Thermal Quarkonia Spectral function from 2+1 flavor Lattice QCD

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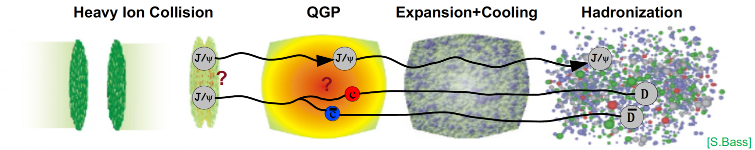
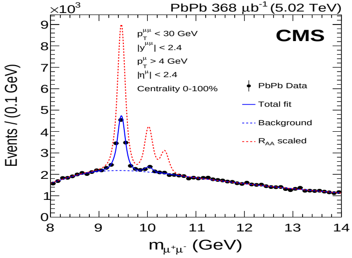
HotQCD

arXiv:Coming Soon



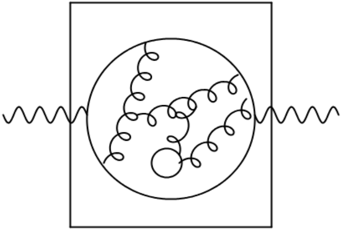
- QGP cause suppression of Quarkonia (bound states of heavy $q\bar{q}$), an important probes to study properties of QGP.
- The in-medium properties of quarkonia bound states is encoded in spectral function.



CMS Collaboration, PLB 790 (2019) 270

Theoretically, the dilepton production rate from a thermally equilibrated plasma is given by:

$$\frac{d\Gamma_{\mu+\mu-}}{d^4Q} \sim \frac{e^2}{Q^2}\, n_b\, \rho_V(Q)$$



ρ_V vector channel spectral function.

- $$\rho_\Gamma(\omega,\vec{k}) = \int dt\, d^3\vec{x} \exp\left[i(\vec{k}.\vec{x} - \omega t)\right] \langle [J_\Gamma(\vec{x},t), J_\Gamma(0,0)]\rangle_T$$

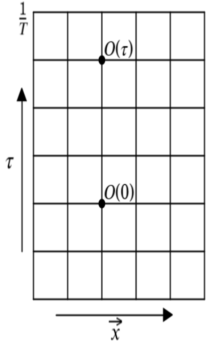
  where $J_\Gamma(\vec{x},t) = \bar{\psi}(x,t)\,\Gamma\,\psi(\vec{x},t)$

- $$\langle \dots \rangle_T = \frac{Tr[\exp(-\beta H_{QCD})\dots]}{Z_{QCD}}$$

- Euclidean-correlation function $G^E(\tau, \vec{k}) = \int \exp\left(i\vec{k}.\vec{x}\right)\langle J_\Gamma(\vec{x},\tau)J_\Gamma(0,0)\rangle$ where $J(\vec{x},t) = \bar{\psi}(\vec{x},\tau)\Gamma\psi(\vec{x},\tau)$

$$G_\Gamma^E(\tau,\vec{k}) = \int_0^\infty \frac{d\omega}{\pi}\rho_\Gamma(\omega,\vec{k})\frac{\cosh[\omega(\tau-\frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

- Numerically ill-posed problem. Small number of data points and statistical errors.



We computed for charm and bottom correlation function on the lattice with $m_\pi \sim 315$ MeV.
Currently we consider $\Gamma = \gamma_5$, γ_i and $\vec{k} = \vec{0}$.
$T \sim 1.2T_c, 1.3T_c, 1.62T_c$ where $T_c = 180$ MeV.
We calculate the bottom and charm correlator at these temperatures.

- $\omega \gg 2M$
  Thermal effects are suppressed. Vacuum perturbation theory will work.
- $\omega \sim 2M$
  Thermal effects are important. Spectral function needs to be calculated using thermal potential.
- $\omega \ll 2M$
  For the pseudoscalar channel, the spectral weights are exponentially suppressed.
  For the vector channel, there is a contribution around $\omega \sim 0$ due to transport.

$$C_>(t;\vec{r},\vec{r}') = \int d^3\vec{x}\,\langle\bar{\psi}(t,x+\frac{\vec{r}}{2})\gamma_5\,U\,\psi(t,x-\frac{\vec{r}}{2})\bar{\psi}(0,-\frac{\vec{r}'}{2})\gamma_5\,U\,\psi(0,-\frac{\vec{r}'}{2})\rangle_T$$

- In the presence of Interaction,

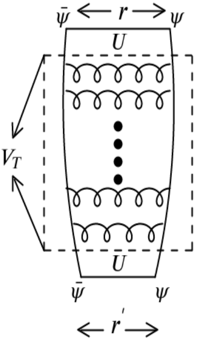
$$\left\{i\partial_t - \left[2M + V_T(r) - \frac{\nabla^2_{\vec{r}}}{M}\right]\right\} C_>(t;\vec{r},\vec{r'}) = 0$$

where V_T is defined in static limit,

$$V_T(r) = i\lim_{t\to\infty}\frac{\partial\log W(r,t)}{\partial t} = V_{re}(r) - iV_{im}(r)$$

with $C_>(0;\vec{r},\vec{r'}) = \delta^3(\vec{r}-\vec{r}')$

$$\rho_p(\omega) \propto \lim_{r\to 0, r'\to 0}\int_{-\infty}^{\infty}\mathrm{d}t\,e^{i\omega t}\,C_>(t;\vec{r},\vec{r'})$$



M.Laine et al, JHEP 0703:054,2007

- Non-perturbative formulation, A. Rothkopf et al., PRL. 108 (2012) 162001

$$W(r,\tau) \;=\; \int_{-\infty}^{\infty} d\omega\, \rho(r,\omega,\,T)\, \exp(-\omega\,\tau)$$

$$W(r,t) \;=\; \int_{-\infty}^{\infty} d\omega\, \rho(r,\omega,\,T)\, \exp(-i\omega\,t)$$

- $\rho(\omega,\,T)$ should have a form which is consistent with potential, $i\,\lim_{t\to\infty}\frac{\partial \log W(r,t)}{\partial t}$ should exist.
- **Gaussian spectral function does not have this limit.** PRD 105, 054513
  Simple Lorentzian does have this limit. But $\rho(r,\omega,\,T)$ depends on the cut-off. PRD 109 ,074504
  Bayesian analysis has a higher systematic error. PRL 114, 082001

$$\log(W(r,\tau)) = -V_{re}(r)\tau - \int_{-\infty}^{\infty} du\, \sigma(r,u) \left[\exp(u\,\tau) + \exp(u\,(\beta-\tau))\right] + \ldots$$

- $i \lim_{t\to\infty} \frac{\partial \log W(r,t)}{\partial t} = \text{finie} \implies \lim_{u\to 0} \sigma(r,u) \sim \frac{1}{u^2}$
- Following HTL PT, $\sigma(r,u) = n_B(u)\left[\frac{V_{im}}{u} + c_1\, u + c_3\, u^3 + \ldots\right]$
- Parametrization

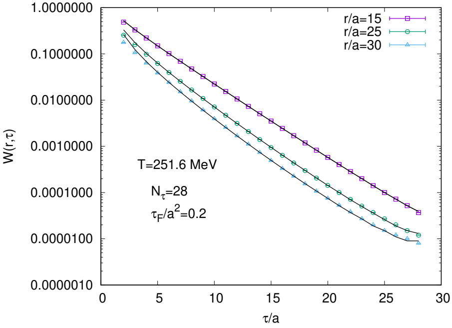
$$W(r,\tau) \;=\; A\exp\left[-V_{re}(r)\tau - \frac{\beta V_{im}(r)}{\pi}\log\left(\sin\left(\frac{\pi\tau}{\beta}\right)\right) + \ldots\right]$$

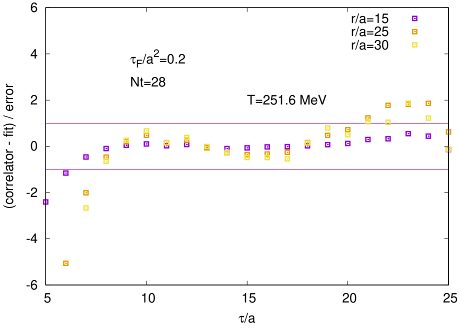
DB and S. Datta, PRD 101, 034507

DB and S. Datta, PRD 103, 014512

DB, O. Kaczmarek, et al., PRD 105, 054513

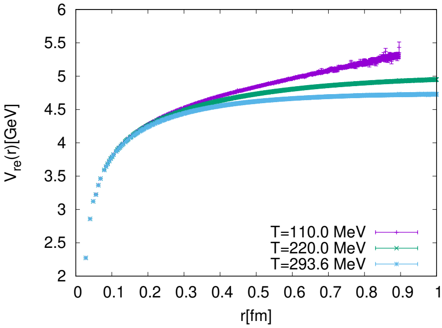
Three parameter fit of Wilson line correlator for different distances.

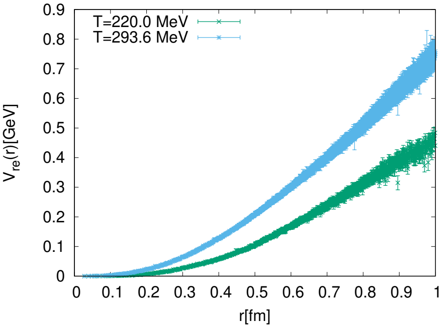




- $\chi^2/ndf \sim 1$

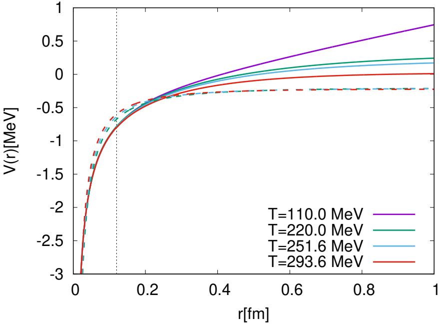
- **Lattice data are perfectly consistent with Color Screening**

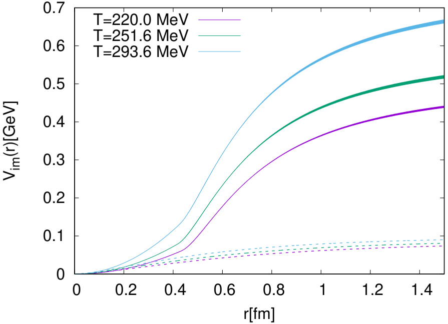




$$V_{re}(r) = \frac{\sigma}{m_d}(1 - \exp(-m_d r)) - \frac{\alpha}{r}\exp(-m_d r) + c$$

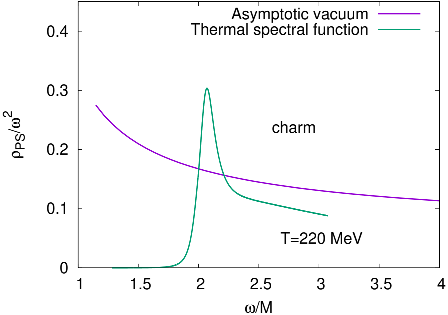
$$V_{im}(r) = \begin{cases} \frac{1}{2}br^2 & \text{for } r < r_0 \\ a_0 - \frac{a_1}{2r^2} - \frac{a_2}{4r^4} & \text{for } r \geq r_0 \end{cases}$$

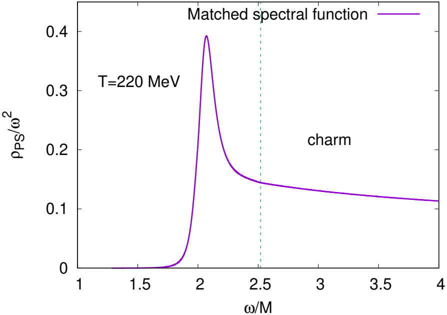




- Non-perturbative thermal potential is very much different from the perturbative potential.

$$\rho_{PS}(\omega) = A_0\, \rho^{T}_{PS}(\omega)\theta(\omega_0 - \omega) + \rho^{T=0}_{PS}(\omega)\theta(\omega - \omega_0)$$





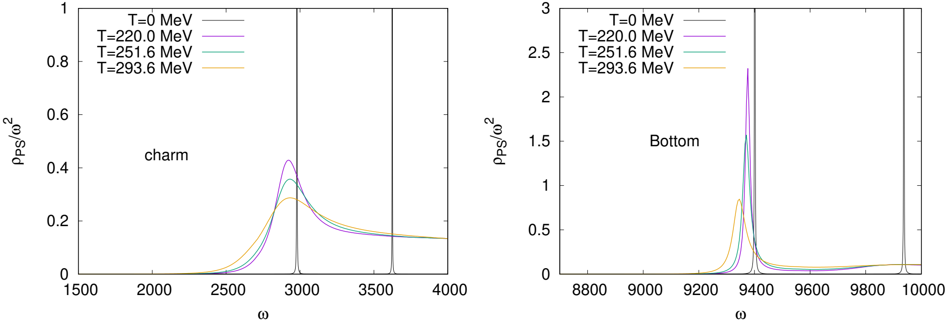
- $A_0 \sim 0.88\, M - 1.2\, M$
- $\omega_0 \sim 2\, M - 3\, M$

Similar spectral function using perturbative potential.

$N_f = 0$ M. Laine et al, JHEP11 (2017) 206

$N_f = 2 + 1$ S. Ali, DB , O.Kaczmarek et al, Few-Body Syst 64, 52 (2023)

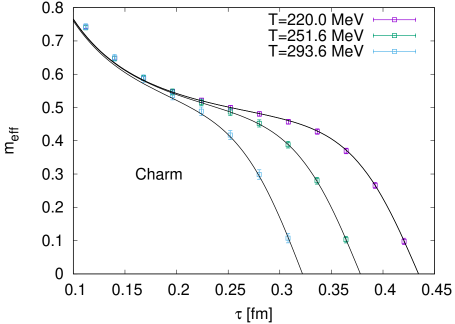
Pole mass: $m_c = 1.35 GeV$ and $m_b = 4.78 GeV$

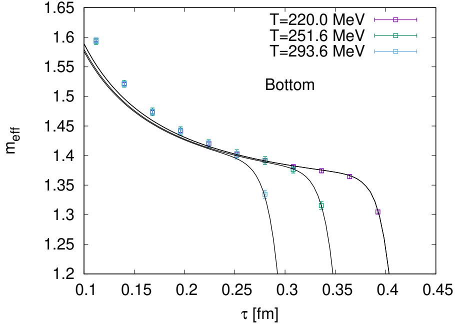


- (1S) state for bottom disappear much after T_c ($T_c = 180 MeV$)
- Significant thermal effects on charmonium state.

$$G^E_{PS}(\tau) = \int_0^\infty \frac{d\omega}{\pi}\rho_{PS}(\omega)\frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

$$m_{eff}(\tau_i) = \log\left(\frac{G^E_{PS}(\tau_i)}{G^E_{PS}(\tau_{i+1})}\right)$$

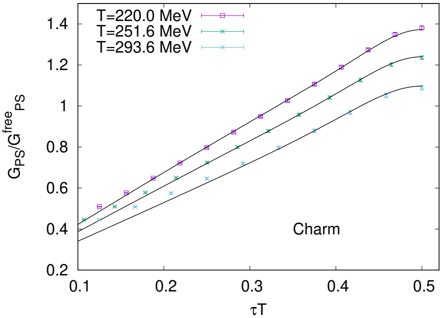


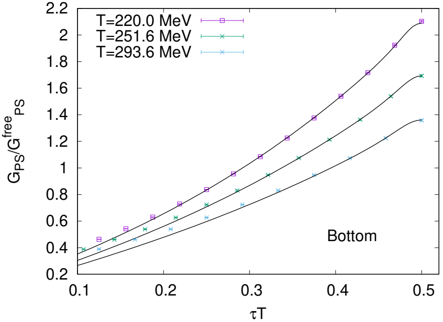


Consistent with lattice data.

$$\rho_{PS}^{model}(\omega, A) = A\,\rho_{PS}(\omega)$$

$$G_{PS}^{E}(\tau, A) = \int_0^\infty \frac{d\omega}{\pi}\rho_{PS}^{model}(\omega, A)\frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$



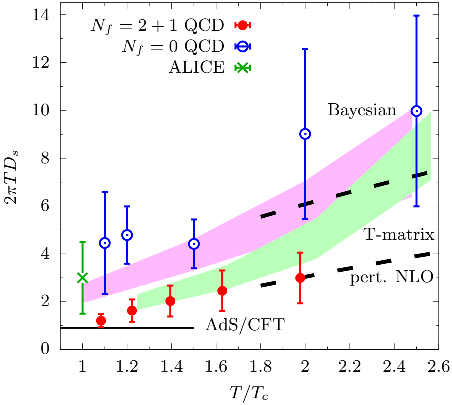


- These spectral functions indeed describe the lattice correlator .

- Transport peak in the vector current correlator:

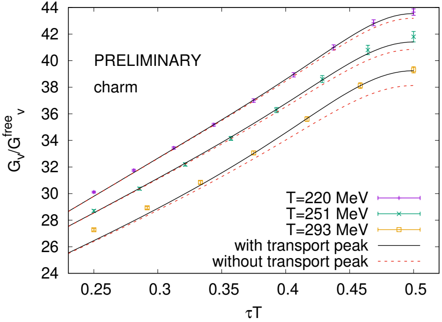
$$\rho_{Transport} = 3\chi_q \frac{T}{M} \frac{\omega \eta_d}{\omega^2 + \eta_d^2}$$

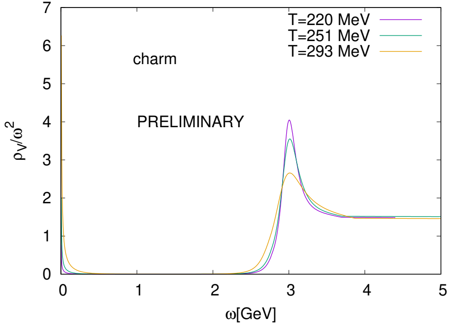
- Extremely narrow transport contribution $\omega \sim \frac{g^4 T^2}{M}$. Bound state contribution $\omega \sim 2M$.



HotQCD, PRL 132 (2024) 5, 051902

$$\rho_v(\omega) = A\rho_{transport}(\omega) + \rho_{boundstate}(\omega)$$





- A small transport contribution is required to fit the lattice data .

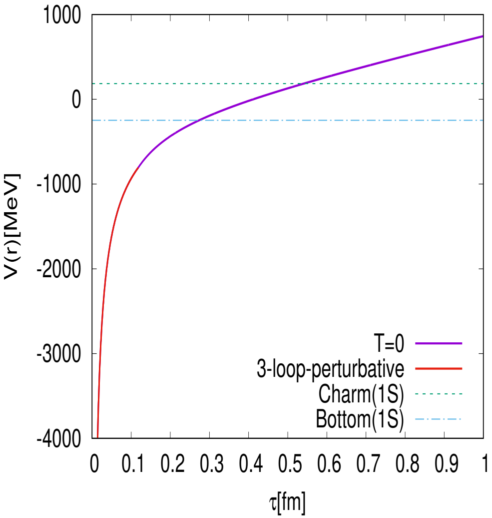
- We calculate the pseudoscalar spectral function from 2+1 flavor lattice QCD correlation functions.
- **Lattice Data supports Color screening of the non-perturbative thermal potential.**
- **We observed a small thermal mass shift for the in-medium $\eta_b(1S)$ and $\eta_c(1S)$ channels and a large thermal width ($\Gamma_c(1S) \gg \Gamma_b(1S)$).**
- The pseudoscalar channel correlator function can be described by the spectral function obtained from the thermal potential.
- For the vector channel, the spectral function needs a small transport contribution in addition to the bound state contribution to describe the lattice data.

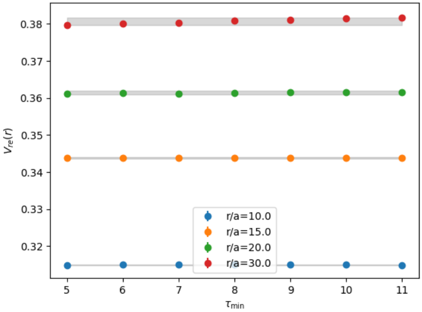
- Cornell fit of $T = 0$ lattice potential.
- Short distance matched renormalon subtracted peruturbative potential.

$$\left[-\frac{\nabla^2}{M} + V(r)\right]\psi_n(r) = E_n\psi_n(r)$$

$$M^{1S} = 2\,M + E_0$$

- $M^b = 4.78$ GeV
- $M^c = 1.35$ GeV




0.38
0.37
0.36
0.35
0.34
0.33
0.32
$V_{re}(r)$
r/a=10.0
r/a=15.0
r/a=20.0
r/a=30.0
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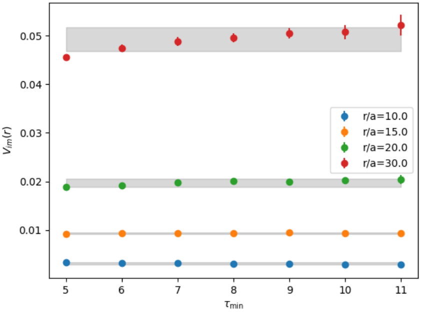
6
7
8
9
10
11
τ_{min}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$V_{im}(r)$
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r/a=30.0
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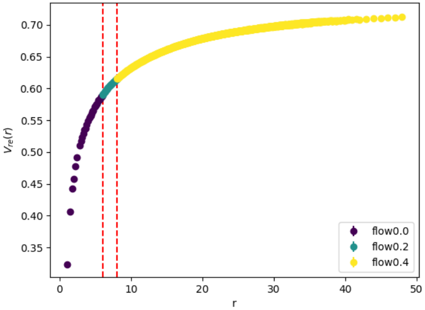
τ_{min}



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0.60
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0.50
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$V_{re}(r)$
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50
r
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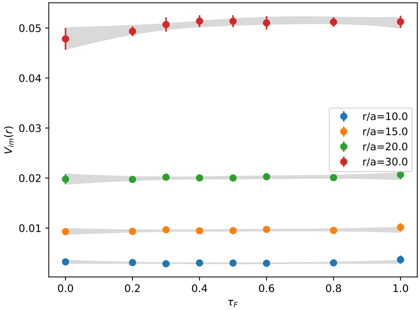
flow0.2
flow0.4



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$V_{im}(r)$
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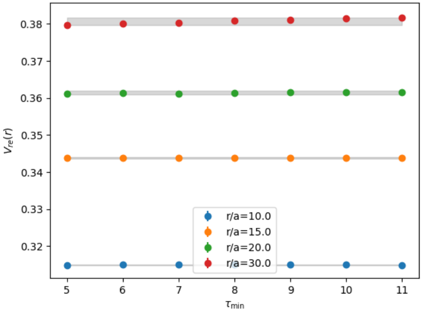
1.0
τ_F
r/a=10.0
r/a=15.0
r/a=20.0
r/a=30.0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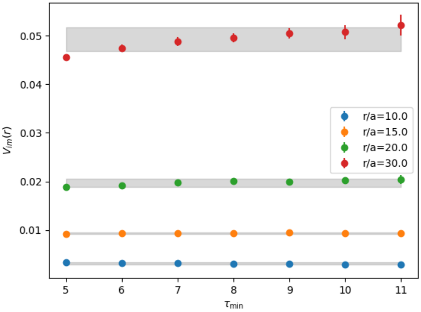
r/a=15.0
r/a=20.0
r/a=30.0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$V_{im}(r)$
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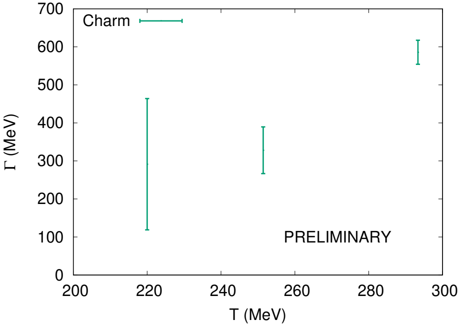
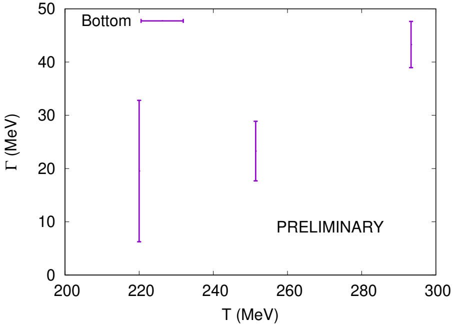
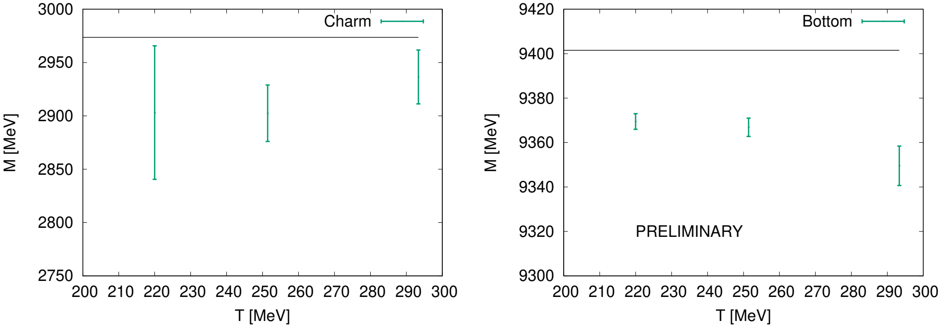
r/a=20.0
r/a=30.0





- We performed skewed Lorentzian fit near the peak.
- $\Gamma_c(1S) \gg \Gamma_b(1S)$



- Mass is identified with peak position of the spectral function.
- Finite mass shift is observed