Quark Matter at High Baryon Density: Conformality and Quarkyonic Matter

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Recent work in collaboration with
Y. Fujimoto T. Kojo; V. Koch, V. Vovchenko and G. Miller
See earlier work with
R. Pisarski, Y. Hidaka, S. Reddy, K. Jeon, D. Duarte, S.Hernandez, , K. Fukushima, M. Praszalowicz, M. Marczenko, K. Redlich and C. Sasaki

Mass and radii of observed neutron stars and data from neutron star collisions give an excellent determination of the equation of state of strongly interacting matter

Such equations of state must be hard
The sound velocity squared is greater than or of the order of 1/3 at only a few times nuclear matter density
This is NOT what one expects from a 1^{st} or 2^{nd} order phase transition
Relativistic degrees of freedom appear to be important

Neutron Star Matter and Some Conjectures on Scale Invariance

From observations of neutron stars masses and radii, one gets very good information about the zero temperature equation of state of nuclear matter

One equates the outward force of matter arising from pressure inward force of gravity. This gives a general relativistic equation of hydrostatic equilibrium.

For a specific equation of state, one obtains a relationship between radii and neutron star masses

Equations of state may be characterized by two dimensionless numbers

Sound velocity:

$$v_s^2 = \frac{dP}{de}$$

and the trace of the stress energy tensor scaled by the energy density

$$\Delta = \frac{1}{3} - \frac{P}{e}$$

$$P = -\frac{dE}{dV}$$

In a scale invariant theory at zero temperature:

$$E \sim (N/V)^{4/3} V \sim N^{4/3} V^{-1/3}$$

$$P = \frac{1}{3}\frac{E}{V} = \frac{1}{3} e$$

$$v_s^2 = \frac{dP}{de} = \frac{1}{3}$$

$$\Delta = \frac{1}{3} - \frac{p}{e} = 0$$

The trace of the stress energy tensor is taken to be a measure of scale invariance. It is anomalous in QCD.

$$T^\mu_\mu = -\frac{\beta(g)}{g}(E^2 - B^2) + m_q(1+\gamma_q)\overline{\psi}\psi$$

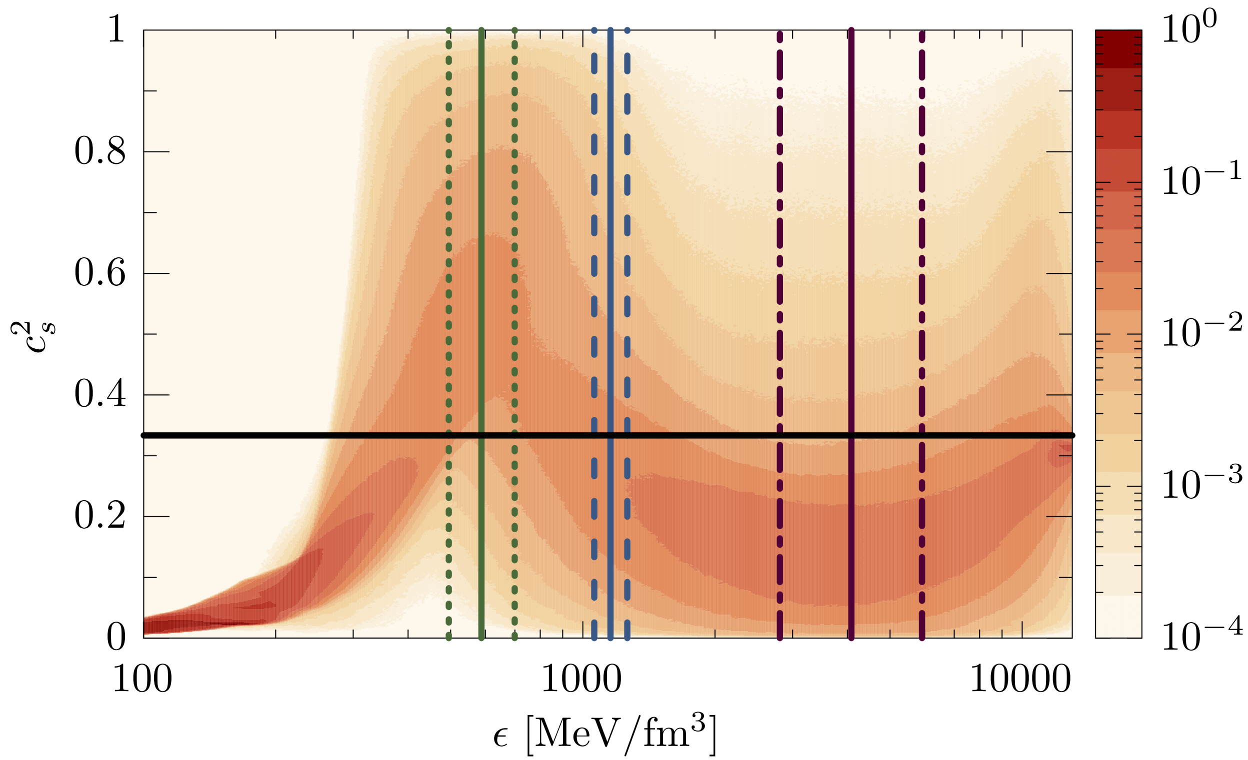
In this equation, the beta function of QCD is negative, and the fermion term is from quarks. The fermion term vanishes in the chiral limit.

If we take matrix elements of single particle states

$$< p \mid T^\mu_\mu \mid p > \sim p^2 = m^2 \geq = 0$$

In the chiral limit, this implies E > B, as we expect for massive quarks, except for the pion, which is very tightly. bound

For dilute systems, the trace anomaly is positive, as it is at high density for a quark gas. In general, we expect it to be positive, except possibly for small effects due to pion condensate, if they exist



As a result of LIGO experiments, and more precise measurement of neutron star masses and radii, the equation of state of nuclear matter at a few times nuclear matter density is tightly constrained

Sound velocity approaches and perhaps exceeds

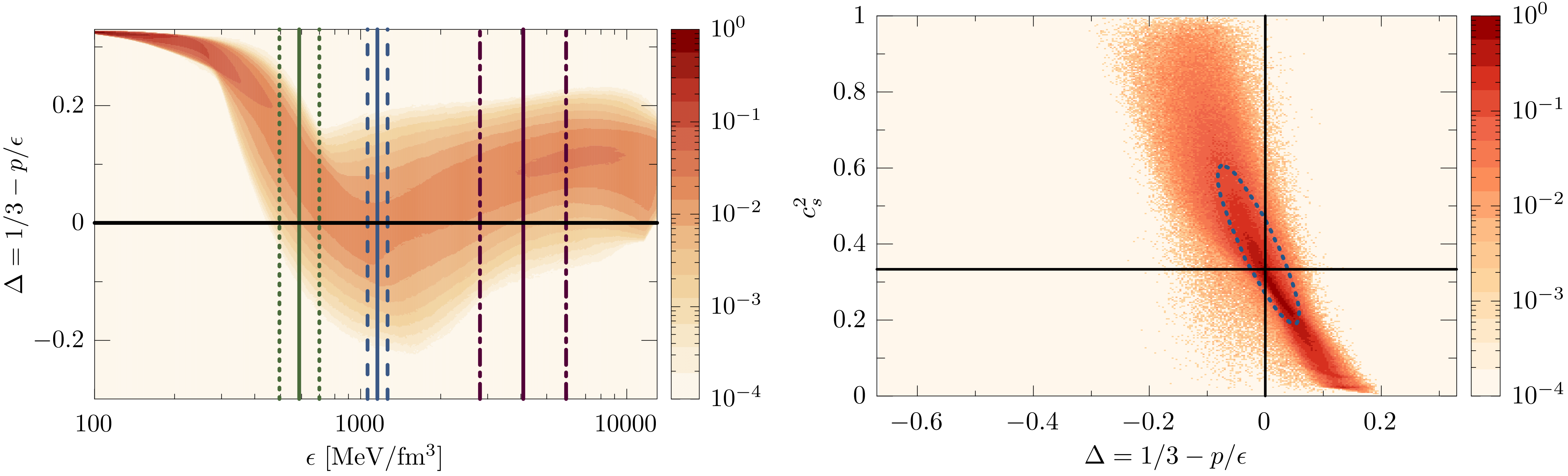
$$v_s^2 = 1/3$$

at a few times nuclear matter density

L. McLerran, M. Marczenko, K. Redlich and C. Sasaki

Y. Fujimoto, K. Fukushima, K. Murase

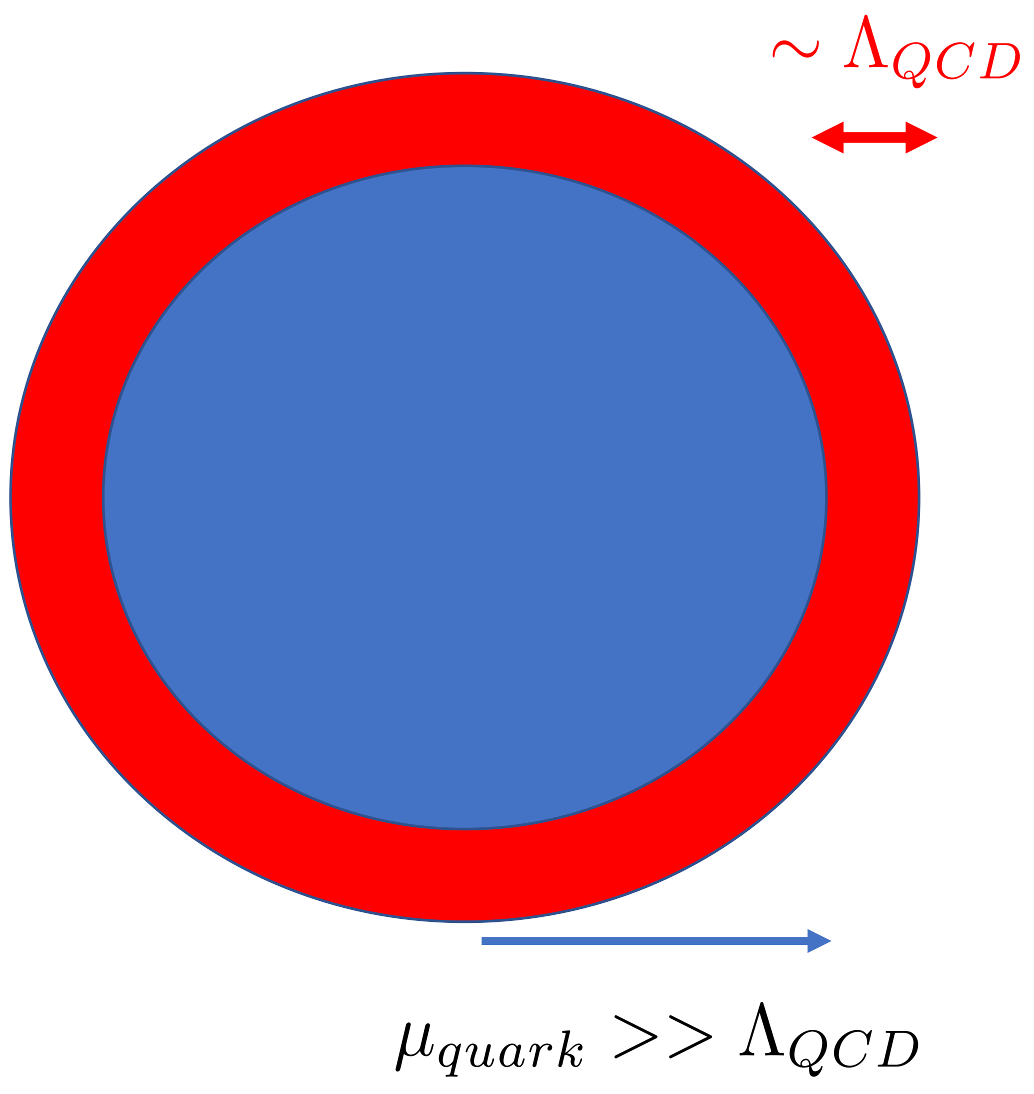
Tews, Carlson, Gandolfi and Reddy; Kojo; Annala, Gorda, Kurkela and Vuorinen



Also, the trace anomaly is approaching zero at highest densities in neutrons stars, where also the sound velocity squared approaches 1/3

Matter is strongly interacting and conformal:
Probably some form of quark matter

Fermi Surface is Non-perturbative



Fermi Surface
Interactions sensitive to infrared
Degrees of freedom: baryons, mesons and glueballs

Fermi Sea: Dominated by exchange interactions which are less sensitive to IR.
Degrees of freedom are quarks

Pisarski and LDM, Reddy and LDM

An Explicit Quantum Mechanical Theory of Quarkyonic Matter

T. Kojo; Y. Fujimoto., LM and T. Kojo

Let occupation number density for nucleons and quarks be

$$f_q, f_N \qquad 0 \leq f_q, f_N \leq 1$$

A duality relation (nucleons are composed of quarks)

$$f_q(k) = \int [dp] \; K(k - p/N_c) f_N(p)$$

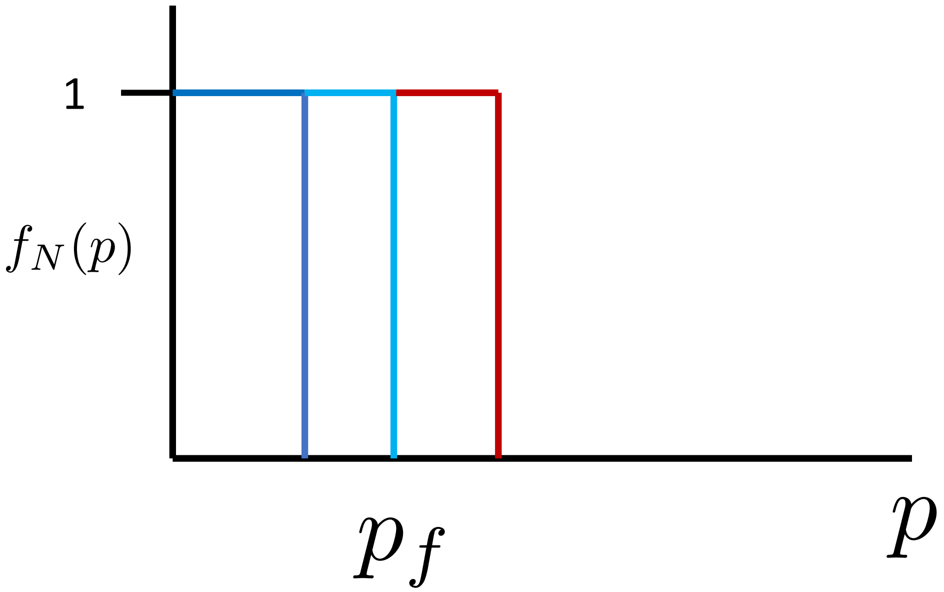
$$\int [dk] \; K(k) = 1$$

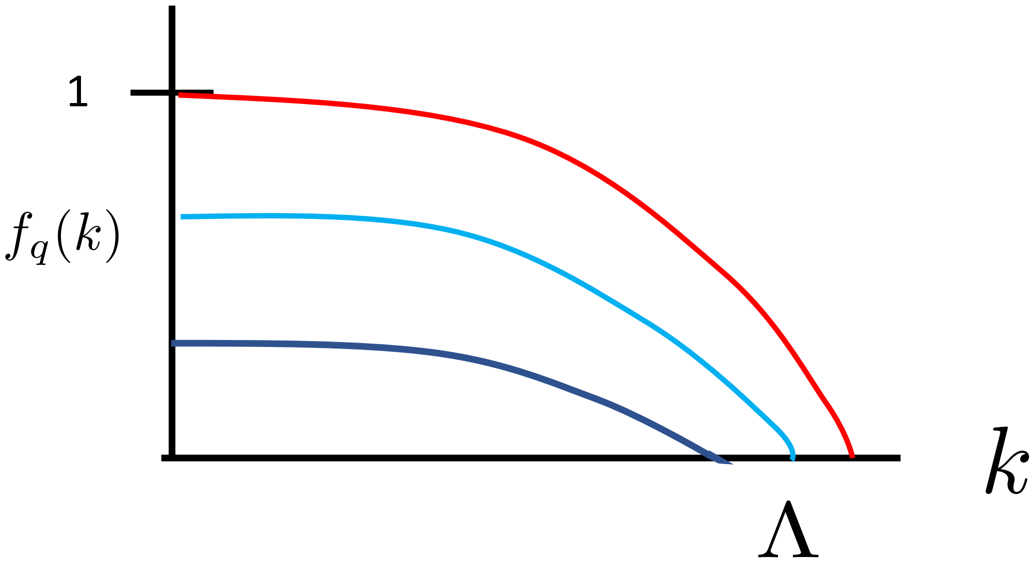
First: Free theory of nucleon and quarks (except for duality relation)

This is a solvable theory with non-trivial solution with two phases:
A nucleonic phase and a quarkyonic phase

Solution looks like:

Low density:





$$1 = \int [dp]\ K(k - p/N_c) f_N(p)$$

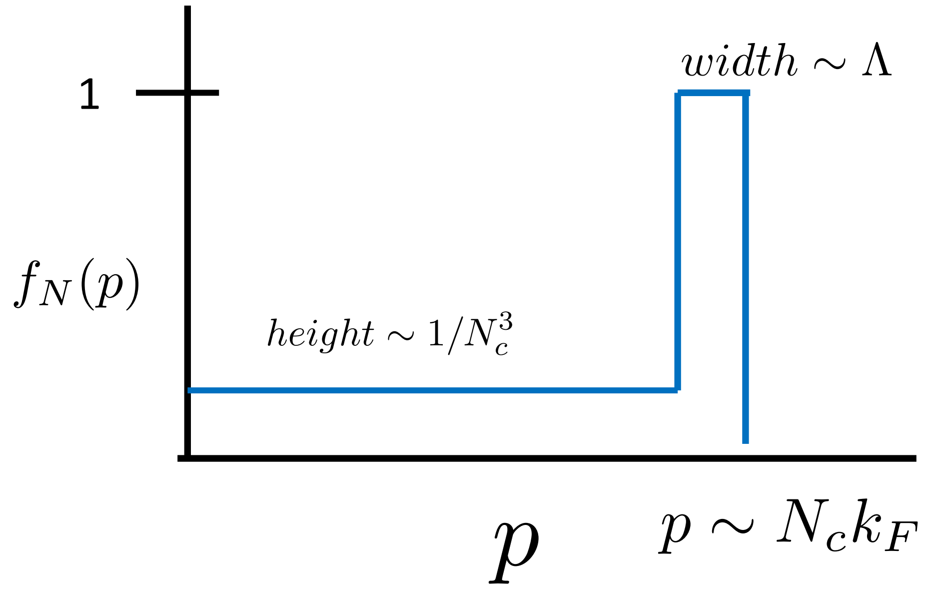
$$\sim K(0) \int [dp]\ f_N(p) = \kappa \frac{n_n}{\Lambda^3}$$

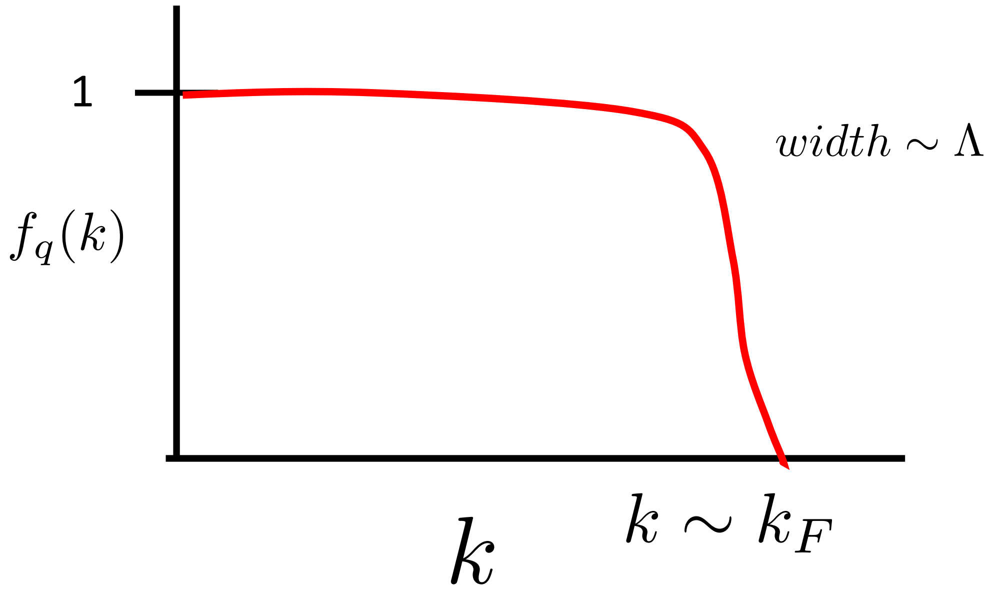
Width of quark distribution determined by intrinsic confinement scale of quarks inside of nucleons

$$\Lambda$$

$$n_N^{crit} \sim \Lambda^3$$

High Density:





At high densities, a thin shell of saturated baryon matter forms surrounded by an underoccupied distribution of nucleons

The quarks make a filled sea of nucleons with an underoccupied tail where the shell of nucleons are not Pauli blocked

Koch, LDM, Miller, and Vovchenko

Might Quarkyonic Matter be Relevant for Nuclear Matter?

General Considerations on the Transmutation Density to Quarkyonic Matter

Transmutation Criterium:

$$1 = f_Q(0) = \int^{k_f} \frac{d^3p}{(2\pi)^3}\ K(p/N_c)\ f_N(p) \sim \frac{1}{4} n_N\ K(0)$$

Using a Gaussian model for the distribution of quarks inside a hadron, and only including valence quarks

$$\phi_{gauss} = 8\pi^{3/2} R^3 e^{-p^2 R^2}$$

Using the RMS charge radius as R

$$R = \sqrt{\frac{2}{3}} r_{RMS}$$

Find transmutation density of twice nuclear matter for measured charge radius
But this ignores the contribution of quark-antiquark pairs: the meson cloud

Including the quark-antiquark pairs and phenomenologically determined

Transverse momentum distribution functions were determined by de Teramond et. al. These distributions functions describe measured integrated valence and sea quark distribution functions, and electromagnetic form factors

$$\frac{dn_Q(k)}{d^3k} = \frac{x}{E} \mid \psi(x, k_T) \mid^2$$

$$\int \frac{dx d^2 k_T}{(2\pi)^3} \mid \psi(x, k_T) \mid^2 = 1$$

$$x = \frac{k^+ + E_k}{p^+ + E_p}$$

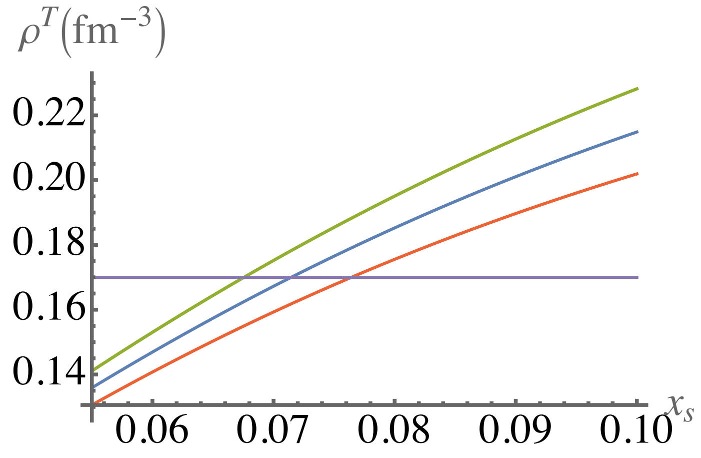
So that for a hadoon at rest and a parton with zero momentum

$$x = E_0/M$$

$$\frac{dn_Q(0)}{d^3k} = \mid \psi(x_0, 0_T) \mid^2 \frac{1}{M}$$

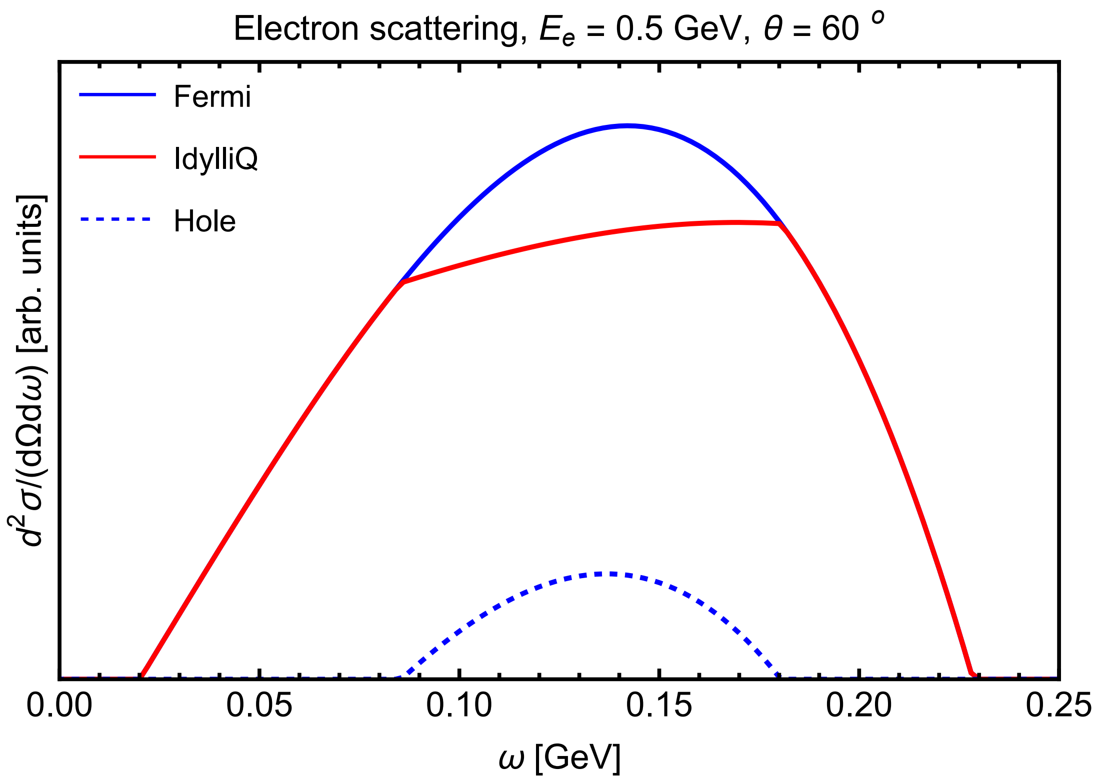
Typical x for valence quarks is about 1/6, because glue carries ½ the momentum, and less than 1/10 for quark-antiquark pairs

Find that for valence quarks alone the transmutation density would be about twice nuclear matter. Sea quark contribution has large uncertainties since it is sensitive to the value of x used.

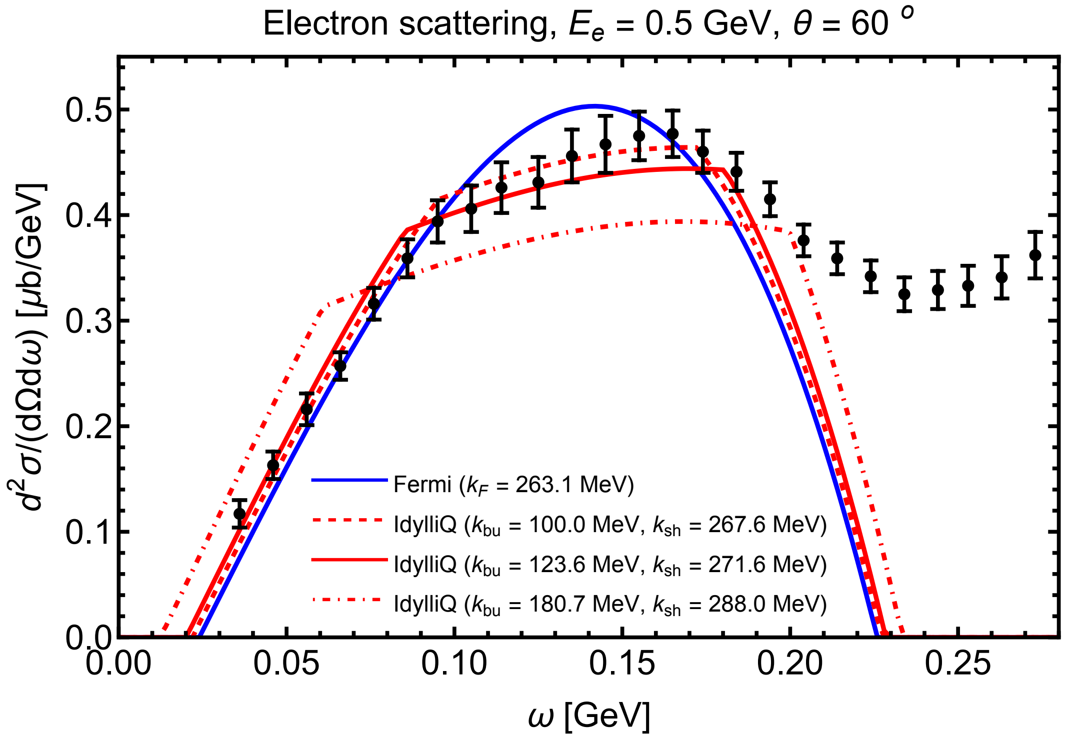


Plausible that the transmutation density is below or less than that of nuclear matter

Experimental Constraint: Electron Scattering:
Quarkyonic Matter has a Fully Occupied Fermi Surface of Nucleons and an underoccupied sea



Hole in Fermi sea lowers cross section near maximum. Peak would be delta function if there was no Fermi momentum



Data is Coulomb corrected and extrapolated to nuclear matter. Energy chosen minimizes final state interactions

Constructing a Theory:

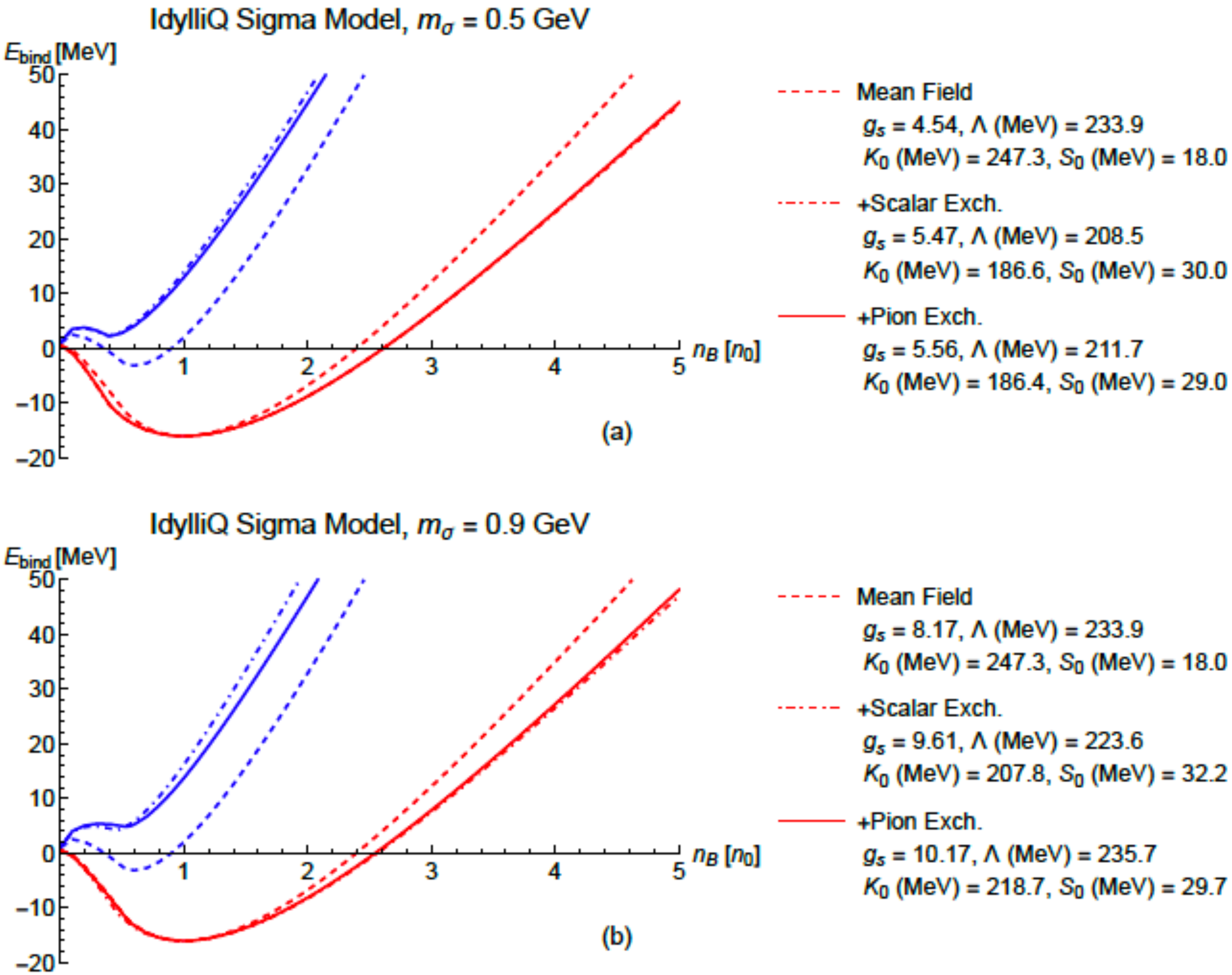
Include only pion and sigma meson interactions. First order correction to free theory of nucleon with constrain on quark occupations number.
Sigma meson interaction in mean field and include exchange interactions.
Note that mean field term is scale invariant at high Fermi momentum:

$$n_{scalar} \sim M \int^{k_f} \frac{d^3p}{2E(2\pi)^3} \sim k_f^2$$

$$\epsilon \sim \frac{g^2}{M^2} n_{scalar}^2 \sim k_f^4$$

Naturally matches to QCD approximate scale invariance.

Vector meson interactions are not scale invariant in mean field.



Reasonable values for nuclear matter. (Too large a hole in the Fermi sea for these parameters)

As isospin increases, minimum moves to lower density and almost disappears for neutron matter. Neutron matter is slightly unbound

How do we test this hypothesis?
Is it more or less true or is it false?