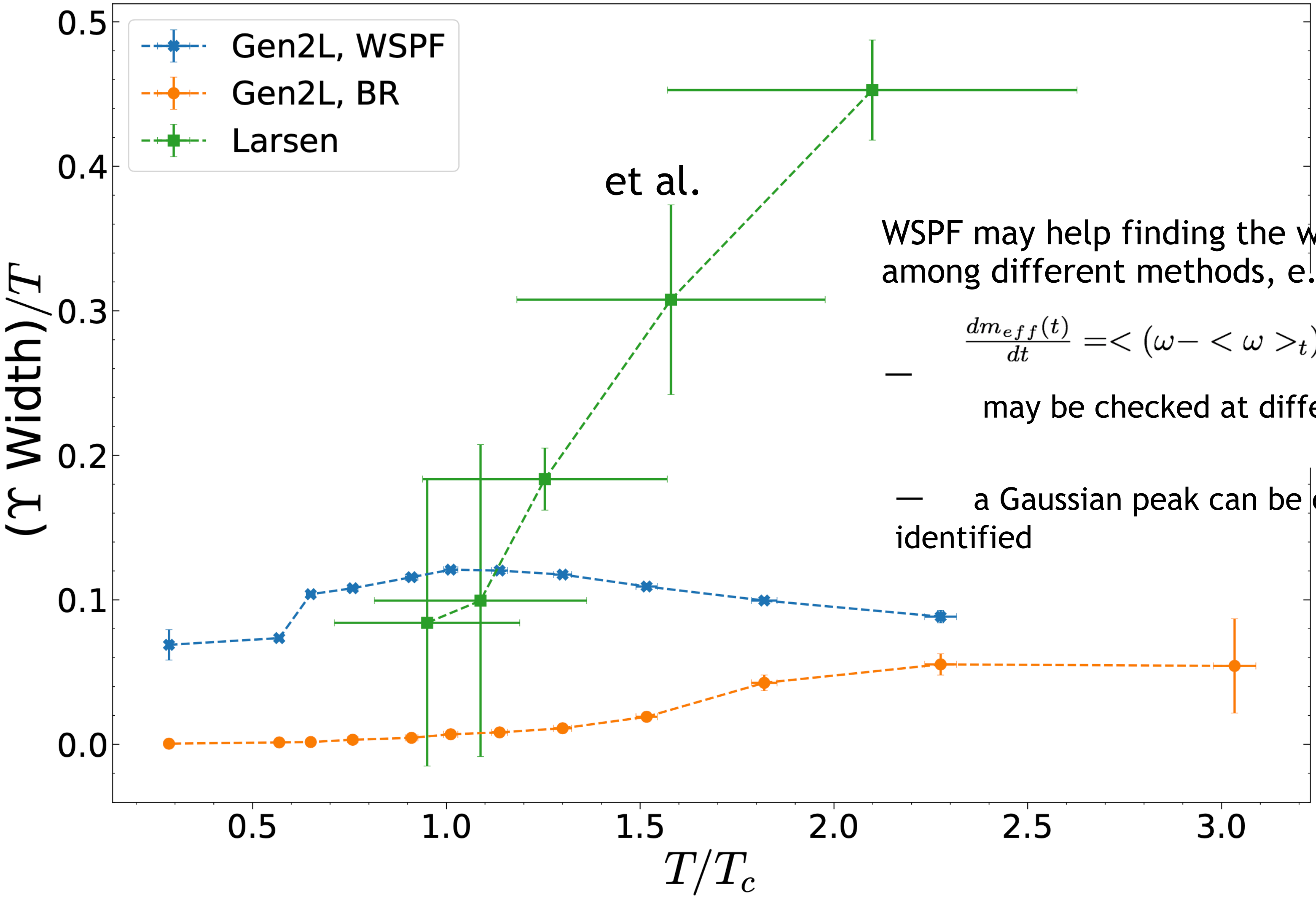
2402.04366



- R. Larsen, S. Meinel, S. Mukherjee, and P. Petreczky Phys. Lett. B 800 (2020) 135119



Plot courtesy of Ryan Bignell



WSPF may help finding the winner among different methods, e.g.

— $\frac{dm_{eff}(t)}{dt} =< (\omega - <\omega>_t)^2 >_{e^{-\omega t}S(\omega)}$ may be checked at different t

— a Gaussian peak can be clearly identified

Summary

A broadly consistent picture of bottomonium sequential suppression

Remaining quantitative differences among inversion methods

Lattice artefacts well under control — fitting models/continuum data needed

Simple sum rules may help identifying the winner among different methods

A good framework for analytic continuation is available and perhaps worth exploring