Curvature of the energy per particle in NSs

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MM, McLerran, Redlich, Sasaki, *PRC* 107 (2023) 2, 025802

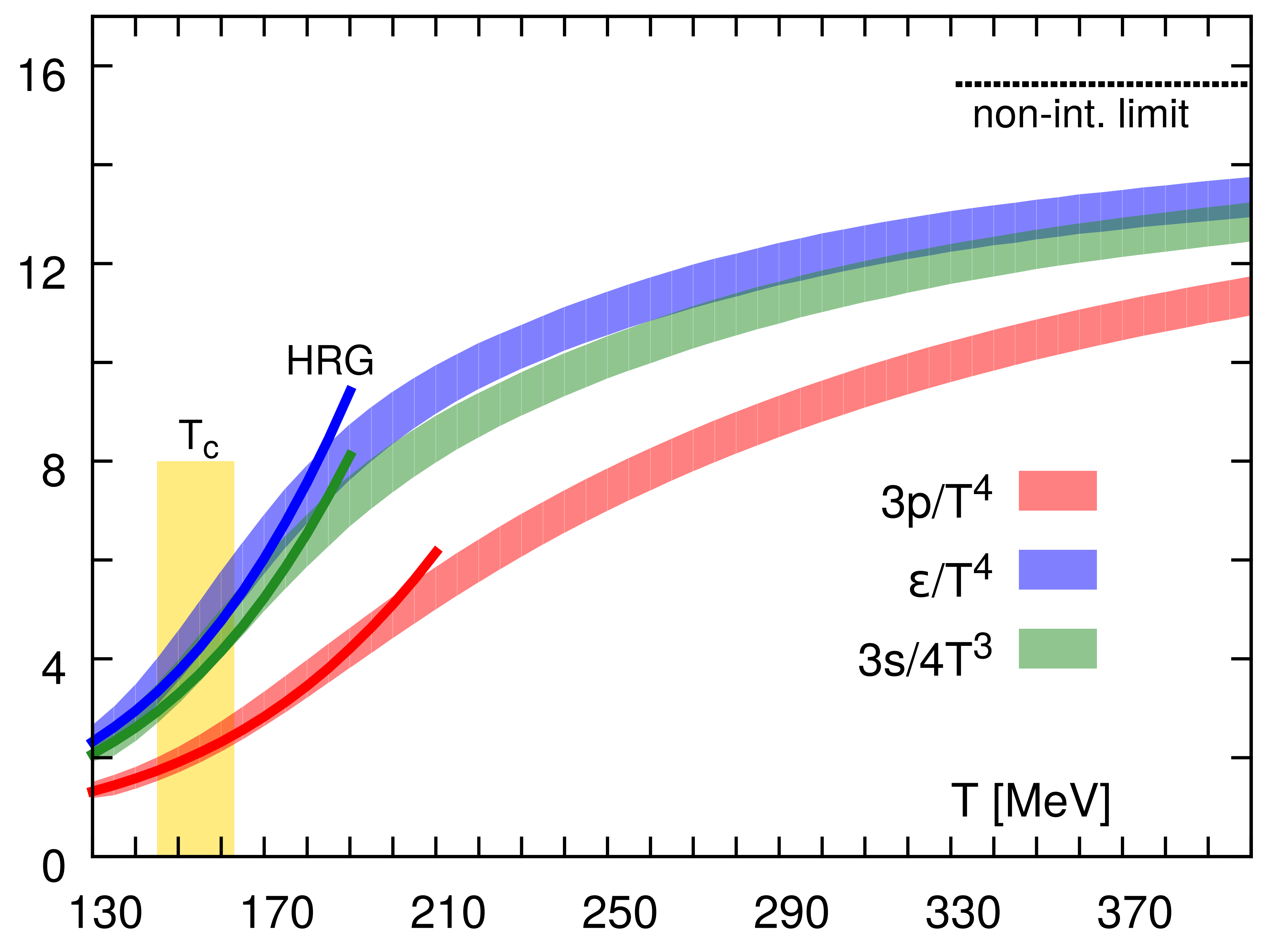
MM, Redlich, Sasaki, *PRD 109 (2024) 4, L041302*

EMMI Workshop - Aspects of Criticality II

02-04.07.2024

Lattice Quantum Chromodynamics

HotQCD, 2014

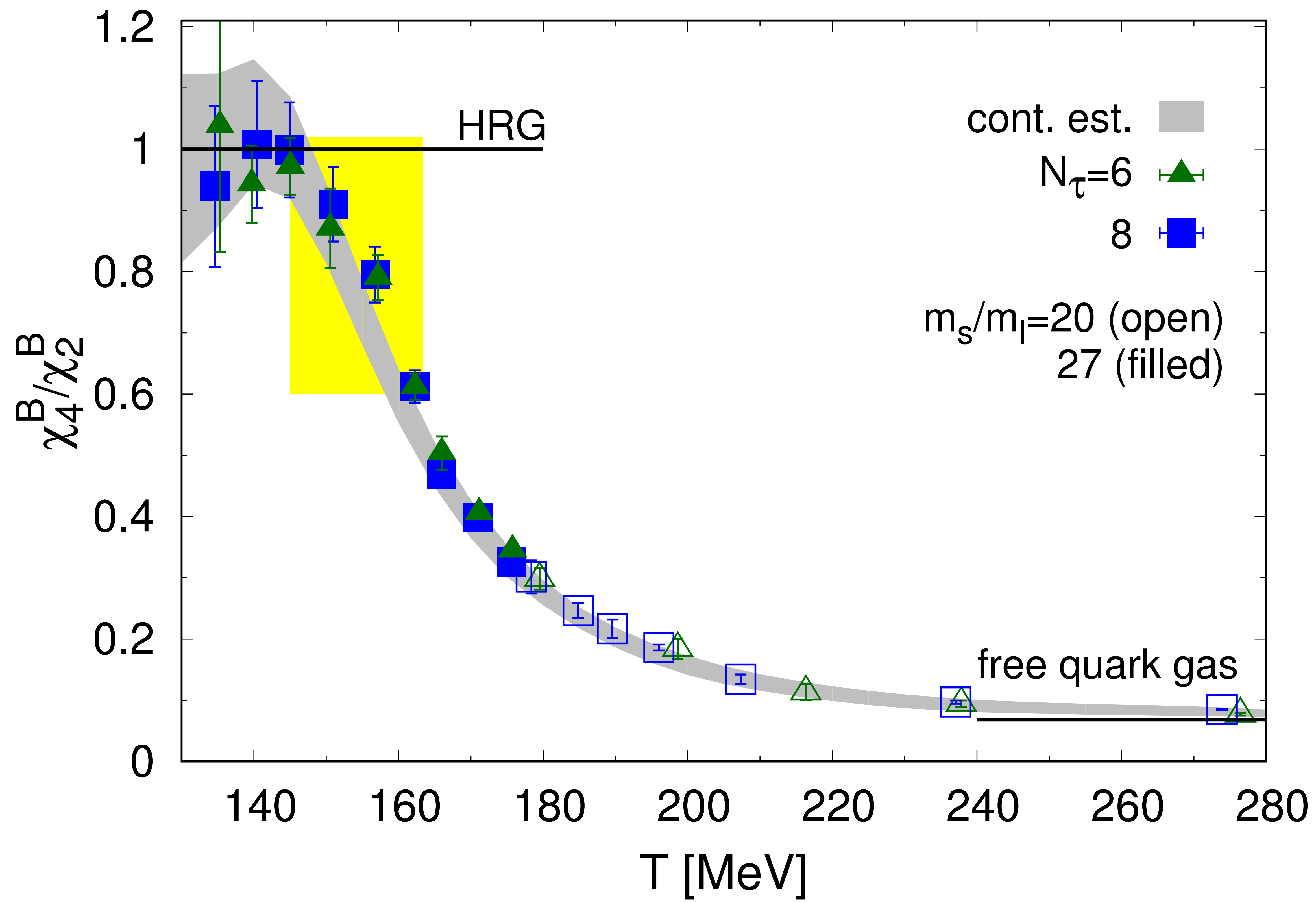


Pressure in the HRG model

$$P^{\mathrm{HRG}} = \sum_{i \in \mathrm{had}} P^{\mathrm{id}} \left(T, \mu_i; m_i\right)$$

Agreement with LQCD EoS up to $\simeq T_c$

HotQCD, 2017



Taylor expansion of LQCD EoS

$$\frac{P}{T^4} = \sum_{k=0}^{\infty} \left(\frac{\mu_B}{T}\right)^k \frac{\chi_k^B}{k!}, \text{ where } \chi_k^B = \frac{\partial^k P/T^4}{\partial \left(\mu_B/T\right)^k}$$

Kurtosis: $\frac{\chi_4^B}{\chi_2^B} \sim B^2$: breakdown $\sim T_c$: changeover to QGP

Quark Matter in Neutron Stars?

Solid Constraints

- Low density: χEFT $(n \lesssim 1.1 n_0)$ Tews et al, 2013
- High density pQCD $(n \gtrsim 40 n_0)$ Gorda et al, 2018

Interpolation methods

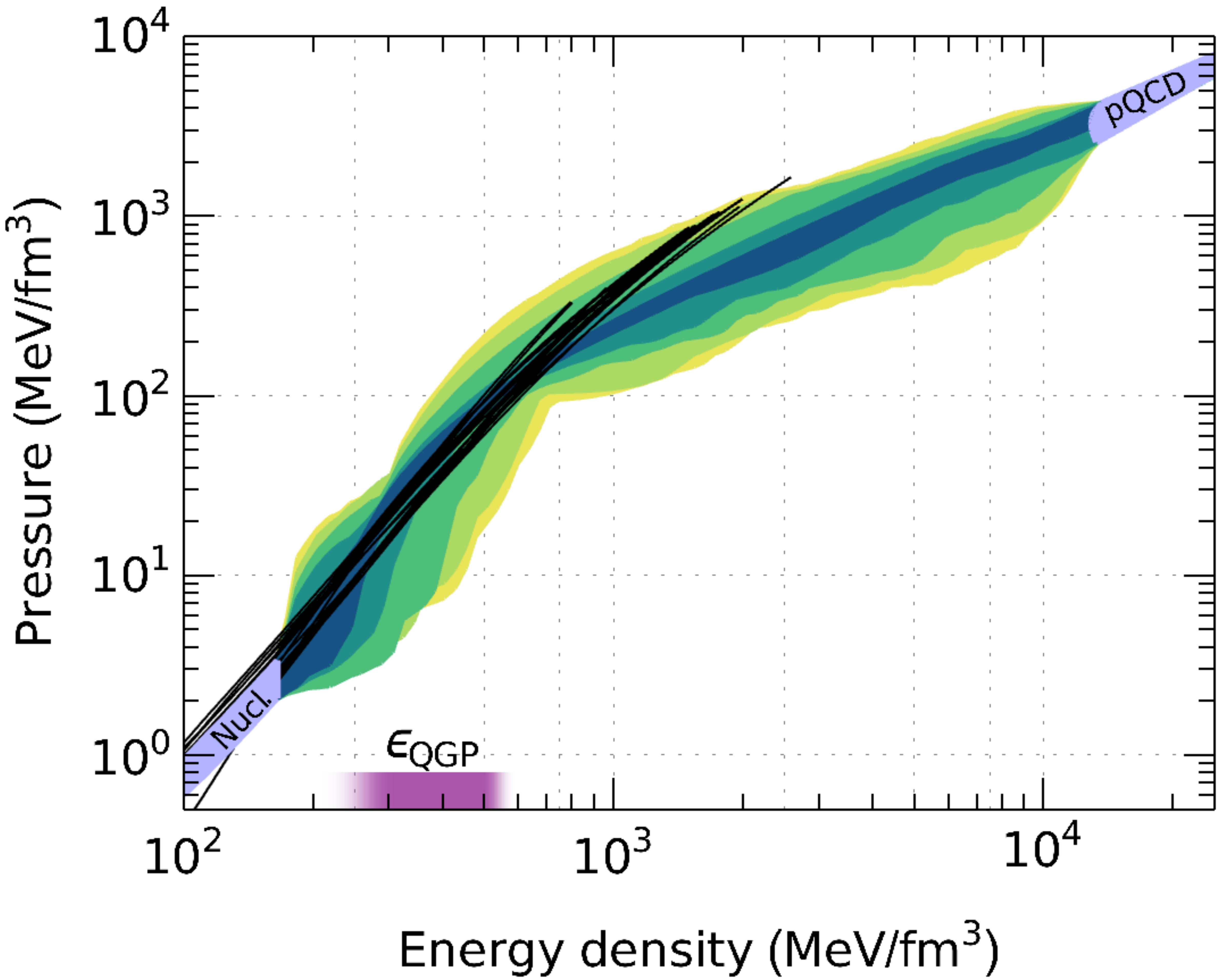
- Polytropes, CSS, Linear Speed of Sound

eg. Annala et al, 2018, 2020; Alford et al 2013, 2017, Li et al 2021

Deconfinement by polytropic index

$$\gamma = \frac{d \log p}{d \log \epsilon} \rightarrow \begin{cases} \gamma > 1.75 \rightarrow \text{Hadrons} \\ \gamma < 1.75 \rightarrow \text{Quarks} \end{cases}$$

Annala et al 2020



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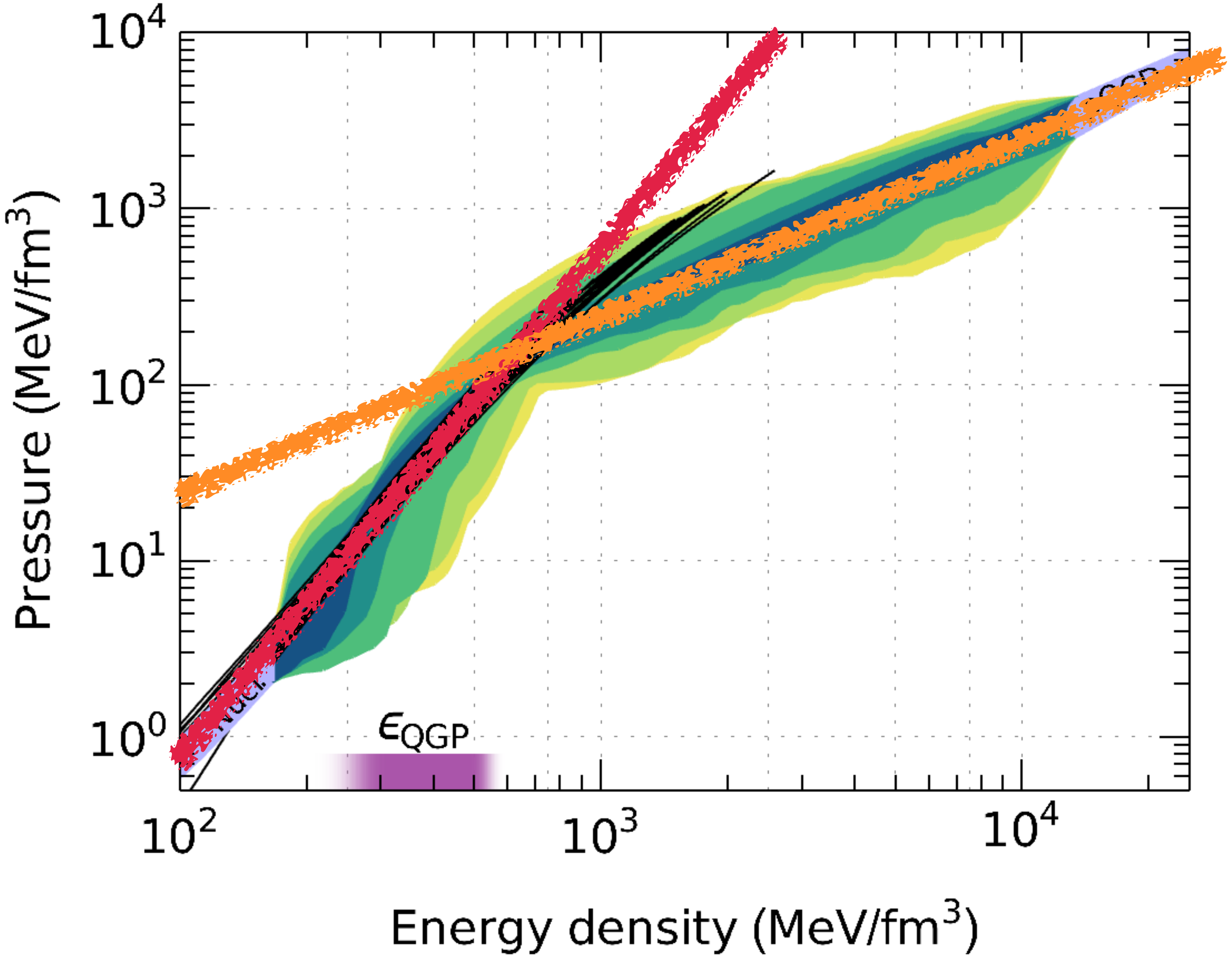


Methodology: Piecewise-linear speed of sound

Annala et al 2020

$$c_s^2 = \frac{n}{\mu}\frac{d\mu}{dn} \qquad c_{s,i}^2 = \frac{(\mu_i - \mu)c_{s,i}^2 + (\mu - \mu_i)c_{s,i+1}^2}{\mu_{i+1} - \mu_i}$$

$+$

χEFT + pQCD

$+$

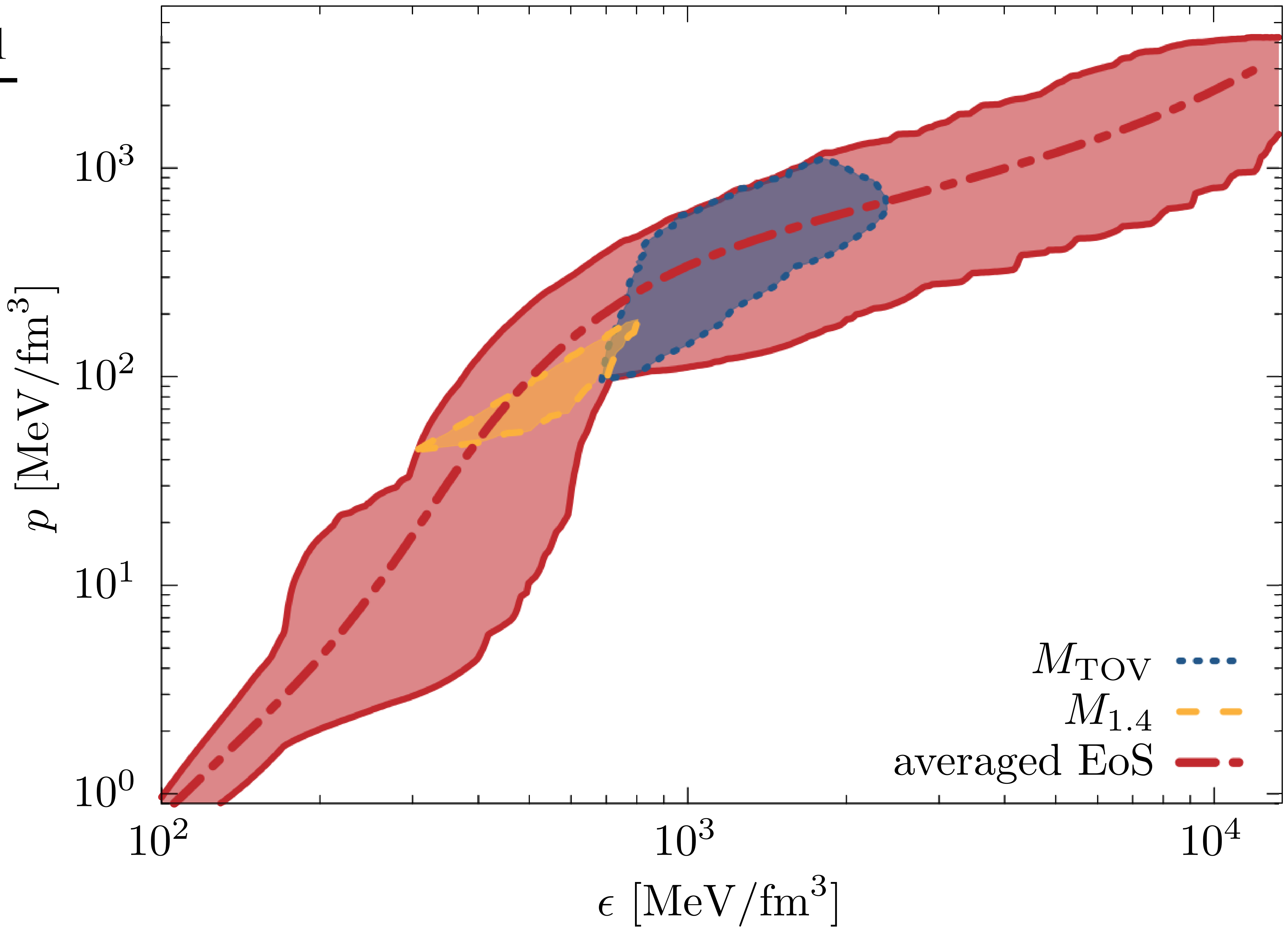
Mass measurement of J0740+6620

$M_{\mathrm{TOV}} \geq (2.08 \pm 0.07)\ M_\odot$ Fonseca et al 2021

$+$

Tidal Deformability from GW170817

$\Lambda_{1.4M_\odot} = 190^{+380}_{-120}$ Abbott et al 2018

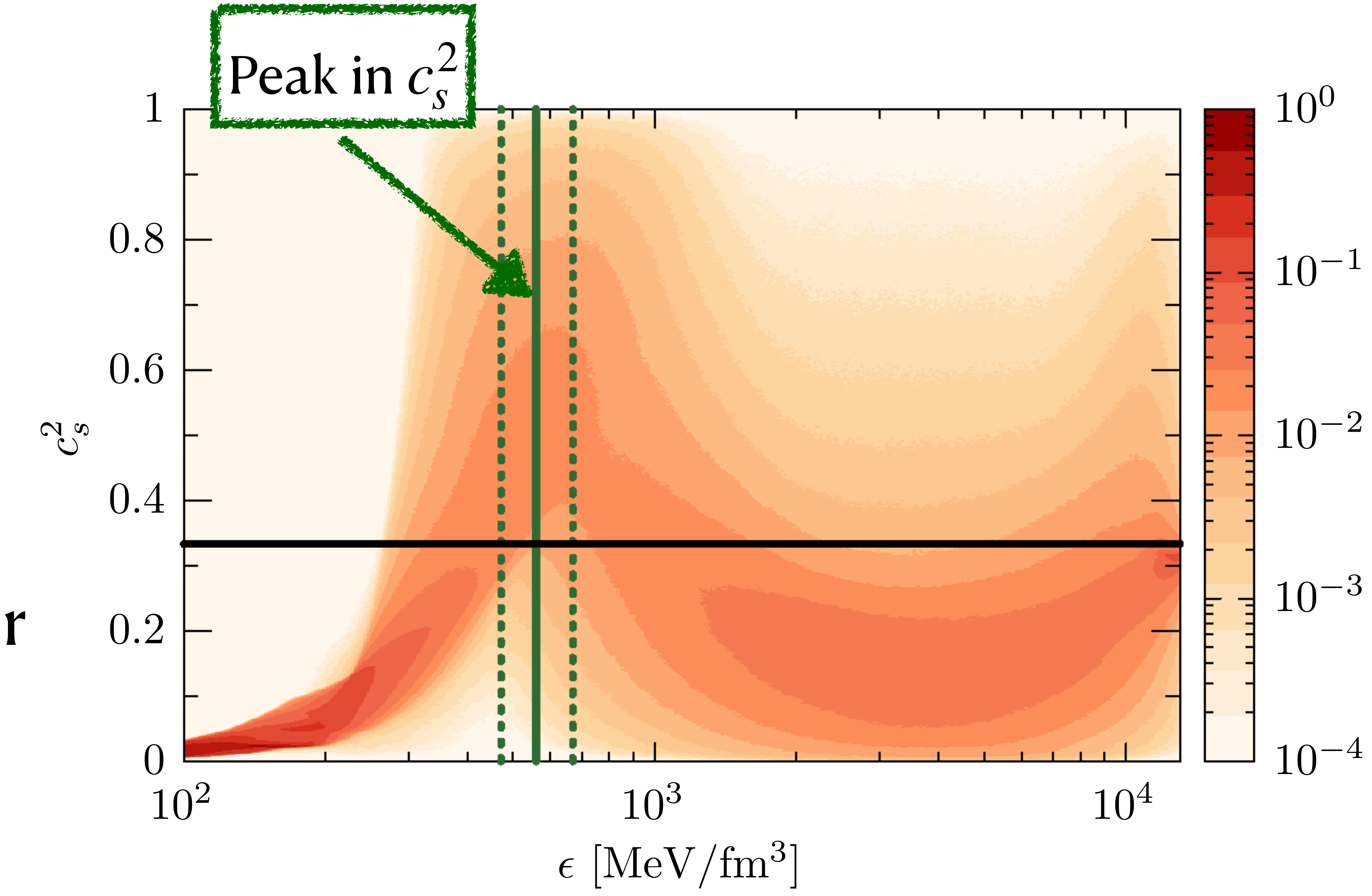


6×10^5 viable Equations of State

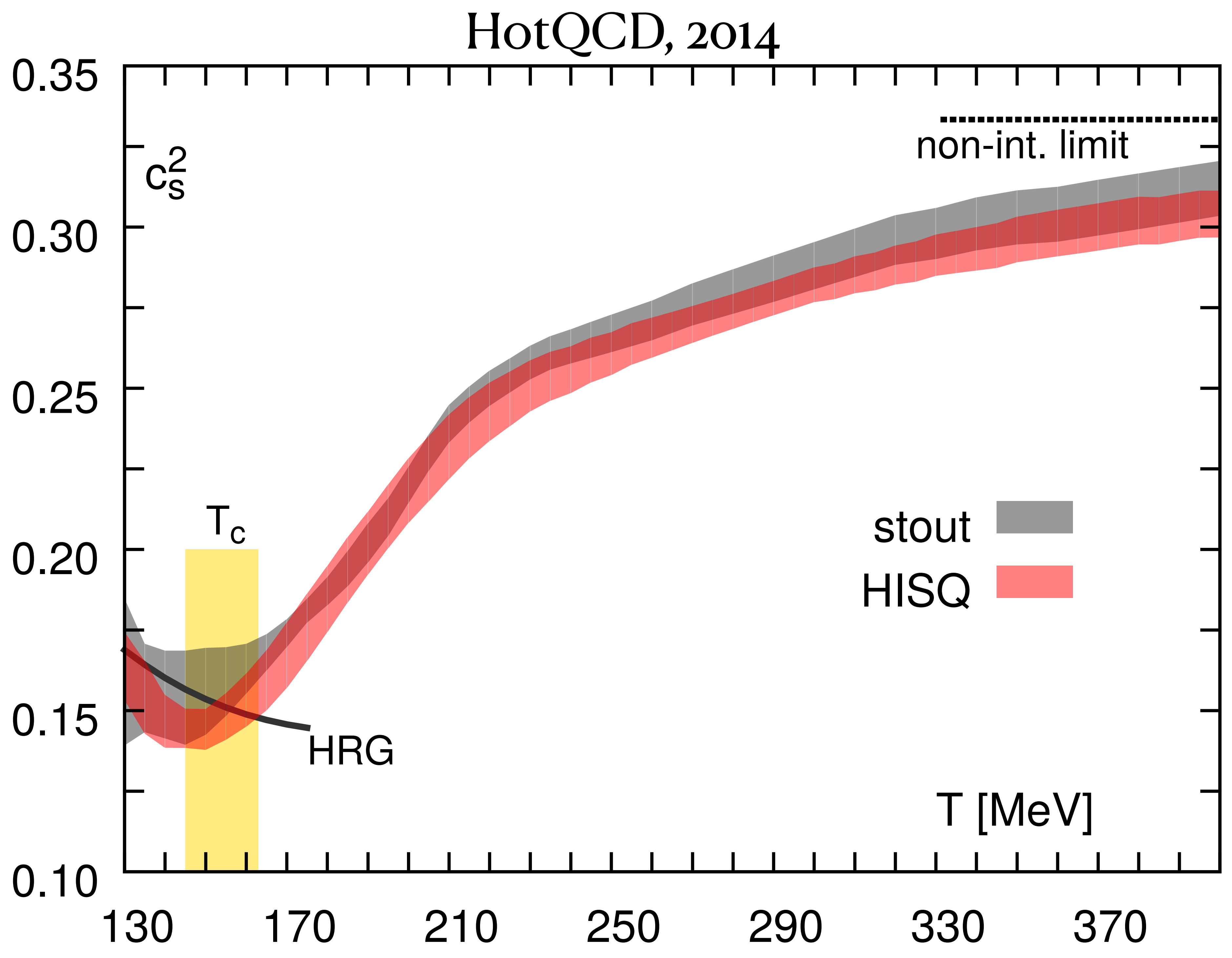
General Structure of Speed of Sound

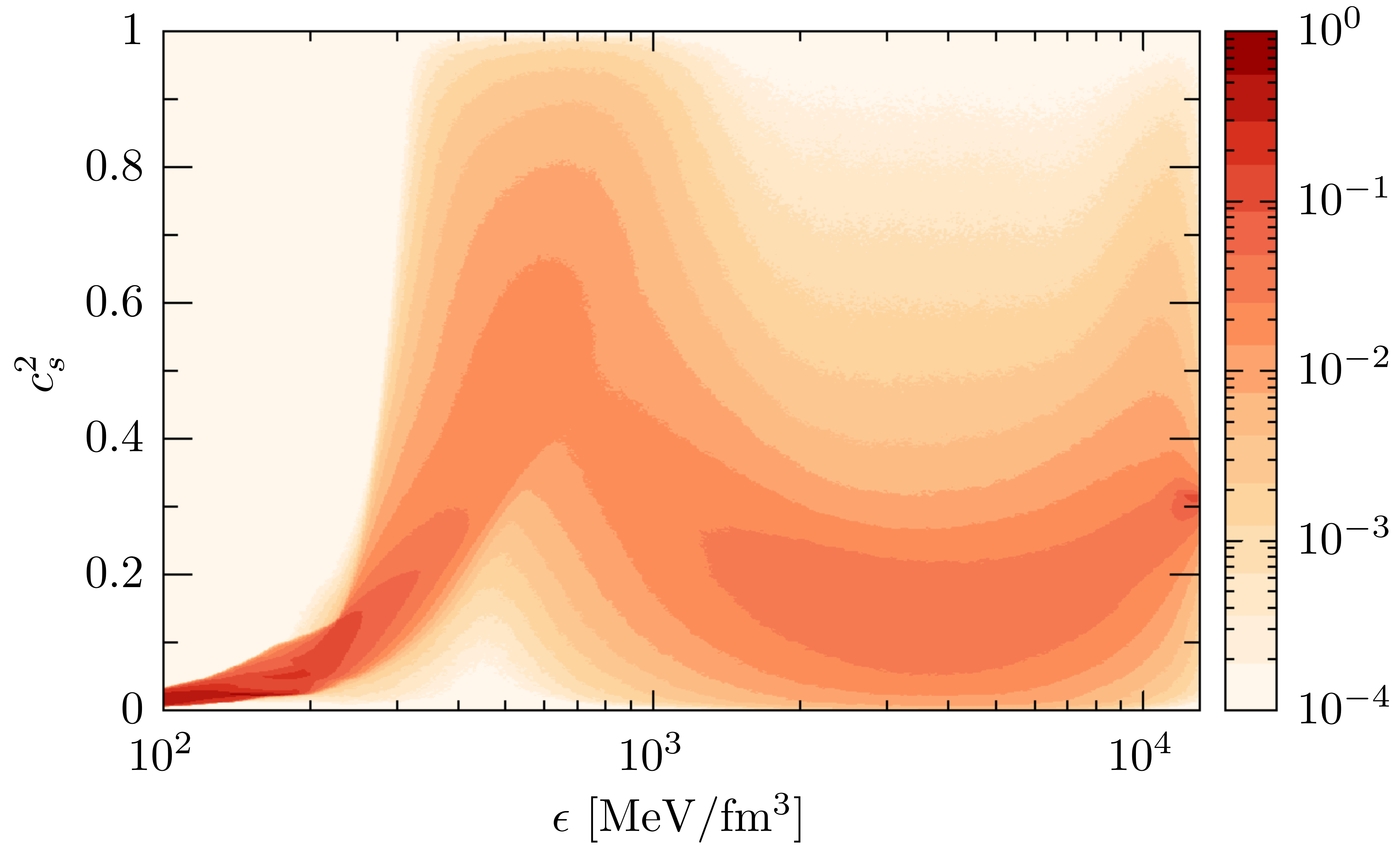
- General peak-dip structure
  Altiparmak et al, 2022

- Peak similar to quarkyonic matter
  McLerran, Reddy, 2019; Pang et al, 2023



Local maximum at $\epsilon_{\text{peak}} = 0.56^{+0.11}_{-0.09}$ GeV/fm^3 with $c_s^2 = 0.82 \pm 0.08$





$$c_s^2 = \frac{S}{T}\frac{dT}{dS} < \frac{1}{3}$$

$$c_s^2 = \frac{n}{\mu}\frac{d\mu}{dn} > \frac{1}{3}$$

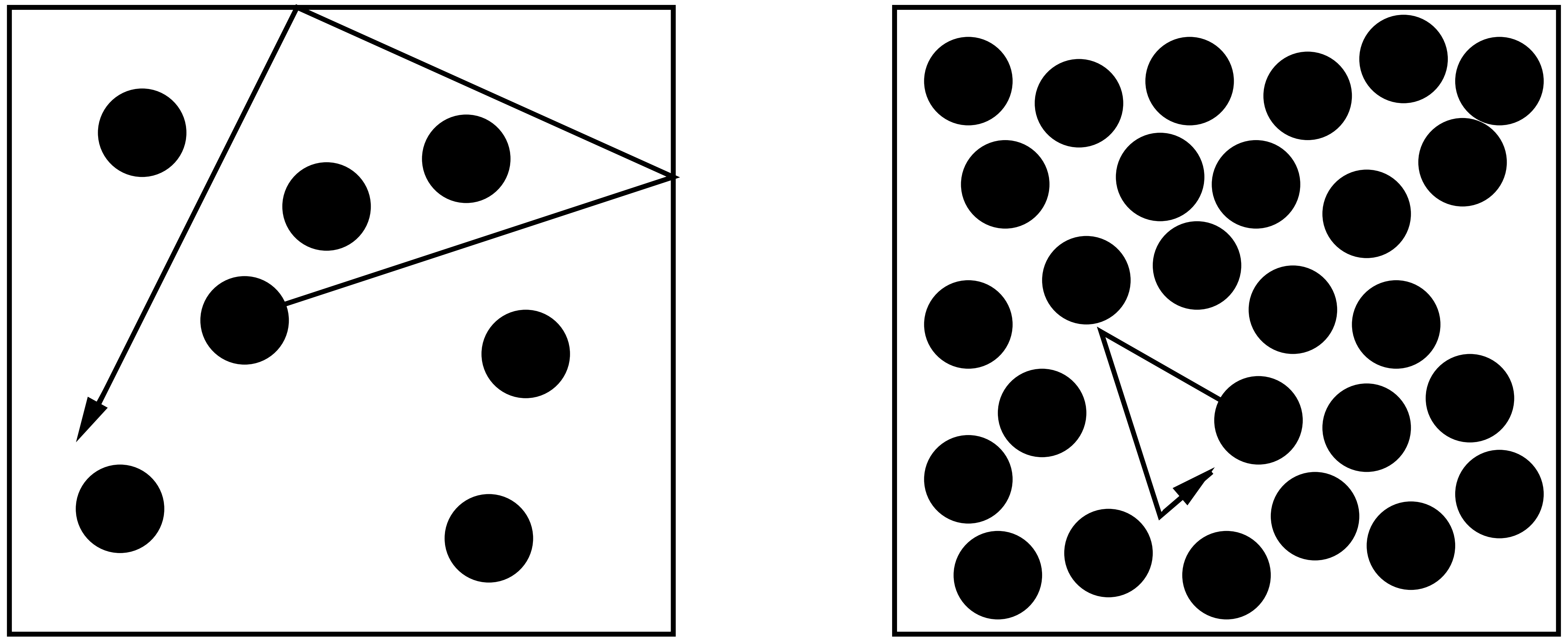
- Attractive interactions with resonance formation
- Chiral symmetry restoration and deconfinement

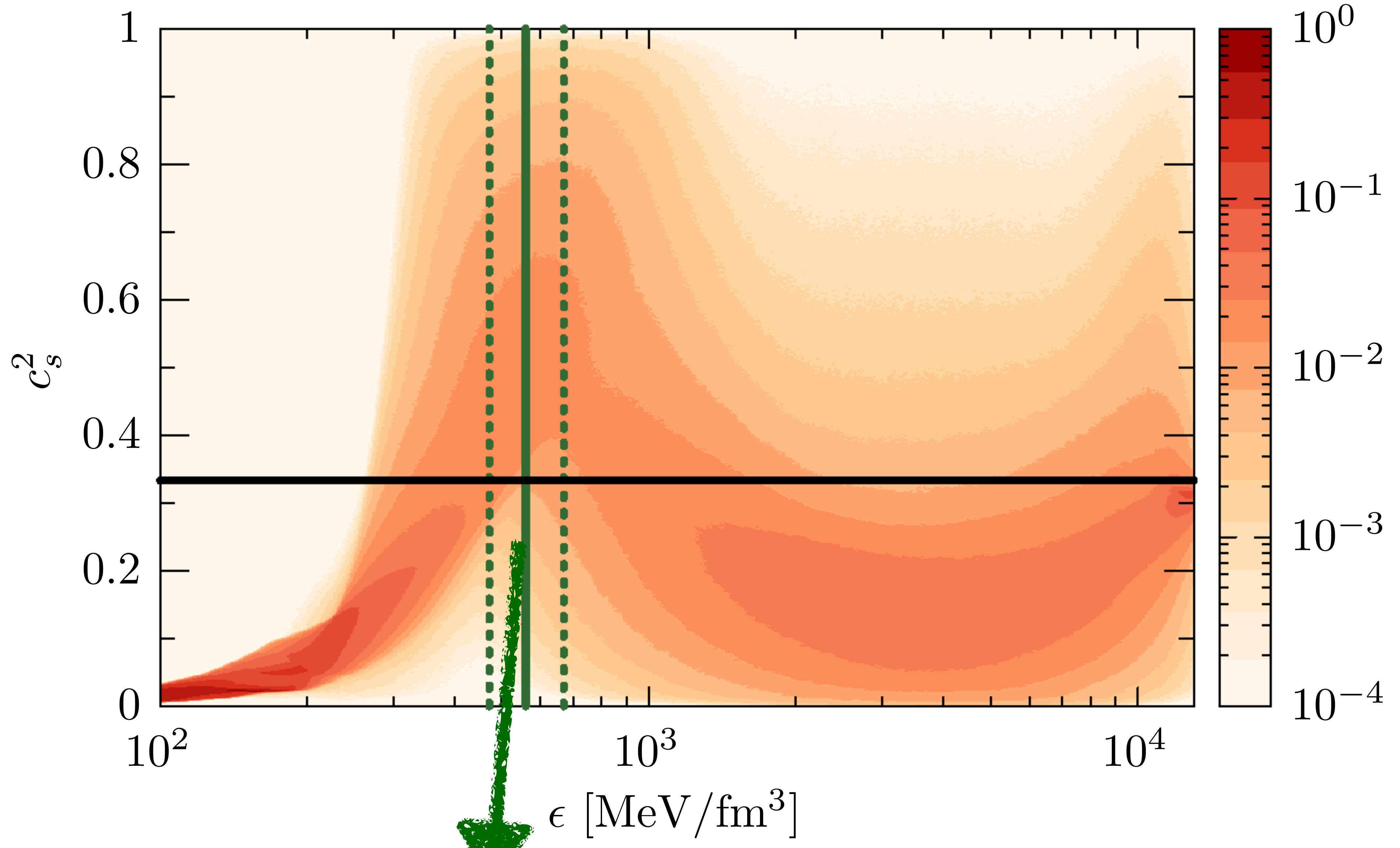
- Dominance of repulsive interactions
- Onset of quark or quarkyonic, or baryquark matter?

Non-monotonicity ⟷ Change of phase

Percolation theory vs speed of sound

see e.g. Satz, 1998; Castorina et al, 2009; Fukushima, 2020





Percolation theory: $n_c = 1.22/V_0$

$$n_{\text{peak}} = 0.54^{+0.09}_{-0.07}\ \text{fm}^{-3}$$

Avg. proton radius: $R_0 = 0.80 \pm 0.05\ \text{fm}$ → $n_c = 0.57^{+0.12}_{-0.09}\ \text{fm}^{-3}$

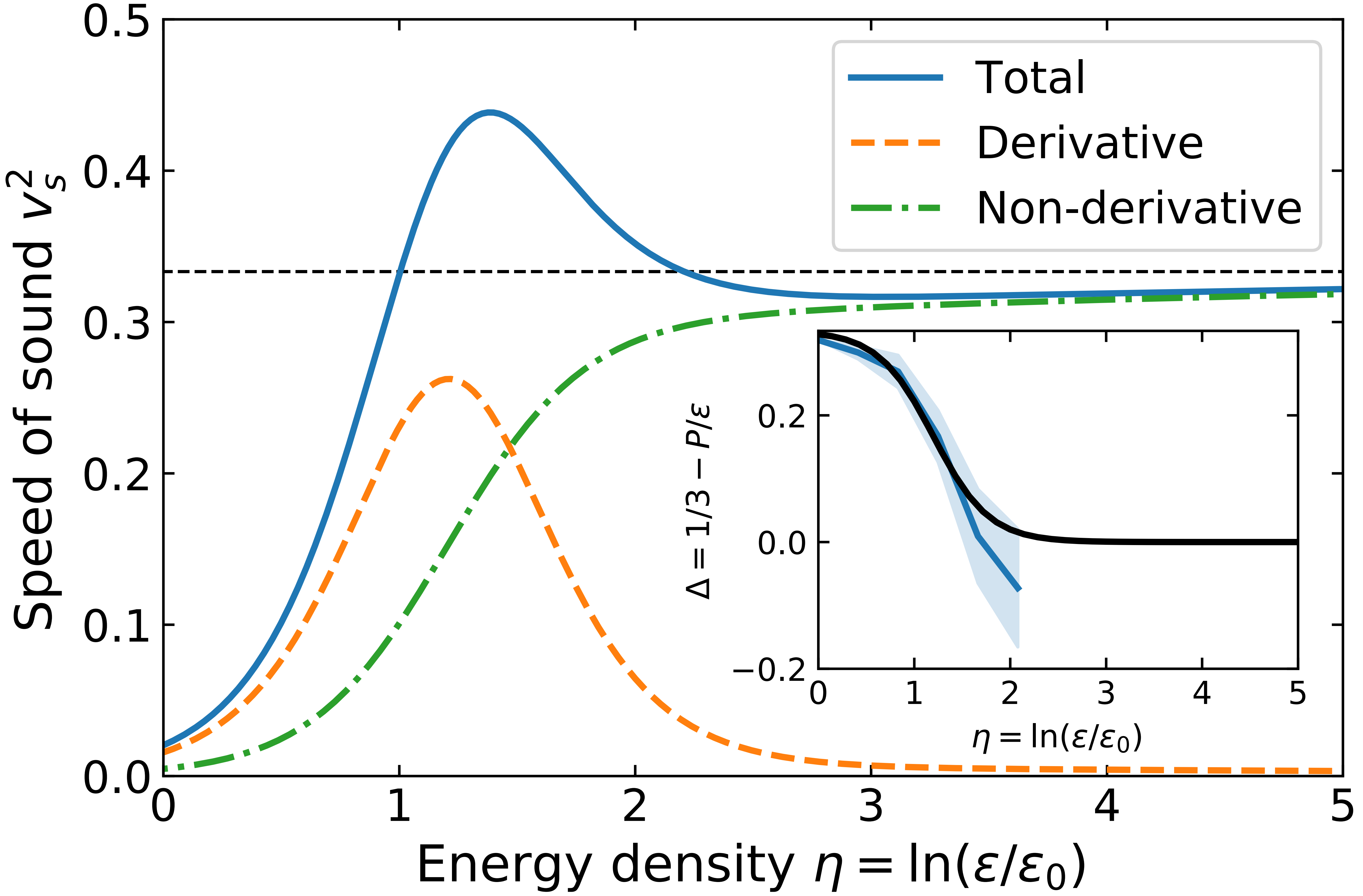
Wang et al 2022

Pb-Pb collisions at $\sqrt{s} = 2.76\ \text{TeV}$ → $n_c = 0.60 \pm 0.07\ \text{fm}^{-3}$

Andronic et al 2018

Speed of Sound as Trace Anomaly

Fujimoto et al 2022



Trace Anomaly: $\Delta = \frac{1}{3} - \frac{p}{\epsilon}$

$$c_s^2 = \frac{d\left(\epsilon \frac{p}{\epsilon}\right)}{d\epsilon} = \frac{1}{3} - \Delta - \epsilon \frac{d\Delta}{d\epsilon}$$

Trace anomaly more informative than speed of sound

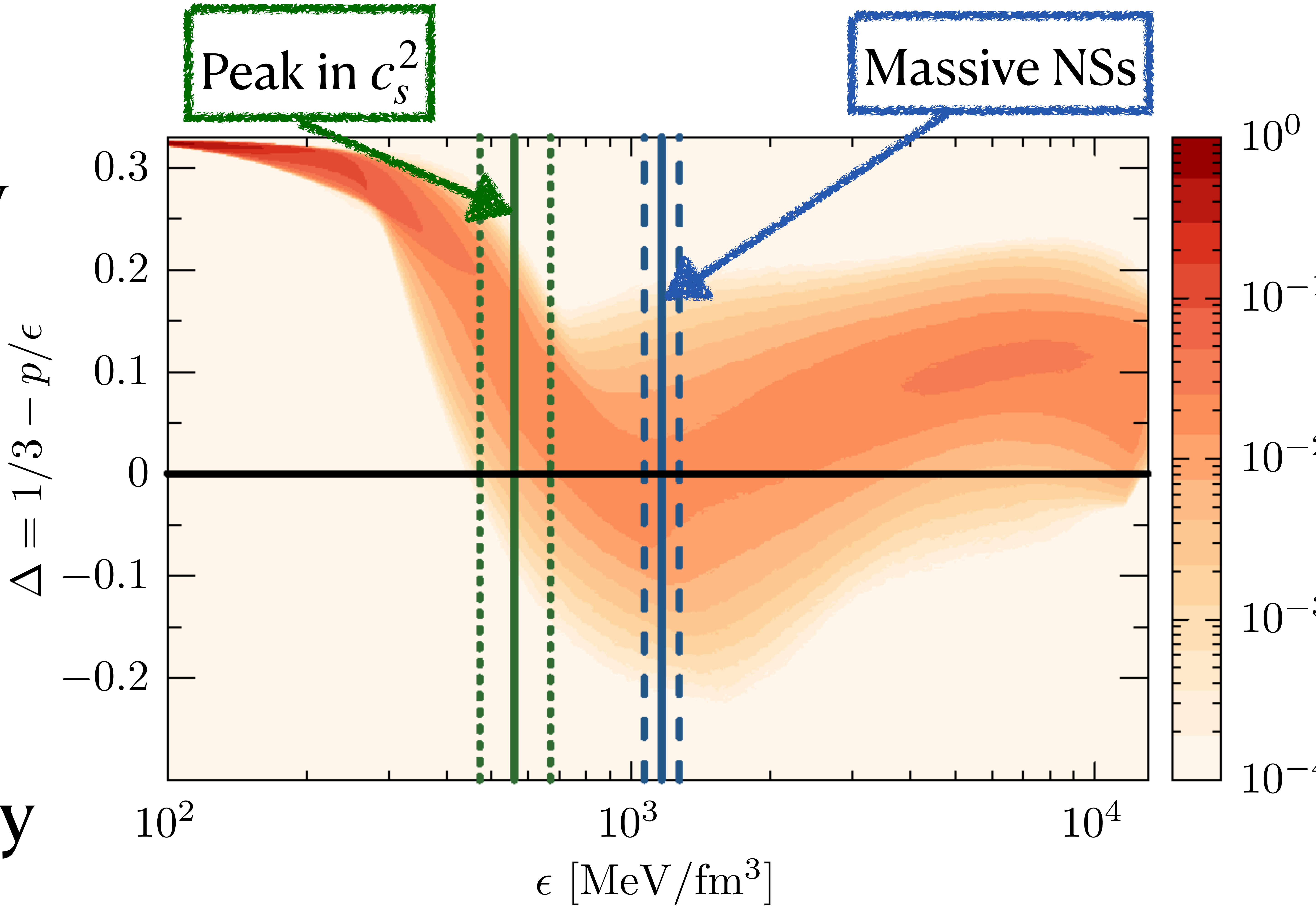
Measure of conformality

Δ monotonic up to $\simeq \epsilon_{\rm TOV}$

$$c_s^2 = \frac{1}{3} - \Delta - \epsilon \frac{d\Delta}{d\epsilon}$$

Maximum in c_s^2

Fast approach to comformality



$\Delta \simeq 0$ at $\epsilon \simeq 1$ GeV/fm^3

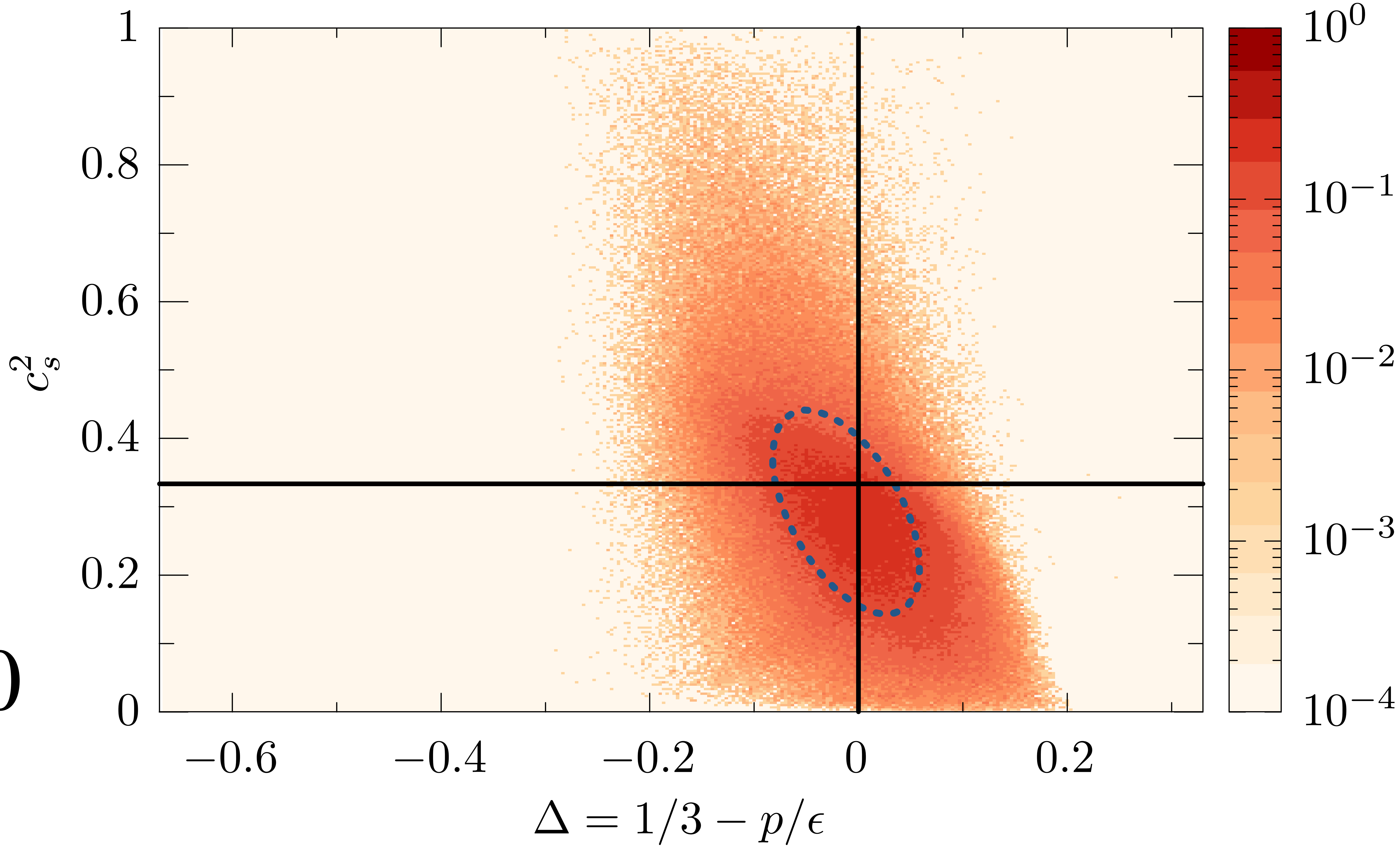
c_s^2 and Δ in Heavy Neutron Stars

Conformality:

$$c_s^2 = \frac{1}{3} \text{ and } \Delta = 0$$

$$c^2_{s,\,\mathrm{TOV}} = 0.28 \pm 0.16 \simeq \frac{1}{3}$$

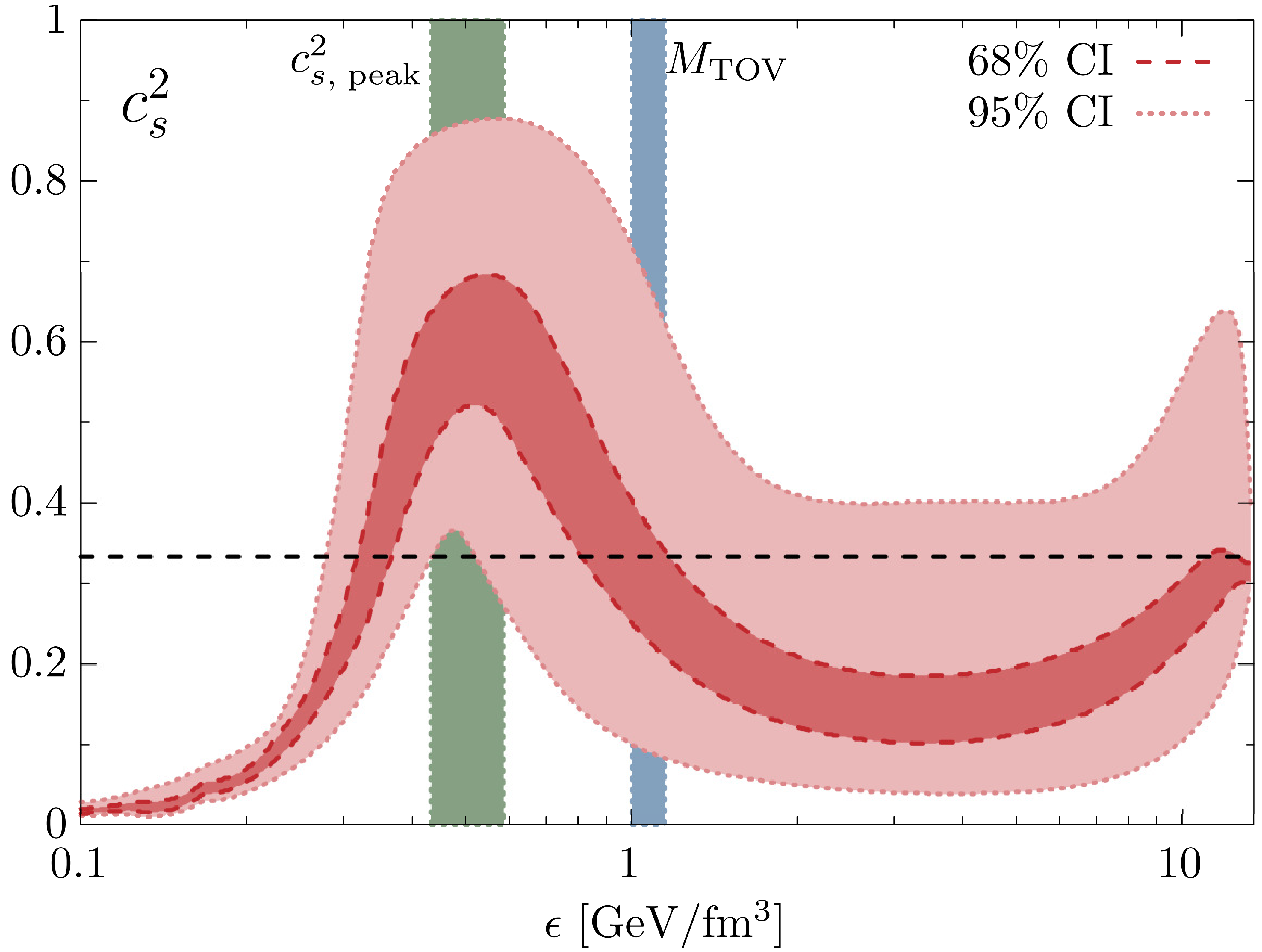
$$\Delta_{\mathrm{TOV}} = -0.01 \pm 0.03 \simeq 0$$



Matter almost conformal in the cores of maximally massive NSs

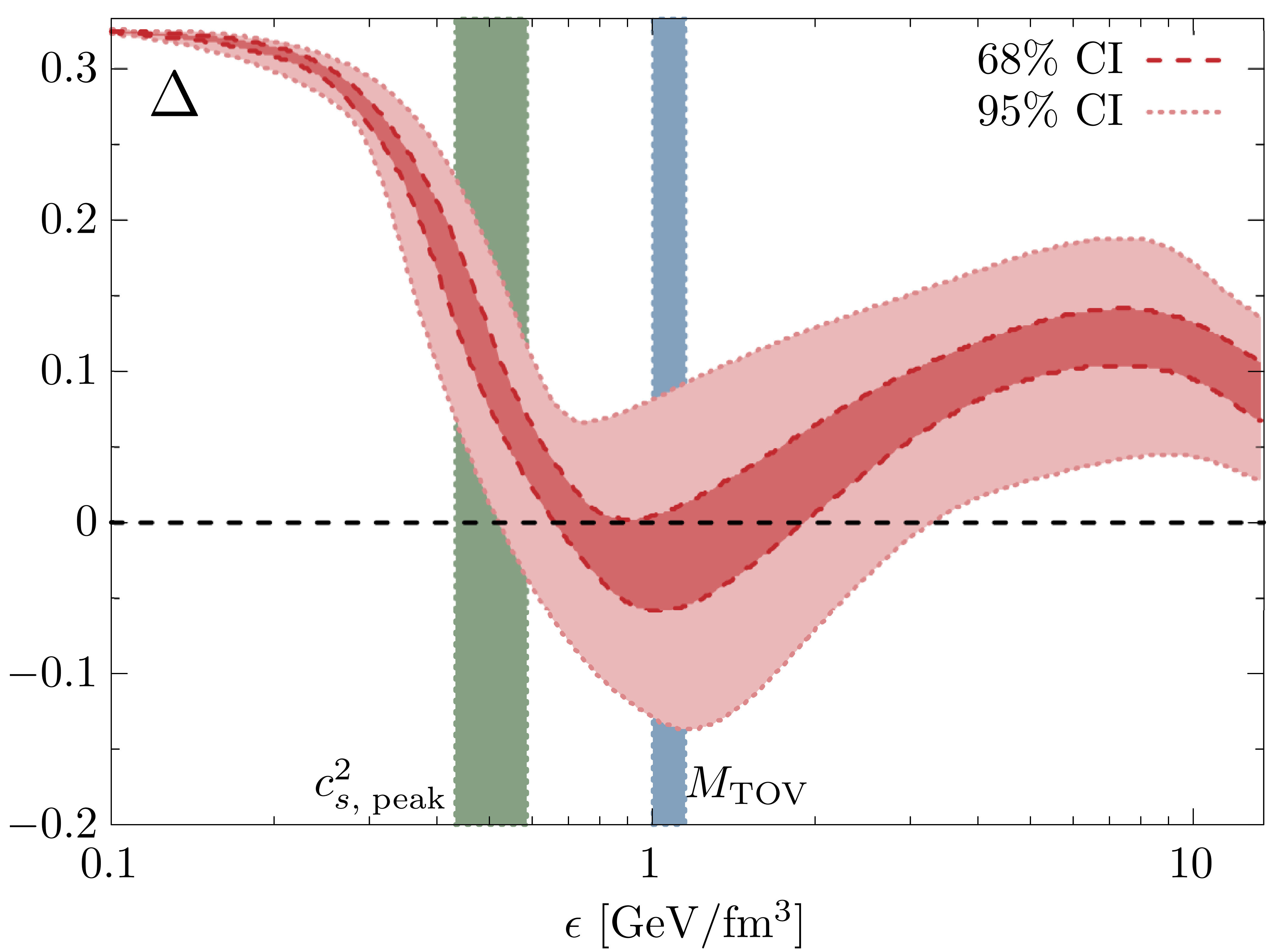
Measures of conformality

$$c_s^2 \simeq \frac{1}{3}$$



Fujimoto et al 2022

$$\Delta = \frac{1}{3} - \frac{p}{\epsilon} \simeq 0$$



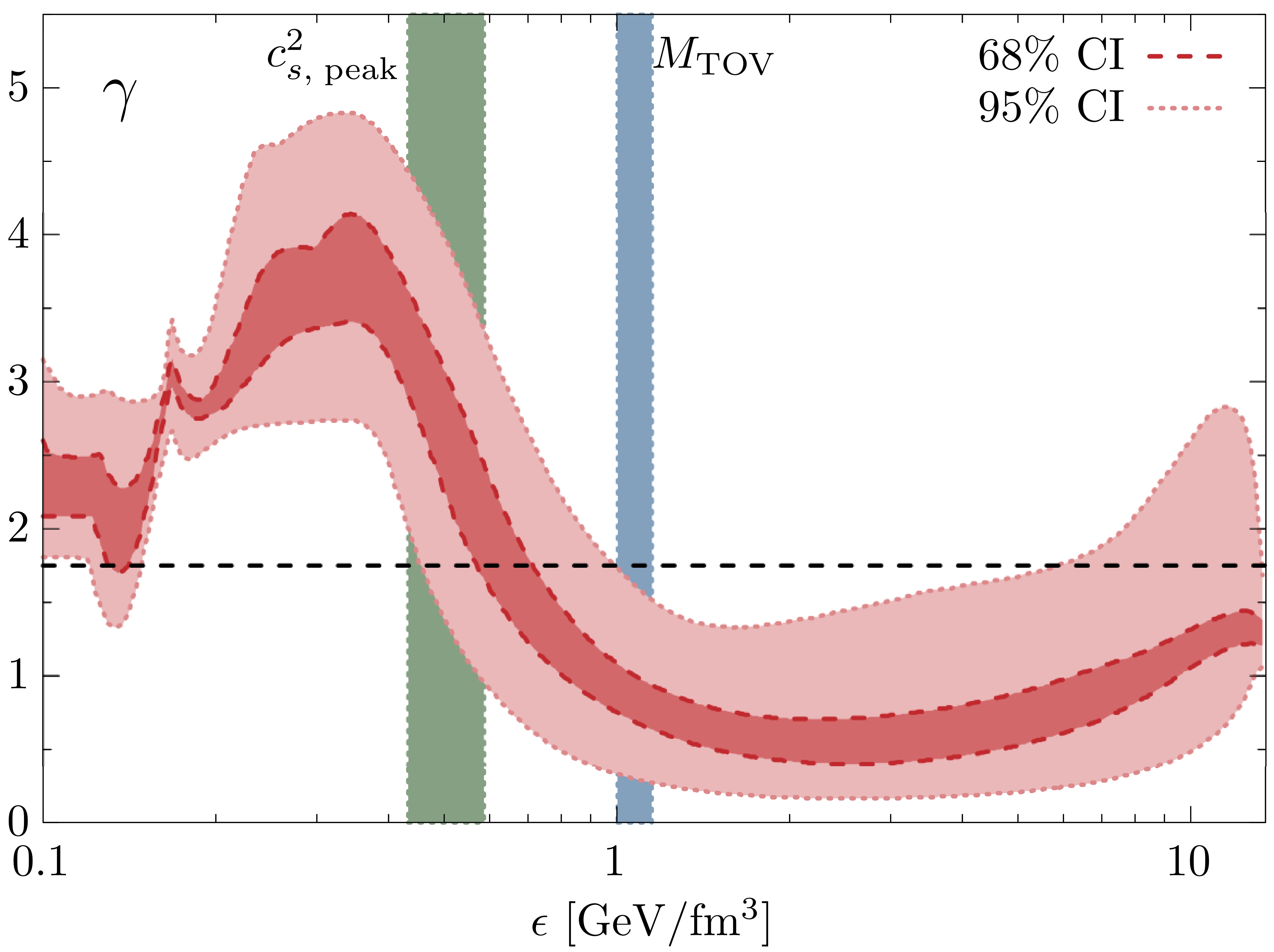
Measures of conformality 2

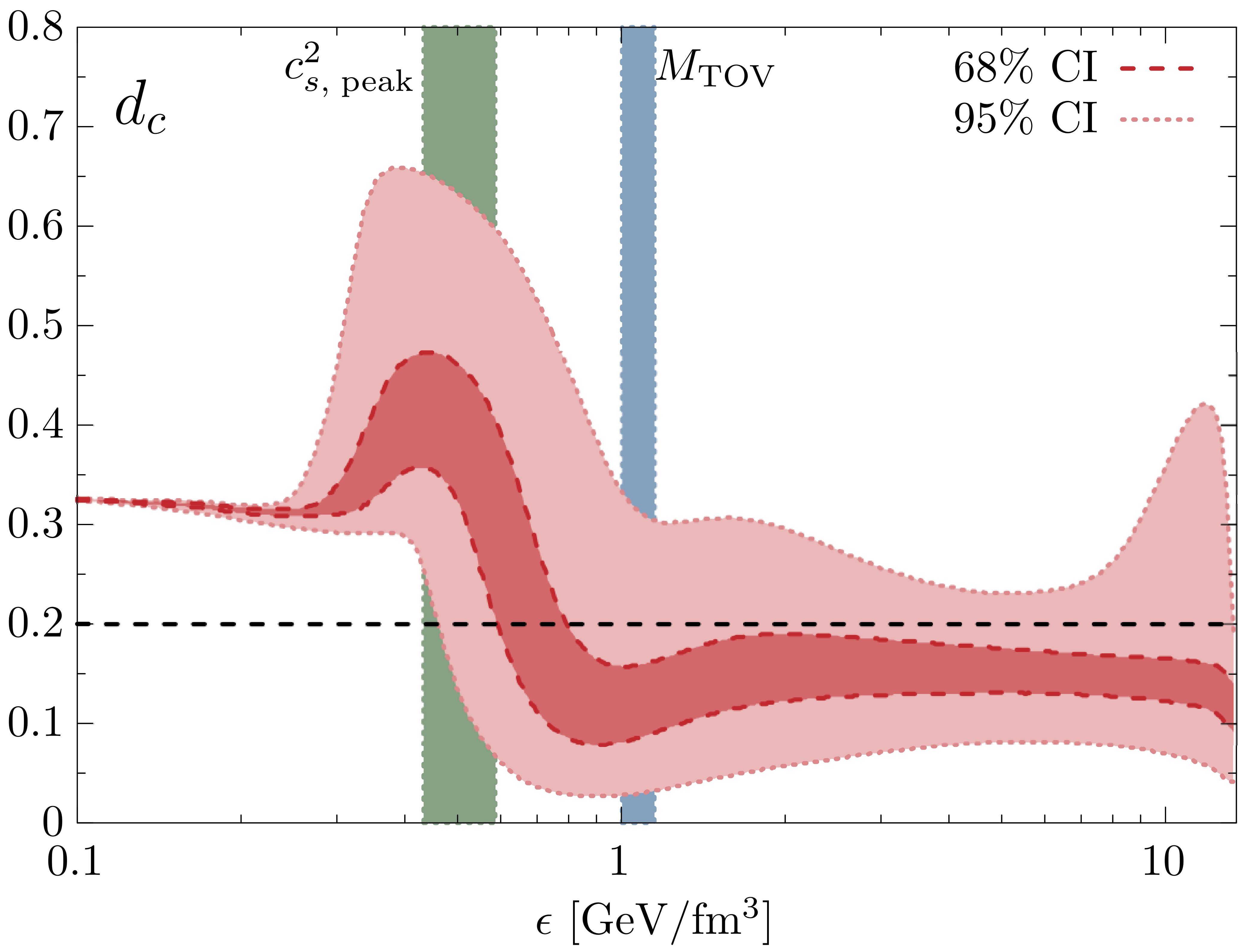
$$\gamma = \frac{\epsilon}{p} c_s^2 = \frac{c_s^2}{1/3 - \Delta} \lesssim 1.75$$

Annala et al 2020

$$d_c = \sqrt{\Delta^2 + (\epsilon \Delta')^2} \lesssim 0.2$$

Annala et al 2023





Curvature of energy per particle

Pressure from $\frac{\epsilon}{n} \rightarrow p = n^2 \frac{d\epsilon/n}{dn}$

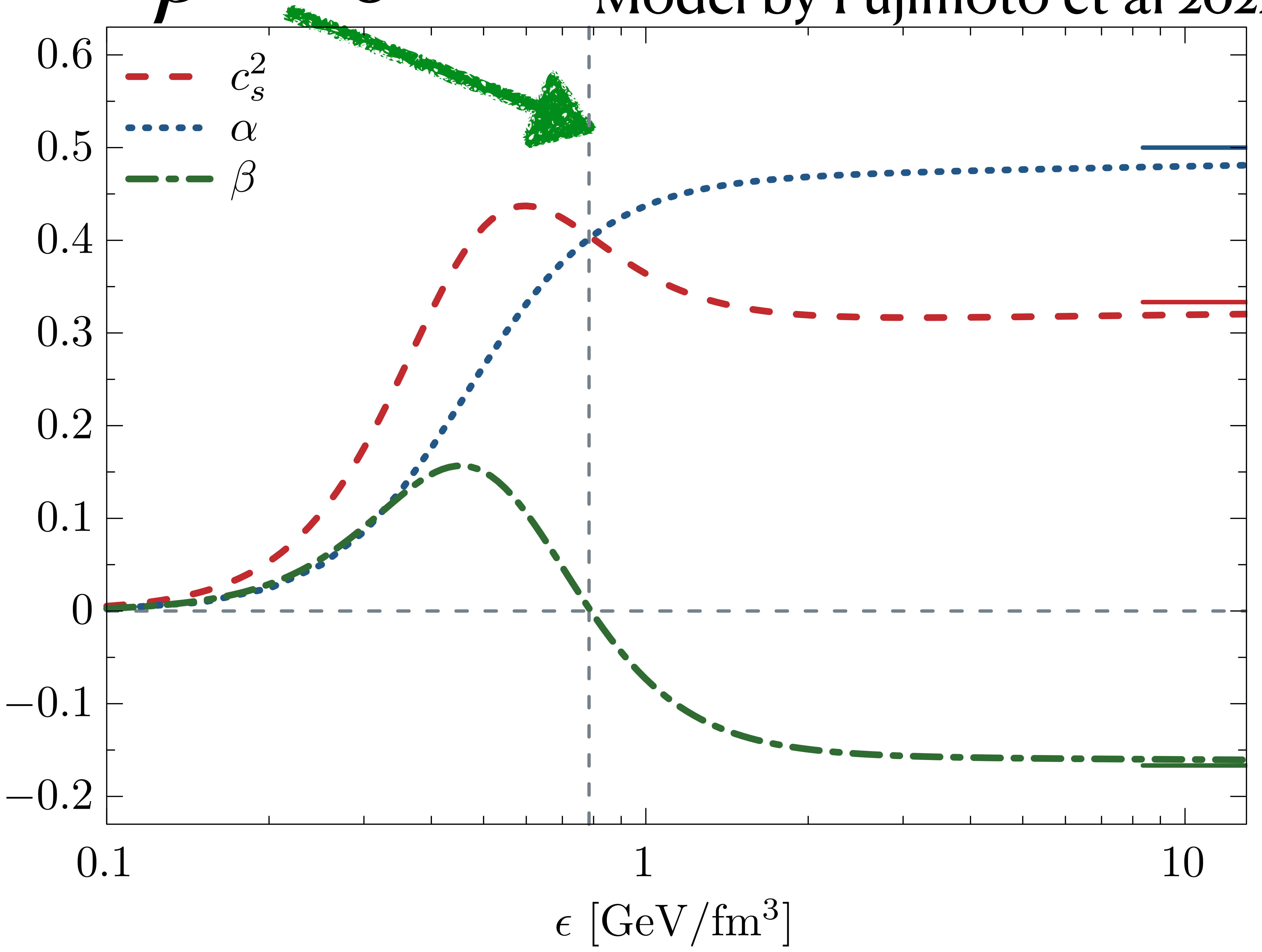
$$c_s^2 = \frac{1}{\mu}\frac{dp}{dn} = \alpha + \beta$$

$$\alpha = 2\frac{n}{\mu}\frac{d\epsilon/n}{dn} = 2\frac{1/3 - \Delta}{4/3 - \Delta}$$

$$\beta = \frac{n^2}{\mu}\frac{d^2\epsilon/n}{dn^2} = c_s^2 - \alpha$$

$\beta = 0$

Model by Fujimoto et al 2022

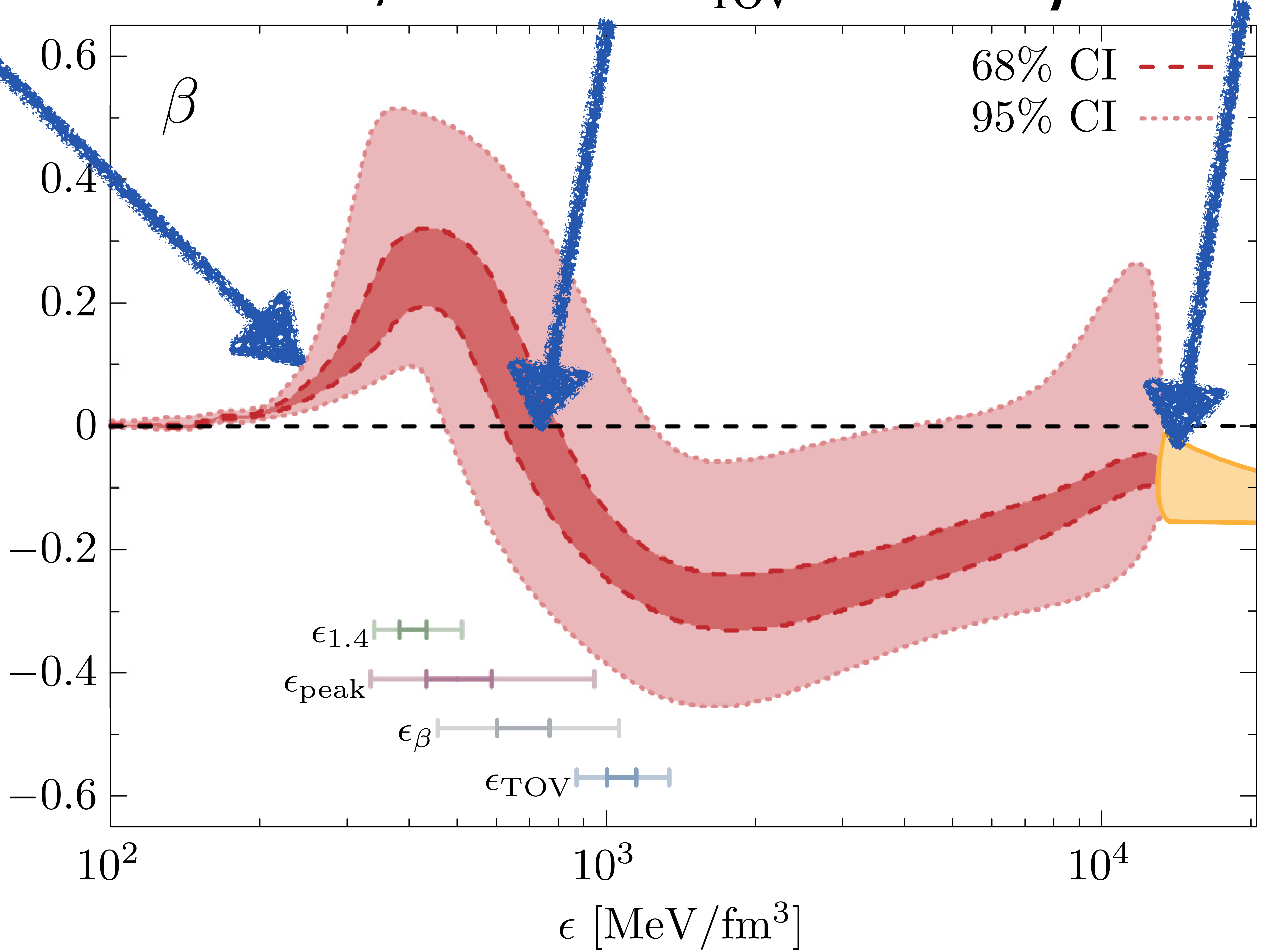


$\beta = 0 \rightarrow$ changeover to conformal regime

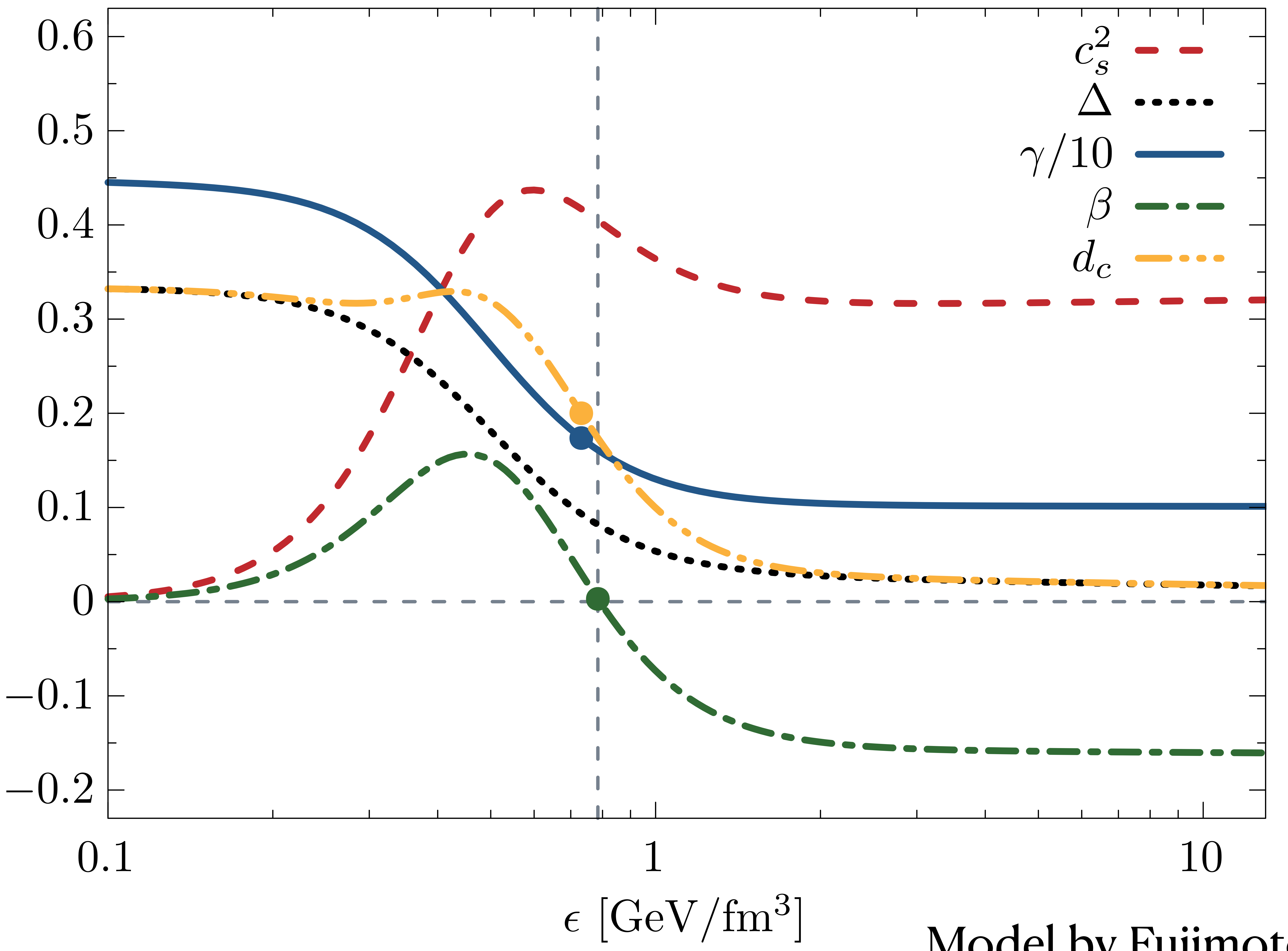
$\beta \simeq c_s^2 > 0$

$\beta < 0$ at $\epsilon \simeq \epsilon_{TOV}$

$\beta \simeq -1/6$



Changeover consistent other measures



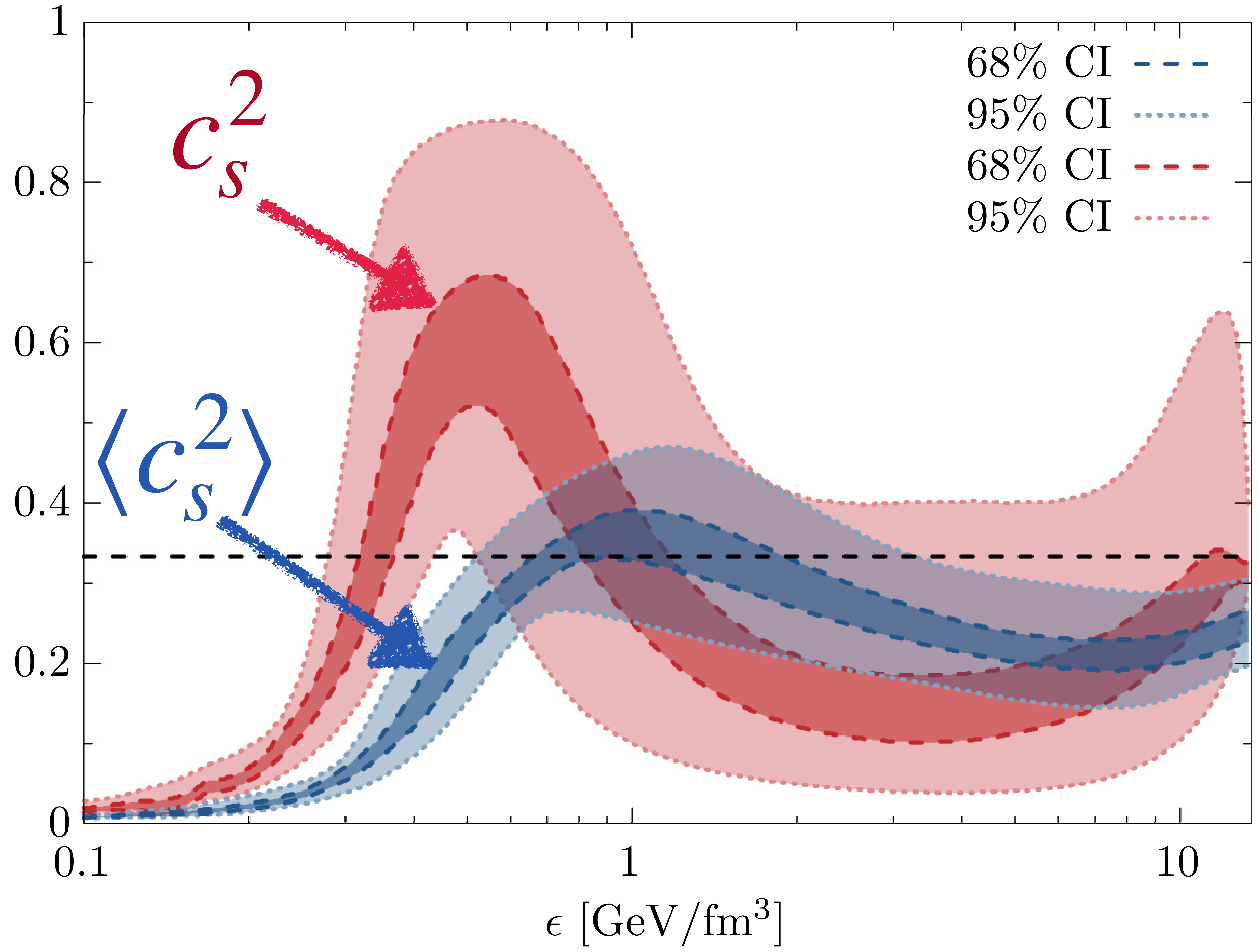
Model by Fujimoto et al 2022

Average Speed of Sound

$$\frac{p}{\epsilon} = \frac{1}{\epsilon}\int_0^{\epsilon} d\epsilon' \frac{dp}{d\epsilon'} = \langle c_s^2 \rangle$$

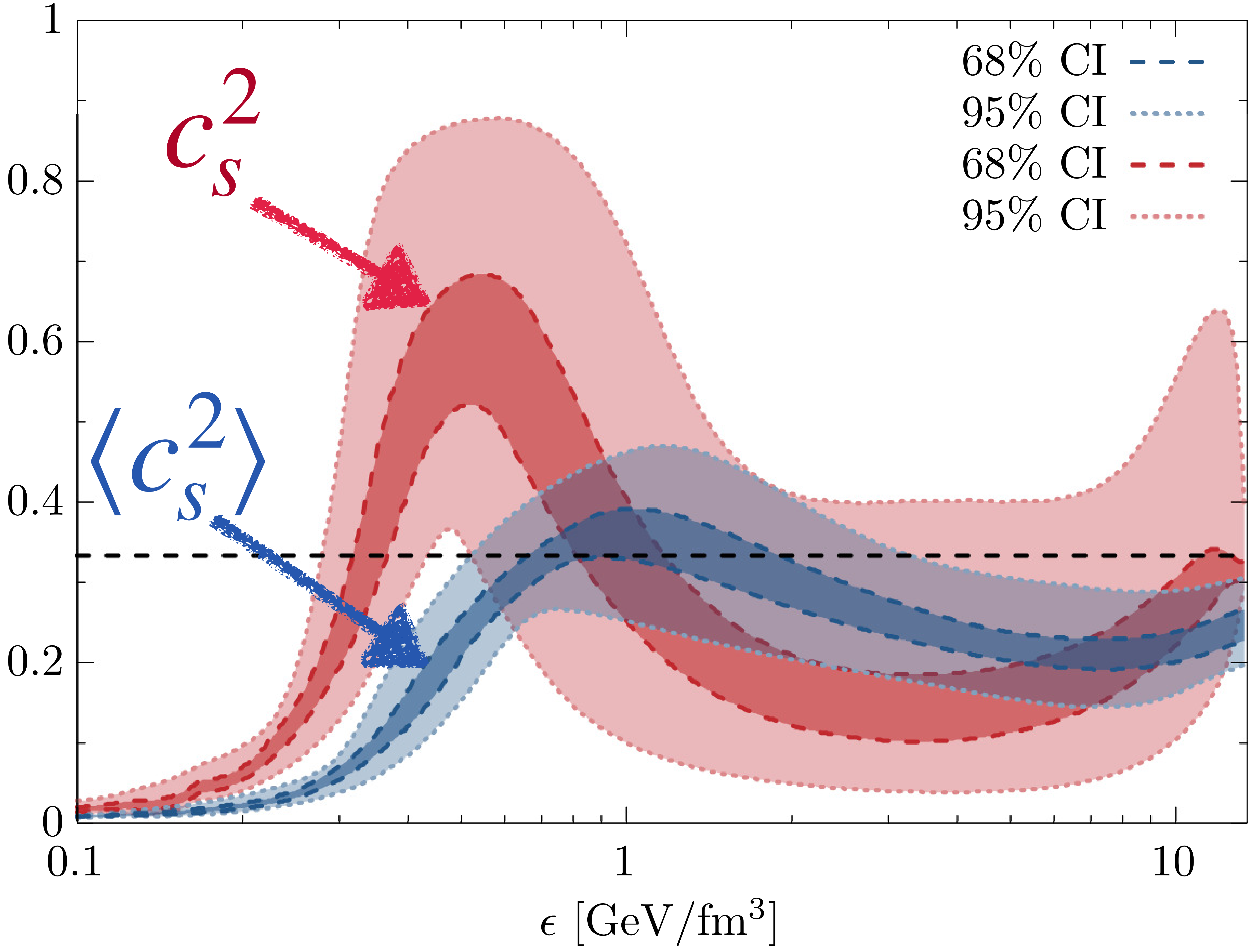
$$\Delta = \frac{1}{3} - \langle c_s^2 \rangle \qquad \epsilon\Delta' = \langle c_s^2 \rangle - c_s^2$$

$$\gamma = c_s^2 / \langle c_s^2 \rangle$$



Conformality : $\Delta \simeq 0 \Leftrightarrow \mathrm{c_s^2} \simeq 1/3$ and $\epsilon\Delta' \simeq 0 \Leftrightarrow \mathrm{c_s^2} \simeq \langle \mathrm{c_s^2} \rangle$

Implications of vanishing trace anomaly

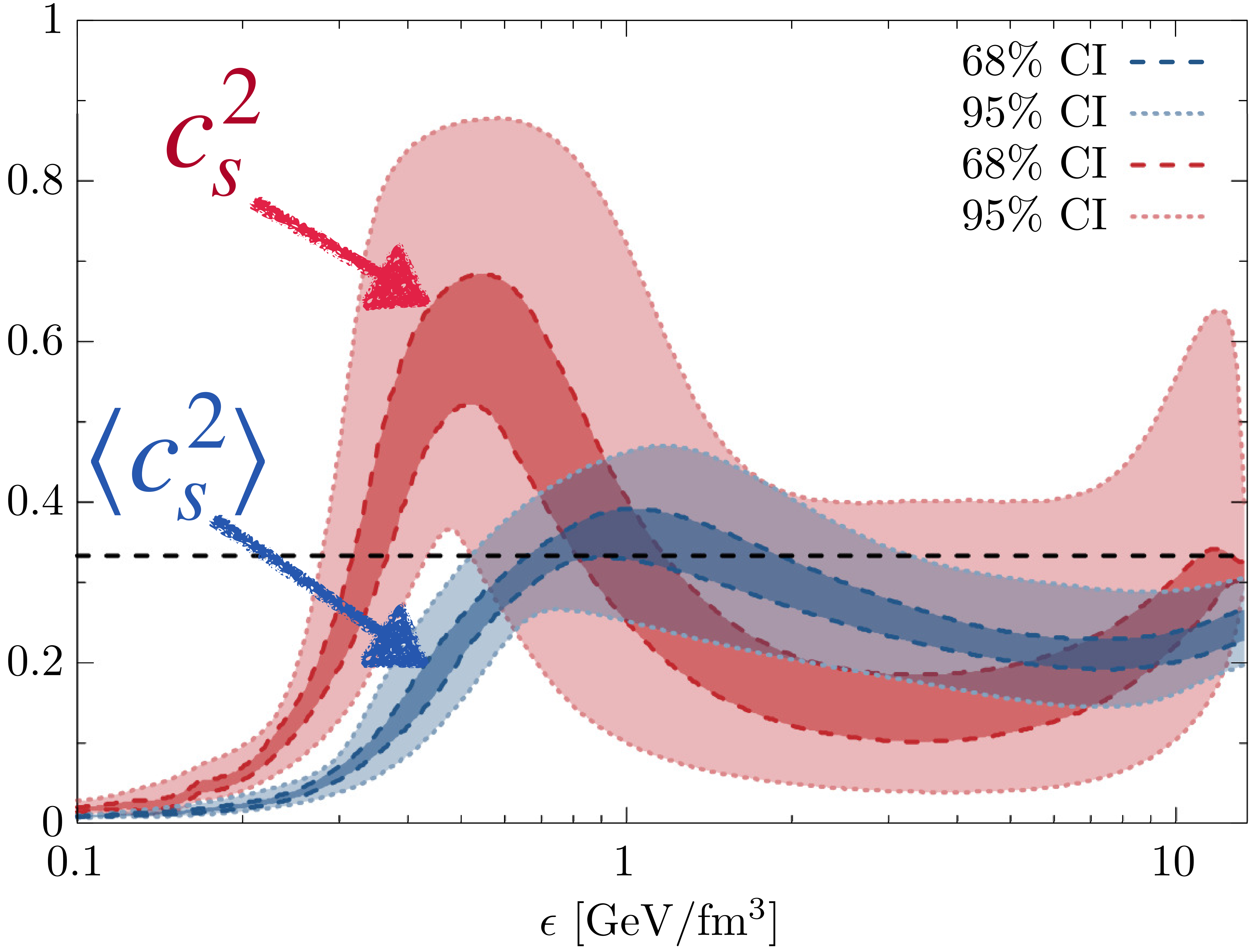


$$\Delta_{\text{TOV}} = -0.01 \pm 0.03 \simeq 0$$

Ansatz 1: $\Delta_{\text{TOV}} = 0 \Leftrightarrow \langle c_s^2 \rangle_{\text{TOV}} = 1/3$

- c_s^2 must exceed 1/3
- c_s^2 features maximum

Implications of vanishing trace anomaly



$$\Delta_{\mathrm{TOV}} = -0.01 \pm 0.03 \simeq 0$$

Ansatz 1: $\Delta_{\mathrm{TOV}} = 0 \Leftrightarrow \langle c_s^2 \rangle_{\mathrm{TOV}} = 1/3$

Ansatz 2: $\Delta \geq 0 \Leftrightarrow \langle c_s^2 \rangle \leq 1/3$

- c_s^2 must exceed 1/3
- c_s^2 features maximum at $\epsilon \leq \epsilon_{\mathrm{TOV}}$
- consequences for NS phenomenology

Fujimoto et al 2022

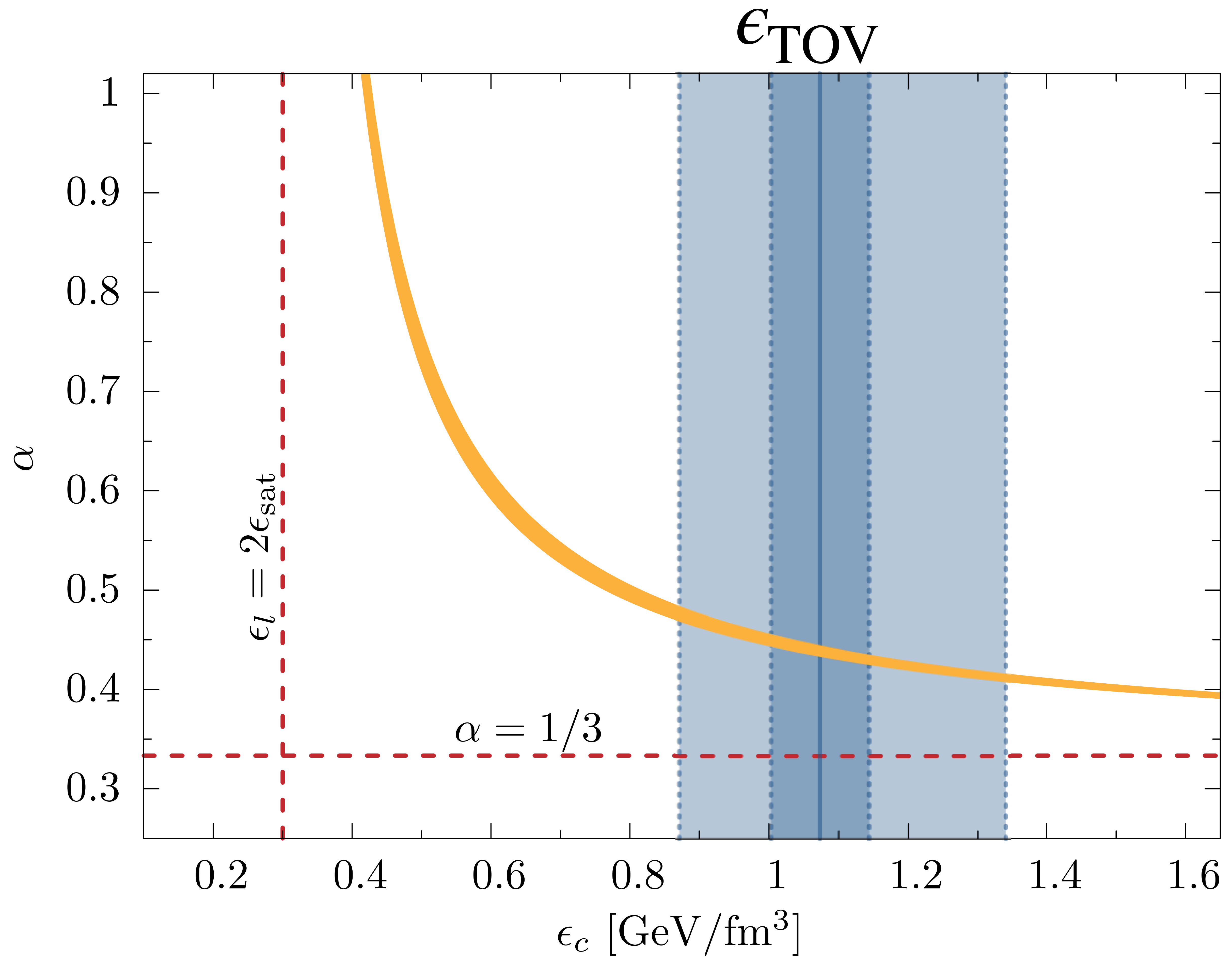
Implications of vanishing trace anomaly 2

$$\langle c_s^2 \rangle_c = \langle c_s^2 \rangle_l \frac{\epsilon_l}{\epsilon_c} + \alpha \left(1 - \frac{\epsilon_l}{\epsilon_c} \right) = \frac{1}{3}$$

Average c_s^2 at $\langle \epsilon_l, \epsilon_c \rangle$

$\langle c_s^2 \rangle_l$ from χEFT at $2\epsilon_0 = 5 - 8 \times 10^{-3}$

Drischler et al (2021)



$$\epsilon_c = \epsilon_{\mathrm{TOV}} \simeq 1 \text{ GeV/fm}^3 \longrightarrow \alpha \simeq 0.4 - 0.5 \longrightarrow c_{s,\max}^2 > \alpha$$

Implications of vanishing trace anomaly 2

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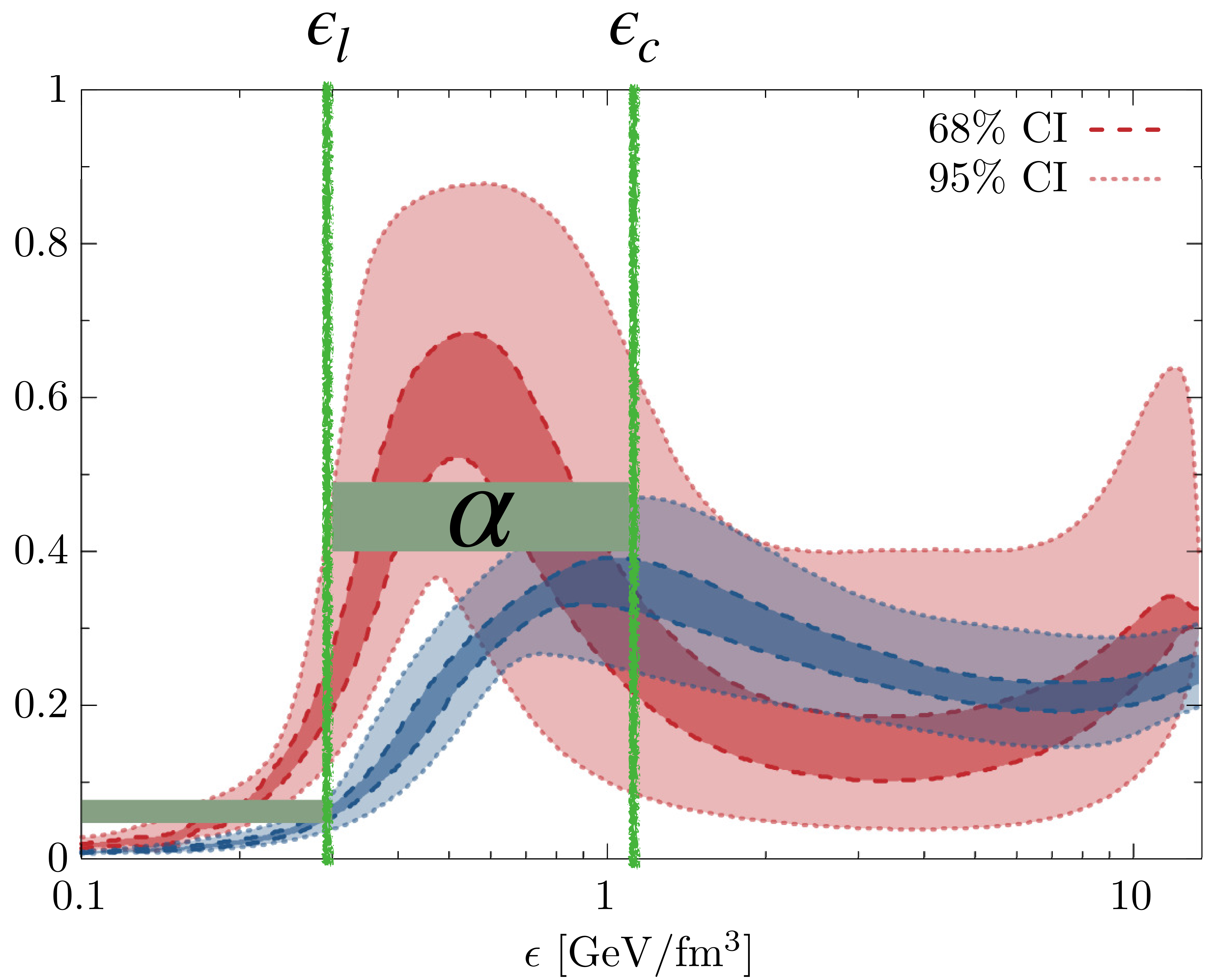


$$\epsilon_c = \epsilon_{\rm TOV} \simeq 1\ {\rm GeV/fm^3} \longrightarrow \alpha \simeq 0.4 - 0.5 \longrightarrow c^2_{s,\rm max} > \alpha$$

Summary

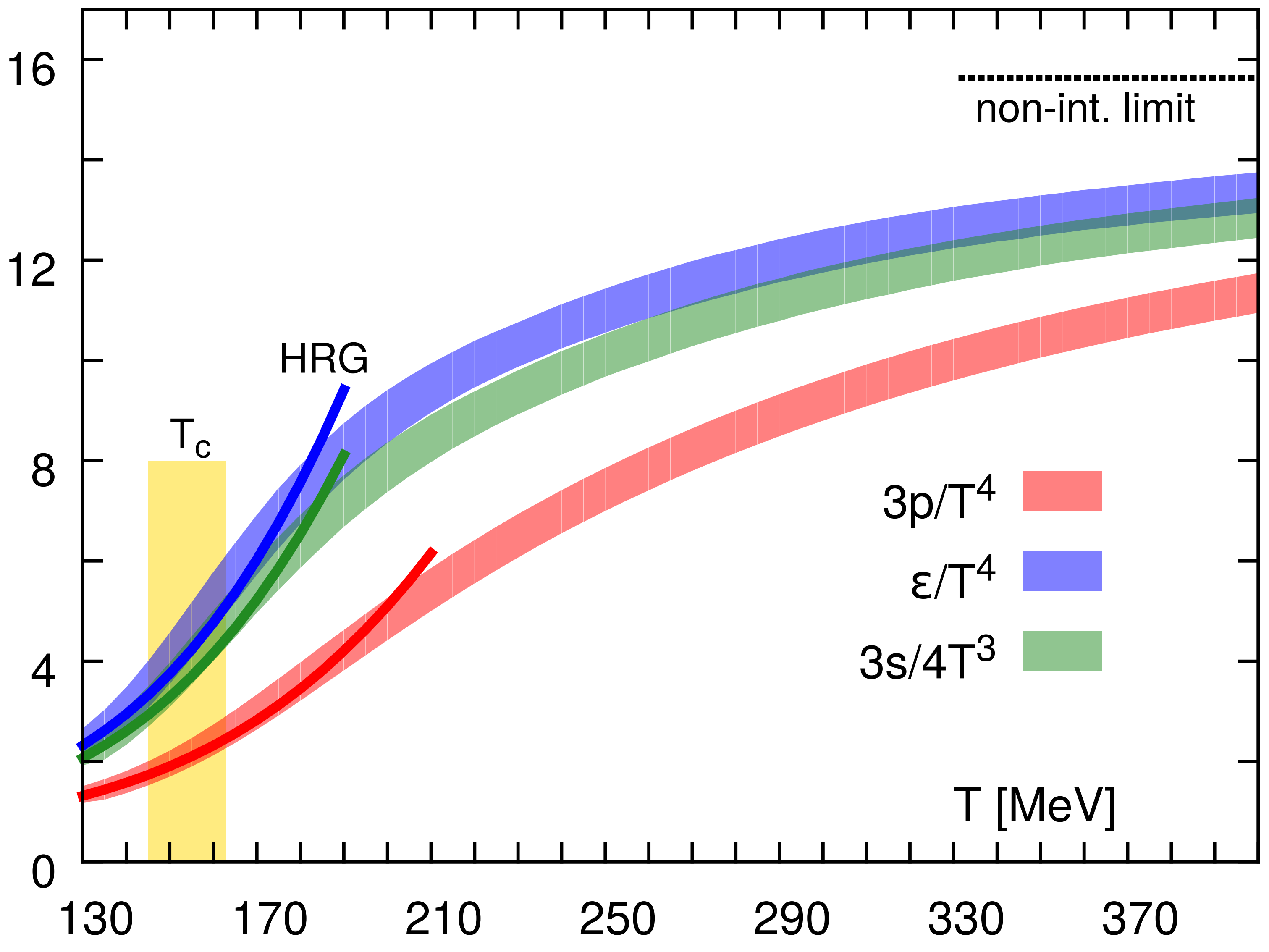
Maximum of c_s^2 consistent with percolation threshold

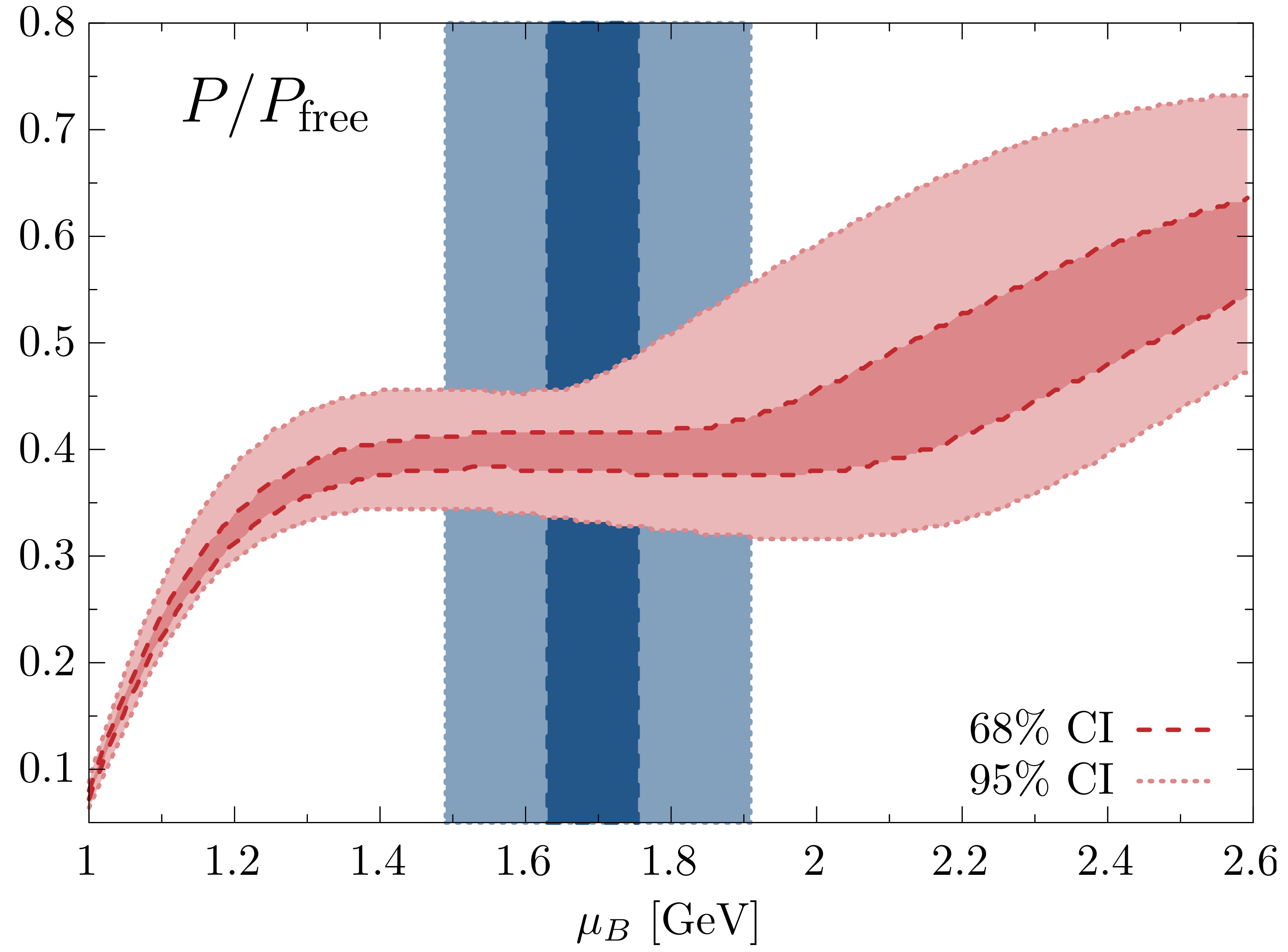
Matter seems to be conformal in the cores of massive NSs

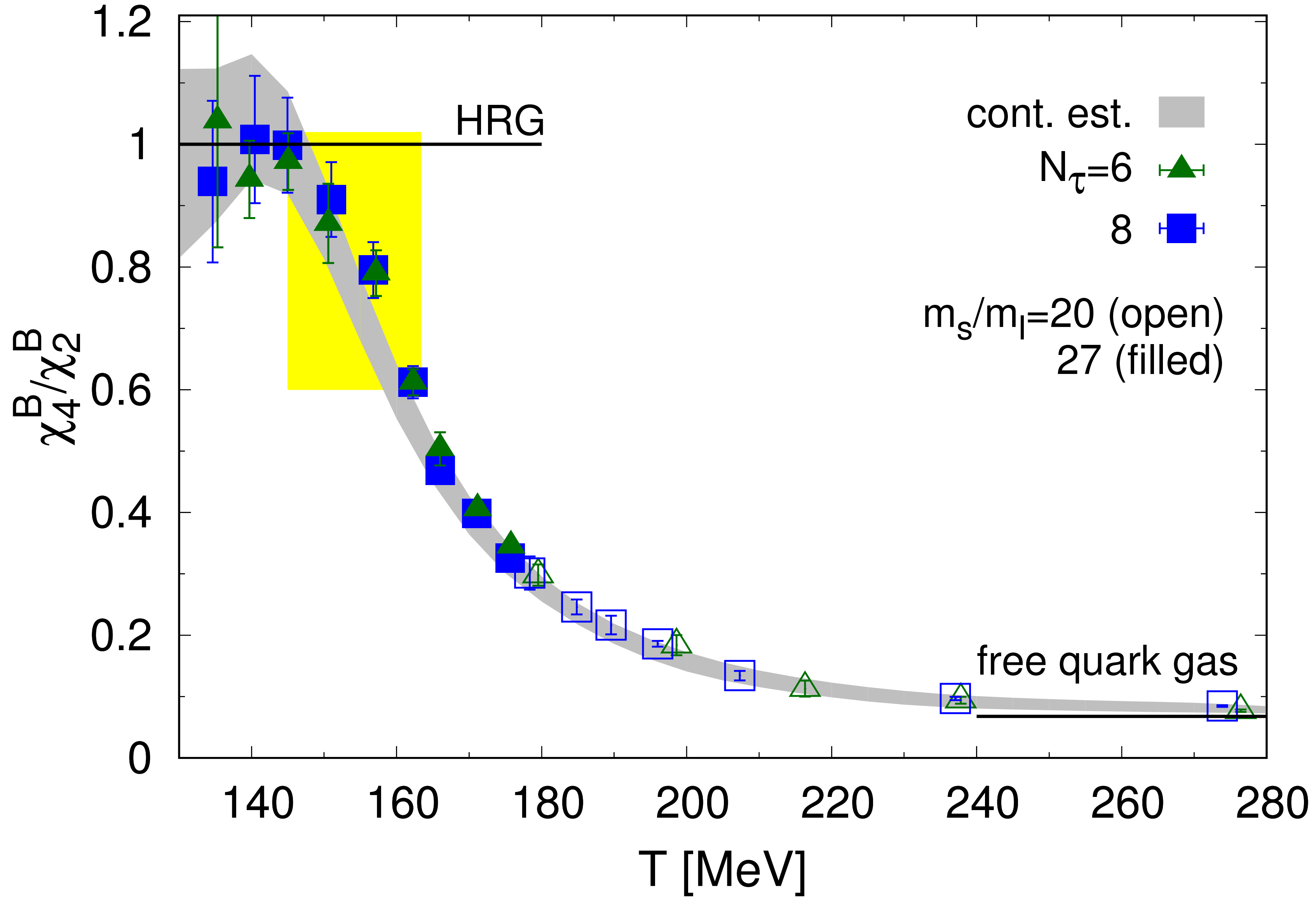
Curvature of energy per baryon can quantify conformal symmetry

Thank You

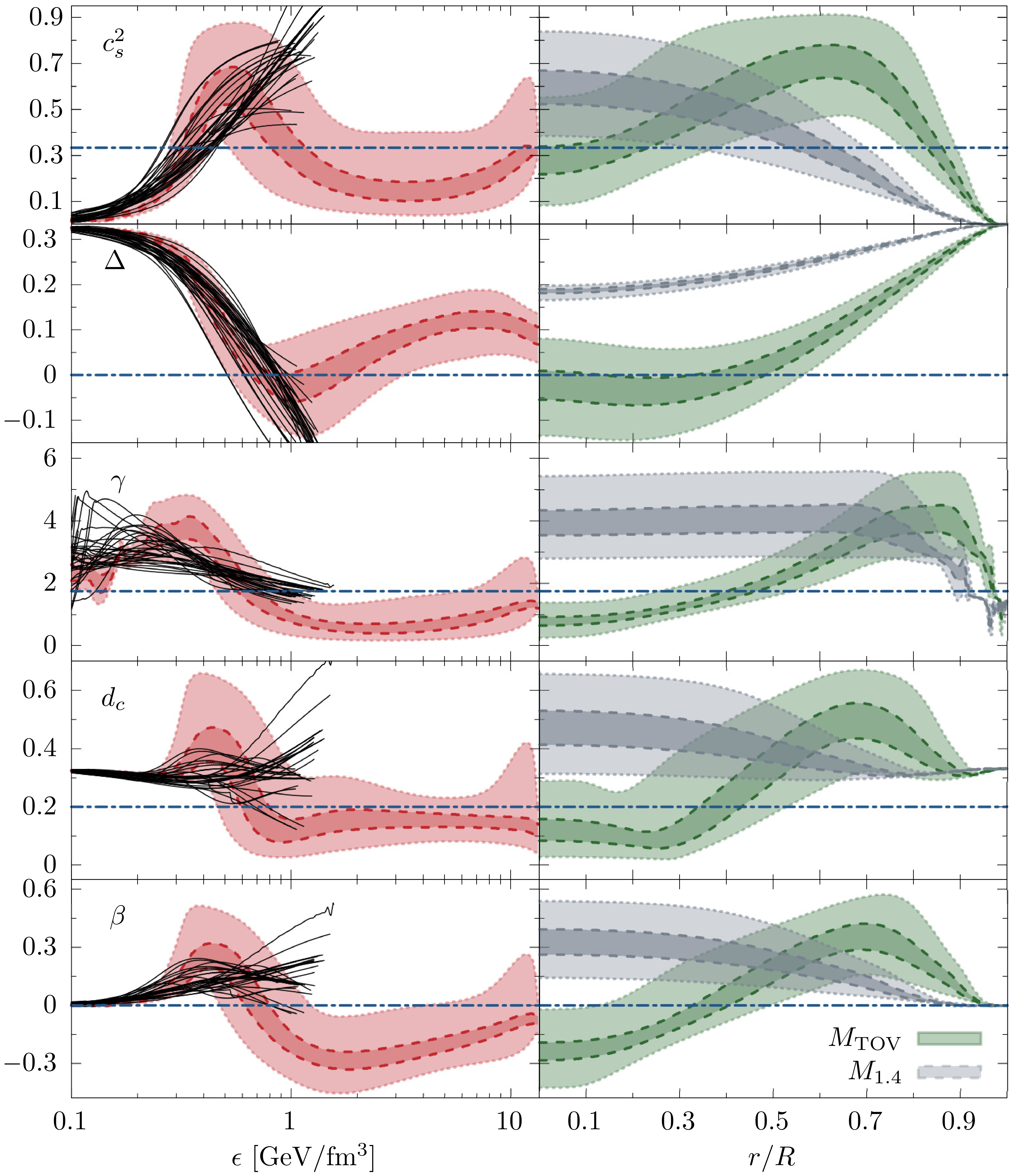
Summary


non-int. limit
HRG
T_c
$3p/T^4$
ε/T^4
$3s/4T^3$
T [MeV]
16
12
8
4
0
130
170
210
250
290
330
370


P/P_{free}
68% CI
95% CI
μ_B [GeV]
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6

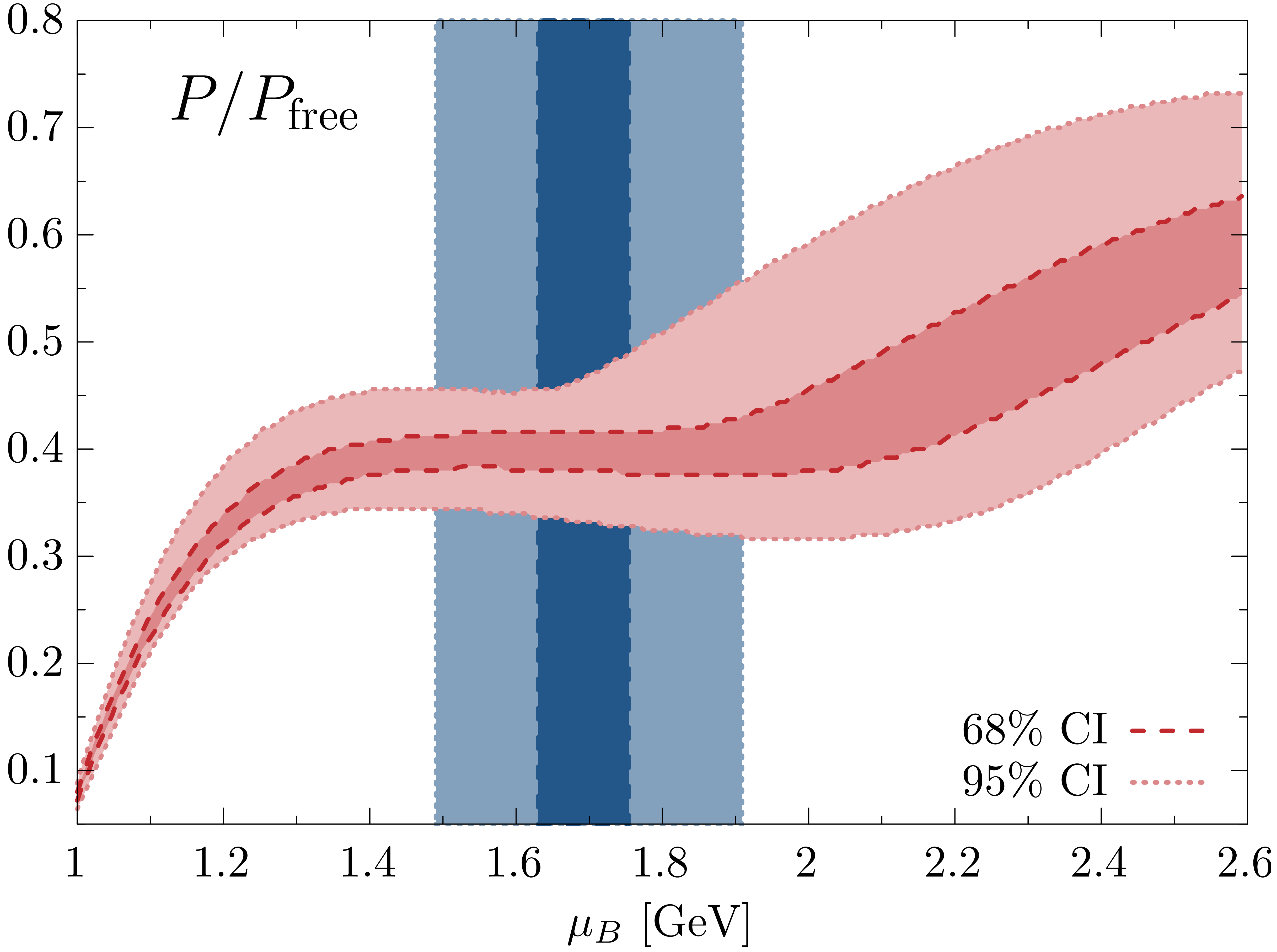

HRG
cont. est.
N_τ=6
8
m_s/m_l=20 (open)
27 (filled)
free quark gas
χ_4^B/χ_2^B
T [MeV]
1.2
1
0.8
0.6
0.4
0.2
0
140
160
180
200
220
240
260
280

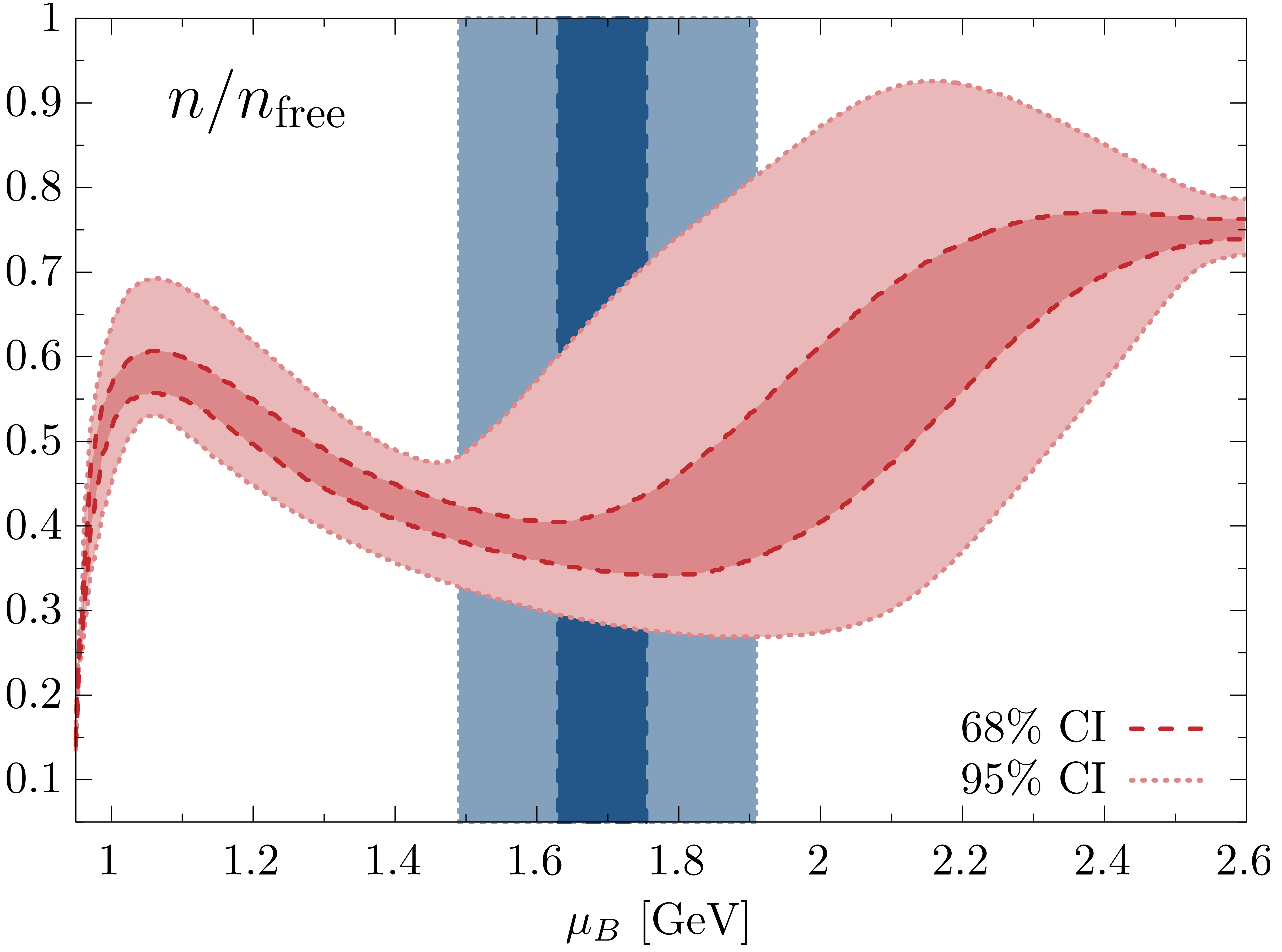
???



c_s^2
Δ
γ
d_c
β
M_{TOV}
$M_{1.4}$
ϵ [GeV/fm^3]
r/R

Equation of State





Net-baryon number susceptibility