Hydrodynamisation of charm quarks in heavy ion collisions

Stefan Floerchinger (Jena University)

work together with **Federica Capellino**, Andrea Dubla, Eduardo Grossi,
Andreas Kirchner and Silvia Masciocchi

EMMI Workshop at the University of Wrocław - Aspects of Criticality II
July 04, 2024.

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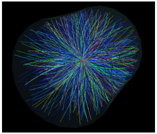
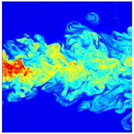
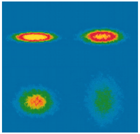
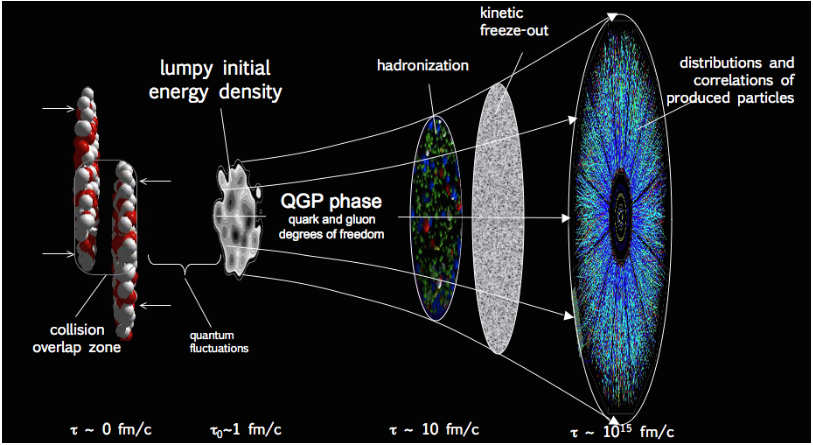
Fluid dynamics

- long distances, long times or strong enough interactions
- quantum fields form a fluid!
- needs **macroscopic** fluid properties
  - thermodynamic equation of state $p(T, \mu)$
  - shear and bulk viscosity η, ζ
  - heat conductivity
  - relaxation times
  - heavy quark diffusion coefficient κ_n
- fixed by **microscopic** properties encoded in Lagrangian $\mathscr{L}_{\text{QCD}}$

High energy nuclear collisions



Relativistic fluid dynamics

Energy-momentum tensor and conserved current

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + (p + \pi_{\text{bulk}})\Delta^{\mu\nu} + \pi^{\mu\nu}$$
$$N^\mu = n\, u^\mu + \nu^\mu$$

- tensor decomposition using fluid velocity u^μ, $\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$
- thermodynamic equation of state $p = p(T, \mu)$

Covariant **conservation laws** $\nabla_\mu T^{\mu\nu} = 0$ and $\nabla_\mu N^\mu = 0$ imply

- equation for energy density ϵ
- equation for fluid velocity u^μ
- equation for particle number density n

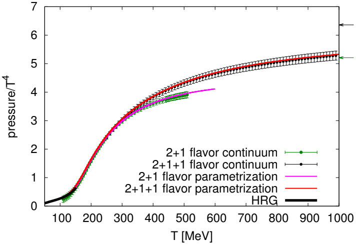
Need **further evolution equations** [e.g Israel & Stewart]

- equation for shear stress $\pi^{\mu\nu}$
- equation for bulk viscous pressure π_{bulk}

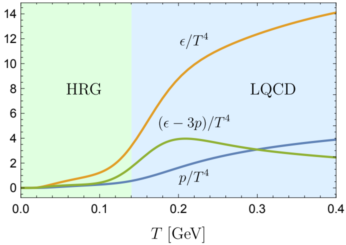
$$\tau_{\text{bulk}}\, u^\mu \partial_\mu \pi_{\text{bulk}} + \ldots + \pi_{\text{bulk}} = -\zeta\, \nabla_\mu u^\mu$$

- equation for diffusion current ν^μ
- non-hydrodynamic degrees of freedom are needed for relativistic causality!

Thermodynamics of QCD



[Borsányi *et al.* (2016), similar Bazavov *et al.* (2014)]



[Floerchinger, Grossi, Lion (2019)]

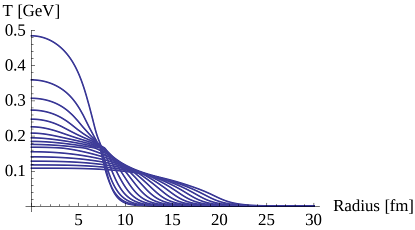
- equation of state at vanishing chemical potential is well known now
- at large temperature lattice QCD
- at small temperature hadron resonance gas approximation
- extensions to non-zero chemical potentials e. g. by Taylor expansion

Flow in heavy ion collisions

Fluid*u*M: Fluid dynamics of heavy ion collisions with Mode expansion

[Floerchinger & Wiedemann, PLB 728, 407 (2014), PRC 88, 044906 (2013), 89, 034914 (2014)]

[Floerchinger, Grossi & Lion, PRC 100, 014905 (2019)]

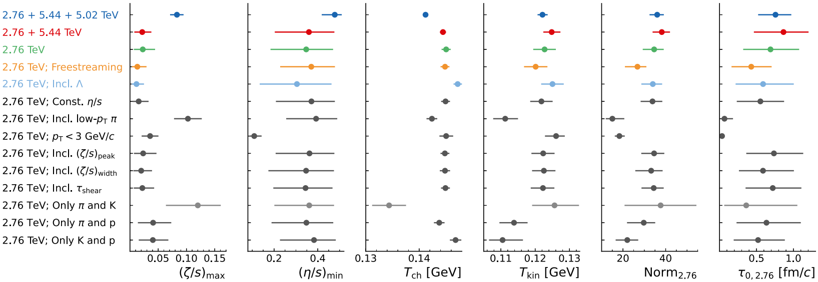


- background-fluctuation splitting + mode expansion
- analogous to cosmological perturbation theory
- substantially improved numerical performance (pseudospectral method)
- resonance decays included
  [Mazeliauskas, Floerchinger, Grossi & Teaney, EPJC 79, 284 (2019)]
- allows fast and precise comparison between theory and experiment

Parameter estimation from theory-experiment comparisson

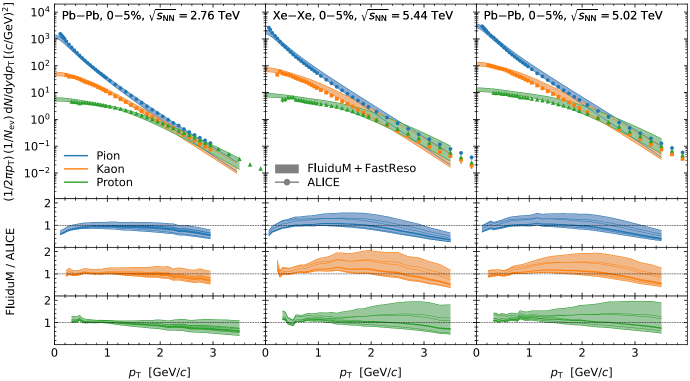
[Vermunt, Seemann, Dubla, Floerchinger, Grossi, Kirchner, Masciocchi, Selyuzhenkov, PRC 108, 064908 (2023)]

- fluid models have parameters
- can be determined with Bayesian analysis from data
- here based on transverse momentum spectra of pions, kaons, protons
- data from Pb-Pb (2.76 TeV), Pb-Pb (5.02 TeV), Xe-Xe (5.44 TeV)



Particle production at the Large Hadron Collider

[Vermunt, Seemann, Dubla, Floerchinger, Grossi, Kirchner, Masciocchi, Selyuzhenkov, PRC 108, 064908 (2023)]

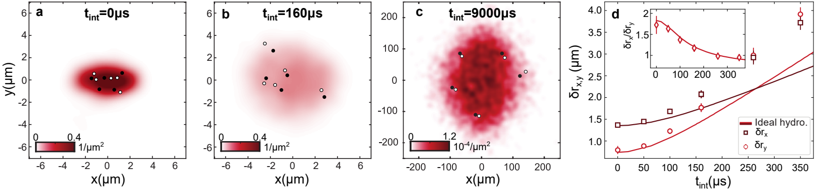


- overall good description
- some deviations for pions at small p_T

Elliptic flow for a few interacting atoms

[S. Floerchinger, G. Giacalone, L. H. Heyen, L. Tharwat, PRC 105, 044908 (2022)]

[S. Brandstetter, P. Lunt, C. Heintze, G. Giacalone, L. H. Heyen, M. Gałka, K. Subramanian, M. Holten, P. M. Preiss, S. Floerchinger, S. Jochim, to appear in Nature Physics]



- elliptic flow of 5+5 strongly interacting fermionic atoms released from anisotropic trap
- qualitative agreement with ideal fluid dynamics

Fluid dynamics for heavy quarks from Fokker-Planck equation

- phase-space distribution function $f(t, \mathbf{x}, p)$
- currents are moments with respect to momenta

$$N^\mu(t, \mathbf{x}) = \int \frac{d^3 p}{(2\pi)^3 p^0} p^\mu f(t, \mathbf{x}, p)$$

- Boltzmann equation for time evolution

$$p^\mu \frac{\partial}{\partial x^\mu} f(t, \mathbf{x}, p) = C[f]$$

- heavy quarks get small “momentum kicks” from light partons
- Fokker-Planck approximation to collision kernel

$$C[f] = k^0 \frac{\partial}{\partial p^j} \left[A^j f + \frac{\partial}{\partial p^k} \left[B^{jk} f \right] \right]$$

- fluid dynamics from taking moments of the Fokker-Planck equation
- approximations justified for slow dynamics

Equations of motion for charm current

- net heavy quark number current $N^\mu_- = N^\mu_Q - N^\mu_{\bar{Q}}$ conserved in QCD but not in electroweak theory
- total integrated net quark number vanishes
- average quark number current $N^\mu_+ = (N^\mu_Q + N^\mu_{\bar{Q}})/2$ approximately conserved for small temperatures $T \ll m_Q$
- we work with

$$N^\mu = N^\mu_+ = n u^\mu + \nu^\mu$$

- conservation law

$$\nabla_\mu N^\mu = u^\mu \partial_\mu n + n \nabla_\mu u^\mu + \nabla_\mu \nu^\mu = 0$$

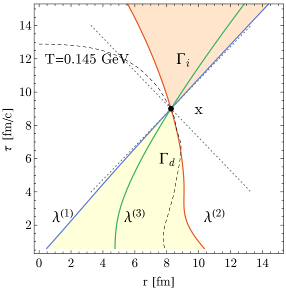
- additional equation of motion

$$\tau_n \Delta^\rho{}_\sigma u^\lambda \nabla_\lambda \nu^\sigma + \nu^\rho + \kappa_n \Delta^{\rho\sigma} \partial_\sigma \left(\frac{\mu}{T}\right) = 0$$

- chemical potential μ conjugate to heavy quark number
- heavy quark diffusion coefficient $\kappa_n = D_s n$
- relaxation time τ_n

Causality

[Floerchinger & Grossi, JHEP 08 (2018) 186]



- dissipative fluid equations *can* be of hyperbolic type
- characteristic velocities depend on fluid fields
- need $|\lambda^{(j)}| < c$ for relativistic causality
- works when relaxation times are large enough

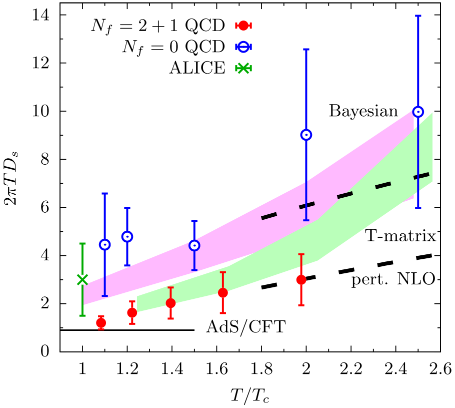
Thermodynamic equation of state for charm

- fluid dynamics needs a thermodynamic equation of state
- dependence of pressure on charm chemical potential not very well known
- we use a hadron resonance model approximation with sum over all measured charmed states

$$n(T,\mu) = \frac{T}{2\pi^2} \sum_{i \in \mathsf{HRGc}} q_i M_i^2 \exp\left(\frac{q_i \mu}{T}\right) K_2\left(\frac{M_i}{T}\right)$$

- yields larger values than gas of free charm quarks
- lattice results would be nice to have

Constraints on charm quark diffusion on the lattice



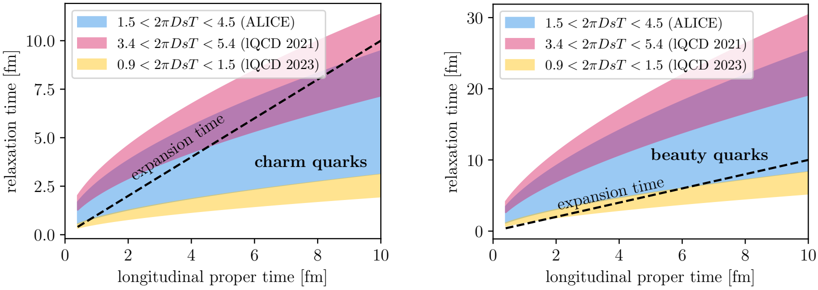
- latest lattice results for heavy quark diffusion coefficient for $N_f = 2+1$ flavor QCD indicate small D_s [HotQCD, PRL 130, 231902 (2023)]
- supports fast hydrodynamization of heavy quarks
- phenomenological analysis based on different transport models and Bayesian analysis based on Langevin dynamics support larger values of D_s [ALICE, JHEP01(2022)174] [Xu, Bernhard, Bass, Nahrgang, Cao, PRC 97, 014907 (2018)]

Applicability of fluid description

[Capellino, Beraudo, Dubla, Floerchinger, Masciocchi, Pawlowski, Selyuzhenkov, PRD 106, 034021 (2022)]

- Fokker-Planck equation yields relation for relaxation time τ_n in terms of diffusion coefficient D_s
- fluid dynamics applicable when the relaxation time is small compared to the dynamics
- for initial Bjorken expansion

$$\tau_n < 1/(\nabla_\mu u^\mu) = \tau$$



Initial conditions for charm current

[Capellino, Dubla, Floerchinger, Grossi, Kirchner, Masciocchi, PRD 108, 116011 (2023)]

- initial density distribution from hard scattering

$$n(\tau_0, r) = \frac{1}{\tau_0} n_{\mathsf{coll}}(r) \frac{1}{\sigma_{pp}^{\mathsf{in}}} \frac{d\sigma^{Q\bar{Q}}}{dy}$$

$\sigma_{pp}^{\mathsf{in}} = 67.6$ mb, $\frac{d\sigma^{Q\bar{Q}}}{dy} = 0.463$ mb [Cacciari, Frixone, Nason, JHEP03(2001)006]

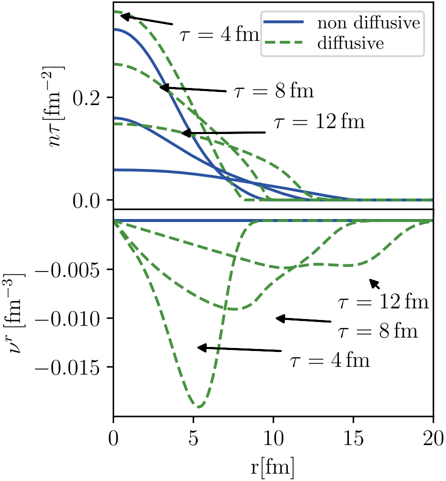
- diffusion current initially assumed to vanish

$$\nu^{\mu}(\tau_0, r) = 0$$

- leads to parameter-free model for initial charm density and current

Evolution of charm density and diffusion current

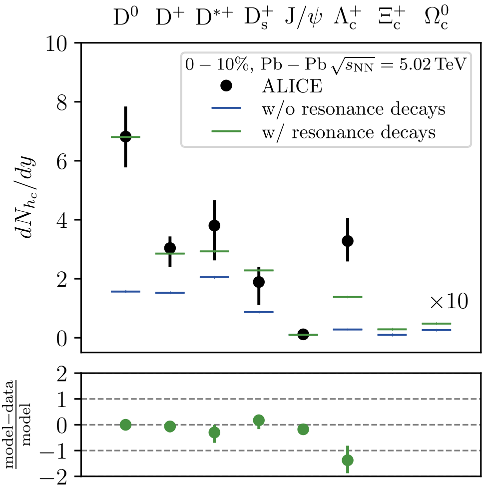
[Capellino, Dubla, Floerchinger, Grossi, Kirchner, Masciocchi, PRD 108, 116011 (2023)]



- Charm density expands and dilutes like energy density
- diffusion leads to further dilution

Yields of charmed hadrons

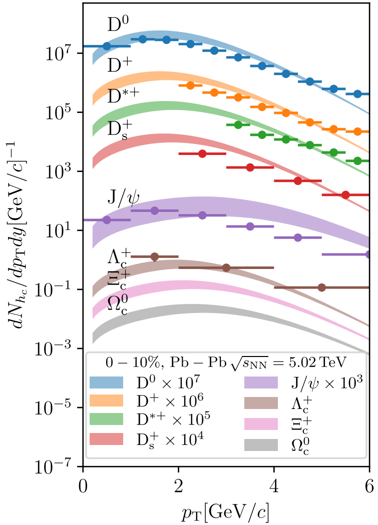
[Capellino, Dubla, Floerchinger, Grossi, Kirchner, Masciocchi, PRD 108, 116011 (2023)]



- resonance decays from FastReso sizeable
- yield of Λ_c^+ underpredicted, possibly missing higher resonances in PDG list?
- prediction for Ξ_c^+ and Ω_c^0

Transverse momentum spectra of charmed hadrons

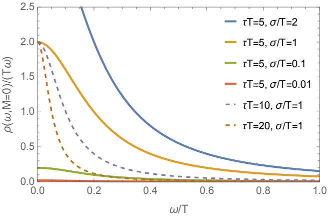
[Capellino, Dubla, Floerchinger, Grossi, Kirchner, Masciocchi, PRD 108, 116011 (2023)]



- good agreement for D-mesons up to $p_{\mathrm{T}} \approx 4 - 5$ GeV
- some deviations for J/Ψ (dissipative correction?)

Electromagnetic spectral function

[S. Floerchinger, C. Gebhardt, K. Reygers, PLB 837 (2023) 137647]



- diffusion law for electric current in fluid dynamic regime

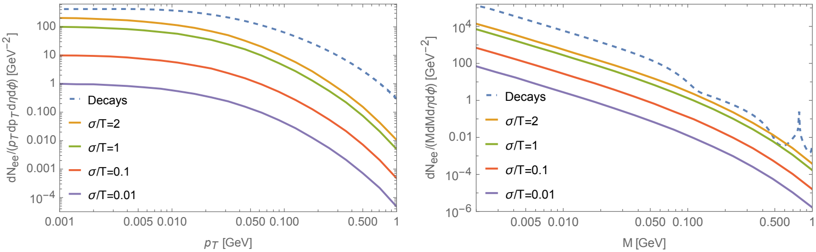
$$J^\alpha + \tau \Delta^\alpha{}_\beta u^\mu \nabla_\mu J^\beta = \rho u^\mu + \sigma \Delta^{\alpha\nu} E_\nu - D\Delta^{\alpha\nu}\partial_\nu n$$

- electric conductivity σ
- diffusion coefficient $D = \sigma/\chi$
- charge susceptibility $\chi = (\partial n/\partial\mu)|_T$
- relaxation time τ constrained by causality $\tau > D = \sigma/\chi$
- allows to determine spectral function at small frequencies and momenta

$$\rho(\omega, \mathbf{p}) = \frac{\sigma\omega(\omega^2 - \mathbf{p}^2)}{(\tau\omega^2 - D\mathbf{p}^2)^2 + \omega^2} + 2\frac{\sigma\omega}{\tau^2\omega^2 + 1}$$

Dielectron transverse momentum and mass spectra

[S. Floerchinger, C. Gebhardt, K. Reygers, PLB 837 (2023) 137647]



- integration over fireball volume at LHC energies
- background contribution from resonance decays dominates
- Hanbury Brown-Twiss correlations could help to distinguish signal from background

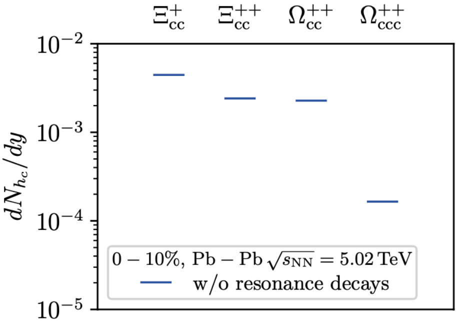
Conclusions

- fluid dynamic description for heavy quark currents
- local *kinetic* equilibrium picture
- on-top description: fluid velocity and temperature governed by QCD fluid with equation of state for $2+1$ light flavors
- spectra of mesons and baryons with charm quarks well described up to transverse momenta of $p_\mathrm{T} \approx 4$ GeV
- total abundances depend on feed-down from resonance decays
- extension to bottom quark current should be attempted
- dissipative corrections at freeze-out seem small but should be studied
- soft photon and dilepton spectra determined by electric conductivity and relaxation time

Backup

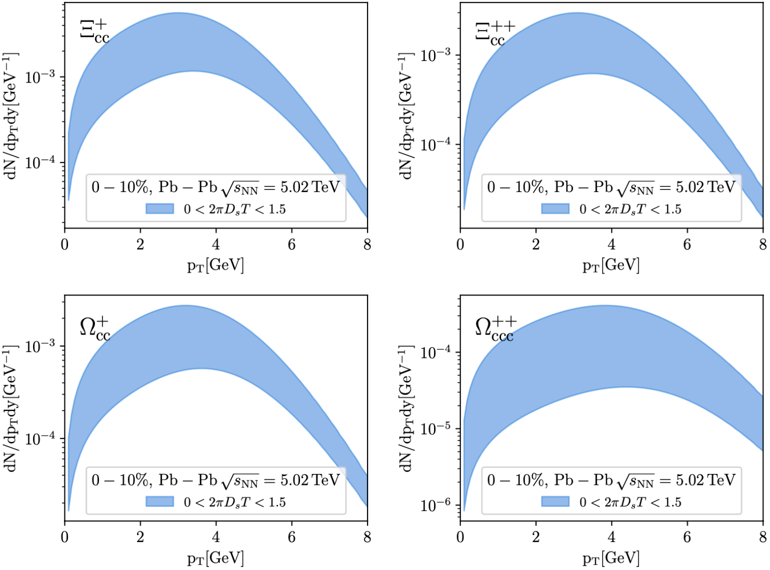
Predictions for yields of multicharmed hadrons

[Capellino, PhD thesis (2024)]



Predictions for transverse momentum spectra of multicharmed hadrons

[Capellino, PhD thesis (2024)]



Fluid dynamics with several conserved quantum numbers

- fluid with conserved quantum number densities $c_m = (\epsilon, n_\mathsf{B}, n_\mathsf{C}, n_\mathsf{S}, \ldots)$
- equation of state in grand canonical ensemble in terms of Massieu potential $w(\beta, \alpha_j) = \beta p(\beta, \alpha_j)$ with $\beta = 1/T$, $\alpha_j = \mu_j/T$,

$$dw = -\epsilon d\beta + n_j d\alpha_j$$

- second derivative yields a matrix of susceptibilities with $\gamma^m = (\beta, \alpha_1, \alpha_2, \ldots)$

$$G_{mn}(\gamma) = \frac{\partial^2 w}{\partial \gamma^m \partial \gamma^n}$$

- fluid evolution equations from conservation laws

$$u^\mu \partial_\mu c_m + f_m = 0$$

- can be written with inverse susceptibility matrix as

$$u^\mu \partial_\mu \gamma^n + (G^{-1}(\gamma))^{nm} f_m = 0$$