Heavy quark momentum broadening in a non-Abelian plasma away from equilibrium

Sayantan Sharma

The Institute of Mathematical Sciences & Homi Bhabha National Institute

July 3, 2024

EMMI Workshop at the University of Wrocław - Aspects of Criticality II

Work in collaboration with Harshit Pandey & Sören Schlichting

Heavy quark diffusion in a quark gluon plasma

- Heavy quarks are formed in the very early stages of a heavy-ion collision event and hence an excellent probe of the medium formed.

- To model how fast their velocities adjust to hydrodynamic (elliptic) flow , heavy quark momentum diffusion coeff. is an important ingredient as $\tau_{m\to\infty} \sim \frac{m}{T} 2\pi D_s = \frac{4\pi \; m \; T}{\kappa}$

- Heavy quarks in quark gluon plasma typically modelled as a Brownian particle

$$\frac{d\mathbf{p}}{dt} = -\eta \mathbf{p}(t) + \vec{\zeta}(t) \; , \;\; \langle \zeta_i(t) \, \zeta_j(t') \rangle = \kappa \; \delta_{ij} \; \delta(t - t') \; .$$

- In a thermal medium, by virtue of fluctuation-dissipation theorem

$$\eta \simeq \frac{\kappa \; \langle \mathbf{v}^2 \rangle}{6T} \; .$$

Heavy quark diffusion in a quark gluon plasma

- Heavy quarks are formed in the very early stages of a heavy-ion collision event and hence an excellent probe of the medium formed.
- To model how fast their velocities adjust to hydrodynamic (elliptic) flow , heavy quark momentum diffusion coeff. is an important ingredient as $\tau_{m\to\infty} \sim \frac{m}{T} 2\pi D_s = \frac{4\pi\ m\ T}{\kappa}$
- Heavy quarks in quark gluon plasma typically modelled as a Brownian particle

$$\frac{d\mathbf{p}}{dt} = -\eta\mathbf{p}(t) + \vec{\zeta}(t)\ , \quad \langle \zeta_i(t)\ \zeta_j(t')\rangle = \kappa\ \delta_{ij}\ \delta(t-t')\ .$$

- In a thermal medium, by virtue of fluctuation-dissipation theorem

$$\eta \simeq \frac{\kappa\ \langle \mathbf{v}^2 \rangle}{6T}\ .$$

Heavy quark diffusion in a quark gluon plasma

- Heavy quarks are formed in the very early stages of a heavy-ion collision event and hence an excellent probe of the medium formed.
- To model how fast their velocities adjust to hydrodynamic (elliptic) flow , heavy quark momentum diffusion coeff. is an important ingredient as $\tau_{m\to\infty} \sim \frac{m}{T} 2\pi D_s = \frac{4\pi \ m \ T}{\kappa}$
- Heavy quarks in quark gluon plasma typically modelled as a Brownian particle

$$\frac{d\mathbf{p}}{dt} = -\eta \mathbf{p}(t) + \vec{\zeta}(t) \ , \ \ \langle \zeta_i(t) \ \zeta_j(t') \rangle = \kappa \ \delta_{ij} \ \delta(t - t') \ .$$

- In a thermal medium, by virtue of fluctuation-dissipation theorem

$$\eta \simeq \frac{\kappa \ \langle \mathbf{v}^2 \rangle}{6T} \ .$$

Heavy quark diffusion in a quark gluon plasma

- Heavy quarks are formed in the very early stages of a heavy-ion collision event and hence an excellent probe of the medium formed.
- To model how fast their velocities adjust to hydrodynamic (elliptic) flow , heavy quark momentum diffusion coeff. is an important ingredient as $\tau_{m\to\infty} \sim \frac{m}{T} 2\pi D_s = \frac{4\pi \ m \ T}{\kappa}$
- Heavy quarks in quark gluon plasma typically modelled as a Brownian particle

$$\frac{d\mathbf{p}}{dt} = -\eta \mathbf{p}(t) + \vec{\zeta}(t) \ , \quad \langle \zeta_i(t) \ \zeta_j(t') \rangle = \kappa \ \delta_{ij} \ \delta(t - t') \ .$$

- In a thermal medium, by virtue of fluctuation-dissipation theorem

$$\eta \simeq \frac{\kappa \ \langle \mathbf{v}^2 \rangle}{6T} \ .$$

Heavy quark diffusion in QGP

- When a particle of mass M and color charge q accelerates in presence of color electric and magnetic fields with a $\langle v_i \; v_j \rangle = \delta_{ij} \; \frac{1}{3}\langle \mathbf{v}^2 \rangle$ given by

$$\mathbf{F} = q \; (\mathbf{E} \; + \; \mathbf{v} \times \mathbf{B}) = \frac{d\mathbf{p}}{dt} \; .$$

- The two-point correlator of the force can be written as an expansion in velocity

$$\langle F_i(t') \; F_j(t) \rangle = q^2 \left[\langle E_i(t') E_j(t) \rangle - \frac{1}{3} \langle \mathbf{v}^2 \rangle \; \langle \delta_{ij} \; B_k(t') B_k(t) - B_i(t') B_j(t) \rangle \right] \; .$$

- The momentum diffusion coefficient κ is defined in the limit $\omega \to 0$ as

$$\kappa = \lim_{\omega \to 0} \int dt' \; e^{i\omega(t-t')} \; \frac{1}{3} \; \sum_{i=1}^{3} \langle F_i(t') \; F_i(t) \rangle \; .$$

Heavy quark diffusion in QGP

- When a particle of mass M and color charge q accelerates in presence of color electric and magnetic fields with a $\langle v_i \ v_j \rangle = \delta_{ij} \ \frac{1}{3}\langle \mathbf{v}^2 \rangle$ given by

$$\mathbf{F} = q \ (\mathbf{E} \ + \ \mathbf{v} \times \mathbf{B}) = \frac{d\mathbf{p}}{dt} \ .$$

- The two-point correlator of the force can be written as an expansion in velocity

$$\langle F_i(t') \ F_j(t) \rangle = q^2 \left[\langle E_i(t') E_j(t) \rangle - \frac{1}{3} \langle \mathbf{v}^2 \rangle \ \langle \delta_{ij} \ B_k(t') B_k(t) - B_i(t') B_j(t) \rangle \right] \ .$$

- The momentum diffusion coefficient κ is defined in the limit $\omega \to 0$ as

$$\kappa = \lim_{\omega \to 0} \int dt' \ \mathrm{e}^{\mathrm{i}\omega(\mathrm{t}-\mathrm{t}')} \ \frac{1}{3} \ \sum_{\mathrm{i}=1}^{3} \langle \mathrm{F_i}(\mathrm{t}') \ \mathrm{F_i}(\mathrm{t}) \rangle \ .$$

Heavy quark diffusion in QGP

- When a particle of mass M and color charge q accelerates in presence of color electric and magnetic fields with a $\langle v_i \; v_j \rangle = \delta_{ij} \; \frac{1}{3} \langle \mathbf{v}^2 \rangle$ given by

$$\mathbf{F} = q \; (\mathbf{E} \; + \; \mathbf{v} \times \mathbf{B}) = \frac{d\mathbf{p}}{dt} \; .$$

- The two-point correlator of the force can be written as an expansion in velocity

$$\langle F_i(t') \; F_j(t) \rangle = q^2 \left[\langle E_i(t') E_j(t) \rangle - \frac{1}{3} \langle \mathbf{v}^2 \rangle \; \langle \delta_{ij} \; B_k(t') B_k(t) - B_i(t') B_j(t) \rangle \right] \; .$$

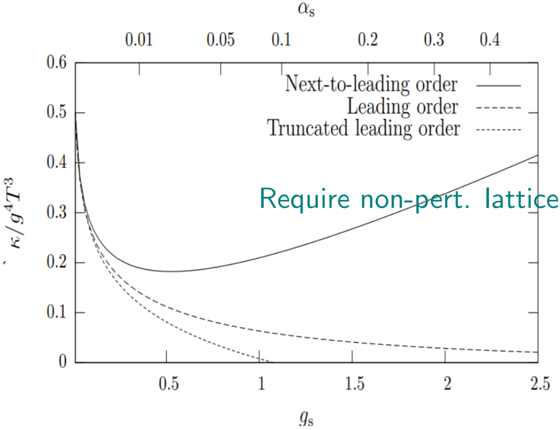
- The momentum diffusion coefficient κ is defined in the limit $\omega \to 0$ as

$$\kappa = \lim_{\omega \to 0} \int dt' \; \mathrm{e}^{\mathrm{i}\omega(\mathrm{t}-\mathrm{t}')} \; \frac{1}{3} \; \sum_{\mathrm{i}=1}^{3} \langle \mathrm{F_i}(\mathrm{t}') \; \mathrm{F_i}(\mathrm{t}) \rangle \; .$$

Heavy quark diffusion in QGP

- In non. relativistic limit $m \gg \pi T$, the velocity sq. $\langle \mathbf{v}^2 \rangle \sim \frac{T}{m} \to 0$ which means that the heavy quark momentum diffusion coefficient can be measured from color electric 2-pt function.

- Perturbative calculations are unreliable! [S. Caron-Huot & G. Moore, 08]



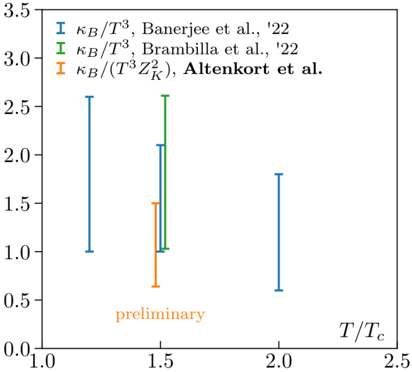
Require non-pert. lattice techniques!

Heavy quark diffusion in QGP

- On the lattice, the corresponding spectral function is constant as a function of ω/T as $\omega \to 0$.
- At $T = 2T_c$ the value of $\kappa_E/T^3 \simeq 1 - 2.1$

  [H. B. Meyer, 09, D. Banerjee et. al., 11, A. Francis et. al., 11,1

- Recently $\mathcal{O}(m/T)$ corrections have been included $\kappa \sim \kappa_E + \frac{2}{3}\kappa_B\langle v^2\rangle$. The contribution from color magnetic part is large! $\to$ for 2+1 QCD both contributions are similar in magnitude at $T = 352$ MeV.



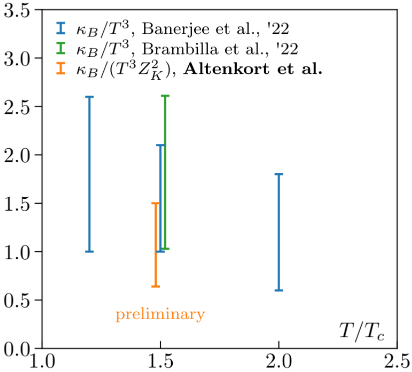
[Bouttefeux & Laine, 20, D. Banerjee, S. Datta, M. Laine, 22, N. Brambilla et. al. 23, Fig. courtesy L. Altenkort et. al. 22, 23]

Heavy quark diffusion in QGP

- On the lattice, the corresponding spectral function is constant as a function of ω/T as $\omega \to 0$.
- At $T = 2T_c$ the value of $\kappa_E/T^3 \simeq 1 - 2.1$

  [H. B. Meyer, 09, D. Banerjee et. al., 11, A. Francis et. al., 11,1

- Recently $\mathcal{O}(m/T)$ corrections have been included $\kappa \sim \kappa_E + \frac{2}{3}\kappa_B\langle v^2\rangle$. The contribution from color magnetic part is large! $\to$ for 2+1 QCD both contributions are similar in magnitude at $T = 352$ MeV.



[Bouttefeux & Laine, 20, D. Banerjee, S. Datta, M. Laine, 22, N. Brambilla et. al. 23, Fig. courtesy L. Altenkort et. al. 22, 23]

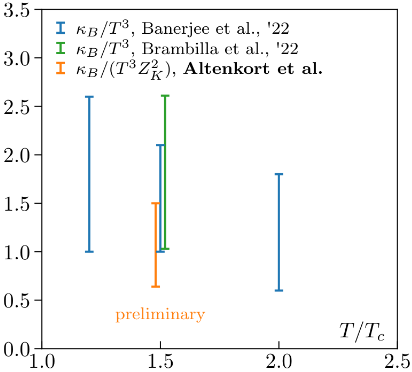
Heavy quark diffusion in QGP

- On the lattice, the corresponding spectral function is constant as a function of ω/T as $\omega \to 0$.

- At $T = 2T_c$ the value of $\kappa_E/T^3 \simeq 1 - 2.1$

  [H. B. Meyer, 09, D. Banerjee et. al., 11, A. Francis et. al., 11,1!

- Recently $\mathcal{O}(m/T)$ corrections have been included $\kappa \sim \kappa_E + \frac{2}{3}\kappa_B\langle v^2\rangle$. The contribution from color magnetic part is large! $\to$ for 2+1 QCD both contributions are similar in magnitude at $T = 352$ MeV.



[Bouttefeux & Laine, 20, D. Banerjee, S. Datta, M. Laine, 22, N. Brambilla et. al. 23, Fig. courtesy L. Altenkort et. al. 22, 23]

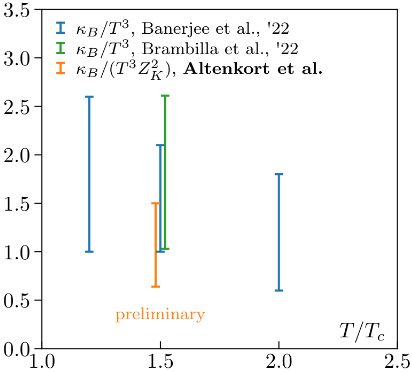
Heavy quark diffusion in QGP

- On the lattice, the corresponding spectral function is constant as a function of ω/T as $\omega \to 0$.

- At $T = 2T_c$ the value of $\kappa_E/T^3 \simeq 1 - 2.1$

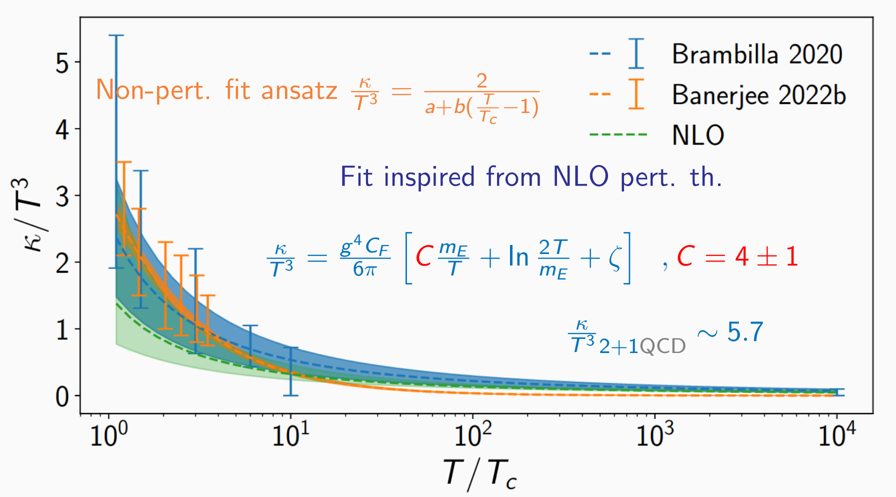
  [H. B. Meyer, 09, D. Banerjee et. al., 11, A. Francis et. al., 11,1!

- Recently $\mathcal{O}(m/T)$ corrections have been included $\kappa \sim \kappa_E + \frac{2}{3}\kappa_B\langle v^2\rangle$. The contribution from color magnetic part is large! $\rightarrow$ for 2+1 QCD both contributions are similar in magnitude at $T = 352$ MeV.



[Bouttefeux & Laine, 20, D. Banerjee, S. Datta, M. Laine, 22, N. Brambilla et. al. 23, Fig. courtesy L. Altenkort et. al. 22, 23]

Heavy quark diffusion in QGP



[Fig. from V. Leino, INT Workshop 22-3, Ref. N. Brambilla et al. 20, D. Banerjee et. al., 22, L. Altenkort et. al., 23]

Diffusion coefficient for charm quarks

- For charm one takes $\langle \mathbf{v}^2 \rangle \sim 0.5$ [L. Altenkort et. al. , 23] $\Rightarrow$ spatial diffusion coeff. at $T \sim 350$ MeV is $2\pi T\ D_s \sim 3.6$
- However for charm quarks an (HQET) expansion in m/T is questionable at these temperatures. Charm quarks also show collective behaviour similar to the light quarks [ALICE Collaboration, S. Acharya et al., 18, See talk by Johanna Stachel]
- Difficult to obtain from the heavy quark vector spectral function as the transport peak is narrow $\omega \sim T^2/m$ [Petreczky & Teaney, 06].
- Nevertheless the kinetic equilibration time estimated from the lattice results

$$\tau \sim \frac{m}{T^2}\ 2\pi T D_s \sim 6 \text{ fm/c}$$

- However during evolution a fairly large time ~ 1 fm/c may be spent in a non-equilibrium phase. How much does it affect the kinetic equilibration time?

Diffusion coefficient for charm quarks

- For charm one takes $\langle \mathbf{v}^2 \rangle \sim 0.5$ [L. Altenkort et. al. , 23] $\Rightarrow$ spatial diffusion coeff. at $T \sim 350$ MeV is $2\pi T\ D_s \sim 3.6$
- However for charm quarks an (HQET) expansion in m/T is questionable at these temperatures. Charm quarks also show collective behaviour similar to the light quarks [ALICE Collaboration, S. Acharya et al., 18, See talk by Johanna Stachel]
- Difficult to obtain from the heavy quark vector spectral function as the transport peak is narrow $\omega \sim T^2/m$ [Petreczky & Teaney, 06].
- Nevertheless the kinetic equilibration time estimated from the lattice results

$$\tau \sim \frac{m}{T^2}\ 2\pi T D_s \sim 6 \text{ fm/c}$$

- However during evolution a fairly large time ~ 1 fm/c may be spent in a non-equilibrium phase. How much does it affect the kinetic equilibration time?

Diffusion coefficient for charm quarks

- For charm one takes $\langle \mathbf{v}^2 \rangle \sim 0.5$ [L. Altenkort et. al. , 23] $\Rightarrow$ spatial diffusion coeff. at $T \sim 350$ MeV is $2\pi T\ D_s \sim 3.6$

- However for charm quarks an (HQET) expansion in m/T is questionable at these temperatures. Charm quarks also show collective behaviour similar to the light quarks [ALICE Collaboration, S. Acharya et al., 18, See talk by Johanna Stachel]

- Difficult to obtain from the heavy quark vector spectral function as the transport peak is narrow $\omega \sim T^2/m$ [Petreczky & Teaney, 06].

- Nevertheless the kinetic equilibration time estimated from the lattice results

$$\tau \sim \frac{m}{T^2}\, 2\pi T D_s \sim 6 \text{ fm/c}$$

- However during evolution a fairly large time ~ 1 fm/c may be spent in a non-equilibrium phase. How much does it affect the kinetic equilibration time?

Diffusion coefficient for charm quarks

- For charm one takes $\langle \mathbf{v}^2 \rangle \sim 0.5$ [L. Altenkort et. al. , 23] $\Rightarrow$ spatial diffusion coeff. at $T \sim 350$ MeV is $2\pi T \ D_s \sim 3.6$

- However for charm quarks an (HQET) expansion in m/T is questionable at these temperatures. Charm quarks also show collective behaviour similar to the light quarks [ALICE Collaboration, S. Acharya et al., 18, See talk by Johanna Stachel]

- Difficult to obtain from the heavy quark vector spectral function as the transport peak is narrow $\omega \sim T^2/m$ [Petreczky & Teaney, 06].

- Nevertheless the kinetic equilibration time estimated from the lattice results

$$\tau \sim \frac{m}{T^2} \, 2\pi T D_s \sim 6 \text{ fm/c}$$

- However during evolution a fairly large time ~ 1 fm/c may be spent in a non-equilibrium phase. How much does it affect the kinetic equilibration time?

Diffusion coefficient for charm quarks

- For charm one takes $\langle \mathbf{v}^2 \rangle \sim 0.5$ [L. Altenkort et. al. , 23] $\Rightarrow$ spatial diffusion coeff. at $T \sim 350$ MeV is $2\pi T\ D_s \sim 3.6$
- However for charm quarks an (HQET) expansion in m/T is questionable at these temperatures. Charm quarks also show collective behaviour similar to the light quarks [ALICE Collaboration, S. Acharya et al., 18, See talk by Johanna Stachel]
- Difficult to obtain from the heavy quark vector spectral function as the transport peak is narrow $\omega \sim T^2/m$ [Petreczky & Teaney, 06].
- Nevertheless the kinetic equilibration time estimated from the lattice results

$$\tau \sim \frac{m}{T^2}\, 2\pi T D_s \sim 6\ \mathrm{fm/c}$$

- However during evolution a fairly large time ~ 1 fm/c may be spent in a non-equilibrium phase. How much does it affect the kinetic equilibration time?

Charm momentum broadening in non-equilibrium gluonic plasma

- We consider a non-equilibrium phase consisting of highly occupied gluon states $n_g \sim 1/\alpha_s$.

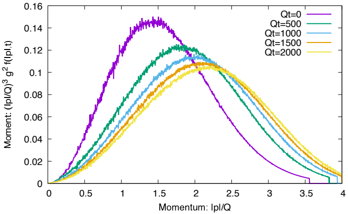
- We have performed our simulations on large fixed volume $N_s^3 = 256^3$ lattice, with lattice spacing $Qa_s = 0.5$, with $N_c = 2$.

- We consider the following initial phase-space distribution of the gluons, motivated from the Color Glass condensate effective theory

[L. McLerran and R. Venugopalan, 94]

$$g^2 f_g(p) = n_0 \frac{Q}{p} e^{-\frac{p^2}{2Q^2}}$$

where $n_0/g^2 \gg 1$.



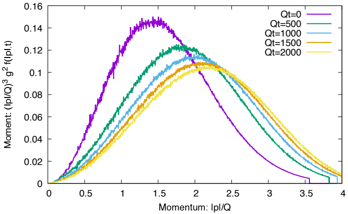
Charm momentum broadening in non-equilibrium gluonic plasma

- We consider a non-equilibrium phase consisting of highly occupied gluon states $n_g \sim 1/\alpha_s$.
- We have performed our simulations on large fixed volume $N_s^3 = 256^3$ lattice, with lattice spacing $Qa_s = 0.5$, with $N_c = 2$.
- We consider the following initial phase-space distribution of the gluons, motivated from the Color Glass condensate effective theory

[L. McLerran and R. Venugopalan, 94]

$$g^2 f_g(p) = n_0 \frac{Q}{p} e^{-\frac{p^2}{2Q^2}}$$

where $n_0/g^2 \gg 1$.



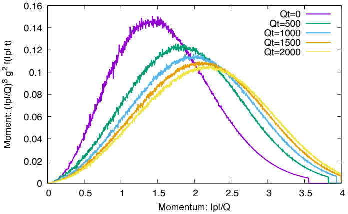
Charm momentum broadening in non-equilibrium gluonic plasma

- We consider a non-equilibrium phase consisting of highly occupied gluon states $n_g \sim 1/\alpha_s$.
- We have performed our simulations on large fixed volume $N_s^3 = 256^3$ lattice, with lattice spacing $Qa_s = 0.5$, with $N_c = 2$.
- We consider the following initial phase-space distribution of the gluons, motivated from the Color Glass condensate effective theory

[L. McLerran and R. Venugopalan, 94]

$$g^2 f_g(p) = n_0 \frac{Q}{p} e^{-\frac{p^2}{2Q^2}}$$

where $n_0/g^2 \gg 1$.

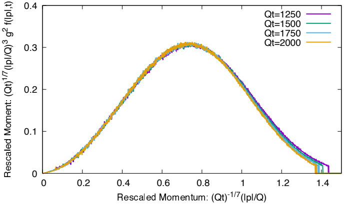


Initial Conditions: Non-Abelian plasma in the self-similar regime

- Starting with this initial condition we have evolved the gauge fields till $Qt = 1500$ where these are described by a self-similar scaling distribution,

$$\left(\frac{\tilde{p}}{Q}\right)^3 f_S(\tilde{p}) = (Qt)^{\frac{1}{7}} \left(\frac{|\mathbf{p}|}{Q}\right)^3 f(|\mathbf{p}|, t)$$

(Here, $\tilde{p} = (Qt)^{-\frac{1}{7}} |\mathbf{p}|$.)



- In analogy to QGP at high enough temperatures, there is a clear separation of scales here as well [J. Berges, K. Boguslavski, S. Schlichting, R. Venugopalan 14]

$$\sqrt{\sigma(t)} \sim Q(Qt)^{-3/10} < m_D(t) \sim Q(Qt)^{-1/7} \ll \Lambda(t) \sim Q(Qt)^{1/7}$$

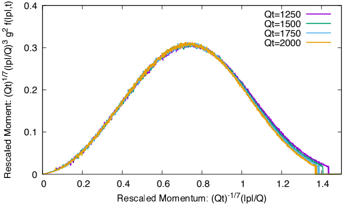
- Plasma in the self-similar regime represents a characteristic non-equilibrium state which we use as our initial state.

Initial Conditions: Non-Abelian plasma in the self-similar regime

- Starting with this initial condition we have evolved the gauge fields till $Qt = 1500$ where these are described by a self-similar scaling distribution,

$$\left(\frac{\tilde{p}}{Q}\right)^3 f_S(\tilde{p}) = (Qt)^{\frac{1}{7}} \left(\frac{|\mathbf{p}|}{Q}\right)^3 f(|\mathbf{p}|, t)$$

(Here, $\tilde{p} = (Qt)^{-\frac{1}{7}} |\mathbf{p}|$.)



- In analogy to QGP at high enough temperatures, there is a clear separation of scales here as well [J. Berges, K. Boguslavski, S. Schlichting, R. Venugopalan 14]

$$\begin{array}{ccccc} \sqrt{\sigma(t)} & < & m_D(t) & \ll & \Lambda(t) \\ \sim Q(Qt)^{-3/10} & & \sim Q(Qt)^{-1/7} & & \sim Q(Qt)^{1/7} \end{array}$$

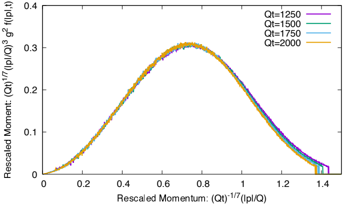
- Plasma in the self-similar regime represents a characteristic non-equilibrium state which we use as our initial state.

Initial Conditions: Non-Abelian plasma in the self-similar regime

- Starting with this initial condition we have evolved the gauge fields till $Qt = 1500$ where these are described by a self-similar scaling distribution,

$$\left(\frac{\tilde{p}}{Q}\right)^3 f_S(\tilde{p}) = (Qt)^{\frac{1}{7}} \left(\frac{|\mathbf{p}|}{Q}\right)^3 f(|\mathbf{p}|, t)$$

(Here, $\tilde{p} = (Qt)^{-\frac{1}{7}} |\mathbf{p}|$.)



- In analogy to QGP at high enough temperatures, there is a clear separation of scales here as well [J. Berges, K. Boguslavski, S. Schlichting, R. Venugopalan 14]

$$\underset{\sim Q(Qt)^{-3/10}}{\sqrt{\sigma(t)}} < \underset{\sim Q(Qt)^{-1/7}}{m_D(t)} \ll \underset{\sim Q(Qt)^{1/7}}{\Lambda(t)}$$

- Plasma in the self-similar regime represents a characteristic non-equilibrium state which we use as our initial state.

Evolving the heavy quarks

- We have implemented for the first time the evolution of heavy quarks as relativistic particles in this gluon background using tree-level improved Wilson-Dirac Hamiltonian on the lattice.

$$\hat{H}_f = \sum_{\mathbf{x}} \hat{\psi}^\dagger_{\mathbf{x}} \gamma^0 (-i\not{D}_W + m)\hat{\psi}_{\mathbf{x}}$$

$$i\gamma^0 \partial_{x^0} \hat{\psi}_{\mathbf{x}} = (-i\not{D}_W + m)\hat{\psi}_{\mathbf{x}} \ .$$

- Our formalism is thus much more general in comparison to studies done earlier in the infinite-mass limit with non-relativistic quarks.

[K. Boguslavski et. al., 21]

- We have chosen a wide set of quark masses, $m/Q = 0.001 - 12.0$. For $Q \sim 1$ GeV, the choice of $m/Q = 1.2$ represents a particle with mass close to that of the charm quark.

Evolving the heavy quarks

- We have implemented for the first time the evolution of heavy quarks as relativistic particles in this gluon background using tree-level improved Wilson-Dirac Hamiltonian on the lattice.

$$\hat{H}_f = \sum_{\mathbf{x}} \hat{\psi}^\dagger_{\mathbf{x}} \gamma^0 (-i \not{D}_W + m) \hat{\psi}_{\mathbf{x}}$$

$$i\gamma^0 \partial_{x^0} \hat{\psi}_{\mathbf{x}} = (-i\not{D}_W + m)\hat{\psi}_{\mathbf{x}} \ .$$

- Our formalism is thus much more general in comparison to studies done earlier in the infinite-mass limit with non-relativistic quarks.

[K. Boguslavski et. al., 21]

- We have chosen a wide set of quark masses, $m/Q = 0.001 - 12.0$. For $Q \sim 1$ GeV, the choice of $m/Q = 1.2$ represents a particle with mass close to that of the charm quark.

Evolving the heavy quarks

- We have implemented for the first time the evolution of heavy quarks as relativistic particles in this gluon background using tree-level improved Wilson-Dirac Hamiltonian on the lattice.

$$\hat{H}_f = \sum_{\mathbf{x}} \hat{\psi}^\dagger_{\mathbf{x}} \gamma^0 (-i \not{D}_W + m) \hat{\psi}_{\mathbf{x}}$$

$$i\gamma^0 \partial_{x^0} \hat{\psi}_{\mathbf{x}} = (-i \not{D}_W + m) \hat{\psi}_{\mathbf{x}} \ .$$

- Our formalism is thus much more general in comparison to studies done earlier in the infinite-mass limit with non-relativistic quarks.

  [K. Boguslavski et. al., 21]

- We have chosen a wide set of quark masses, $m/Q = 0.001 - 12.0$. For $Q \sim 1$ GeV, the choice of $m/Q = 1.2$ represents a particle with mass close to that of the charm quark.

Calculating the quark spectral functions

- Quark spectral function, in terms of quark fields, is defined as $\rho^{\alpha\beta}(x,y) = \left\langle \left\{ \hat{\psi}^{\alpha}(x), \hat{\bar{\psi}}^{\beta}(y) \right\} \right\rangle$.
- Can be decomposed into Scalar, Pseudo-scalar, Vector, Axial-vector, Tensor components

$$\rho = \rho_S + i\gamma_5\rho_P + \gamma_\mu\rho_V^\mu + \gamma_\mu\gamma_5\rho_A^\mu + \frac{1}{2}\sigma_{\mu\nu}\rho_T^{\mu\nu}$$

- Within our non-equilibrium plasma, typical momenta of gluons $\sim 2-3\ Q$, hence HTL approximation is justified for heavy quarks with momenta $p = |\mathbf{p}| << m_{bare}$.
- For heavy quarks within HTL approx,

$$\mathrm{Re}\rho_V^0(t) = e^{-\gamma(m_{bare})t}\cos m_{eff}\,t\ ,\ \ \rho_S^0(t) = e^{-\gamma(m_{bare})t}\sin m_{eff}\,t.$$

- For light quarks with zero momenta ($p=0$), expression for the HTL spectral function simplifies to $\mathrm{Re}\rho_V^0(t, p=0) \approx e^{-\gamma(m_{eff}, p=0)t}\cos[\omega(m_{eff}, 0)t]$.

Calculating the quark spectral functions

- Quark spectral function, in terms of quark fields, is defined as $\rho^{\alpha\beta}(x,y) = \left\langle \left\{ \hat{\psi}^{\alpha}(x), \hat{\bar{\psi}}^{\beta}(y) \right\} \right\rangle$.
- Can be decomposed into Scalar, Pseudo-scalar, Vector, Axial-vector, Tensor components

$$\rho = \rho_S + i\gamma_5\rho_P + \gamma_\mu\rho_V^\mu + \gamma_\mu\gamma_5\rho_A^\mu + \frac{1}{2}\sigma_{\mu\nu}\rho_T^{\mu\nu}$$

- Within our non-equilibrium plasma, typical momenta of gluons $\sim 2-3\ Q$, hence HTL approximation is justified for heavy quarks with momenta $p = |\mathbf{p}| << m_{bare}$.
- For heavy quarks within HTL approx,

$$\mathrm{Re}\rho_V^0(t) = e^{-\gamma(m_{bare})t}\cos m_{eff}\,t\ ,\ \ \rho_S^0(t) = e^{-\gamma(m_{bare})t}\sin m_{eff}\,t.$$

- For light quarks with zero momenta ($p=0$), expression for the HTL spectral function simplifies to $\mathrm{Re}\rho_V^0(t, p=0) \approx e^{-\gamma(m_{eff}, p=0)t}\cos[\omega(m_{eff}, 0)t]$.

Calculating the quark spectral functions

- Quark spectral function, in terms of quark fields, is defined as $\rho^{\alpha\beta}(x,y) = \left\langle \left\{ \hat{\psi}^{\alpha}(x), \hat{\bar{\psi}}^{\beta}(y) \right\} \right\rangle$.
- Can be decomposed into Scalar, Pseudo-scalar, Vector, Axial-vector, Tensor components

$$\rho = \rho_S + i\gamma_5\rho_P + \gamma_\mu\rho_V^\mu + \gamma_\mu\gamma_5\rho_A^\mu + \frac{1}{2}\sigma_{\mu\nu}\rho_T^{\mu\nu}$$

- Within our non-equilibrium plasma, typical momenta of gluons $\sim 2-3\ Q$, hence HTL approximation is justified for heavy quarks with momenta $p = |\mathbf{p}| << m_{bare}$.
- For heavy quarks within HTL approx,

$$\mathrm{Re}\rho_V^0(t) = e^{-\gamma(m_{bare})t}\cos m_{eff}\,t\ ,\ \ \rho_S^0(t) = e^{-\gamma(m_{bare})t}\sin m_{eff}\,t.$$

- For light quarks with zero momenta ($p = 0$), expression for the HTL spectral function simplifies to $\mathrm{Re}\rho_V^0(t, p=0) \approx e^{-\gamma(m_{eff}, p=0)t}\cos[\omega(m_{eff}, 0)t]$.

Calculating the quark spectral functions

- Quark spectral function, in terms of quark fields, is defined as $\rho^{\alpha\beta}(x,y) = \left\langle \left\{ \hat{\psi}^{\alpha}(x), \hat{\bar{\psi}}^{\beta}(y) \right\} \right\rangle$.
- Can be decomposed into Scalar, Pseudo-scalar, Vector, Axial-vector, Tensor components

$$\rho = \rho_S + i\gamma_5\rho_P + \gamma_\mu\rho_V^\mu + \gamma_\mu\gamma_5\rho_A^\mu + \frac{1}{2}\sigma_{\mu\nu}\rho_T^{\mu\nu}$$

- Within our non-equilibrium plasma, typical momenta of gluons $\sim 2-3\ Q$, hence HTL approximation is justified for heavy quarks with momenta $p = |\mathbf{p}| << m_{bare}$.
- For heavy quarks within HTL approx,

$$\mathrm{Re}\rho_V^0(t) = e^{-\gamma(m_{bare})t}\cos m_{eff}\, t\ ,\ \ \rho_S^0(t) = e^{-\gamma(m_{bare})t}\sin m_{eff}\, t.$$

- For light quarks with zero momenta ($p = 0$), expression for the HTL spectral function simplifies to $\mathrm{Re}\rho_V^0(t, p=0) \approx e^{-\gamma(m_{eff}, p=0)t}\cos[\omega(m_{eff}, 0)t]$.

Calculating the quark spectral functions

- Quark spectral function, in terms of quark fields, is defined as $\rho^{\alpha\beta}(x,y) = \left\langle \left\{ \hat{\psi}^{\alpha}(x), \hat{\bar{\psi}}^{\beta}(y) \right\} \right\rangle$.
- Can be decomposed into Scalar, Pseudo-scalar, Vector, Axial-vector, Tensor components

$$\rho = \rho_S + i\gamma_5\rho_P + \gamma_\mu\rho_V^\mu + \gamma_\mu\gamma_5\rho_A^\mu + \frac{1}{2}\sigma_{\mu\nu}\rho_T^{\mu\nu}$$

- Within our non-equilibrium plasma, typical momenta of gluons $\sim 2-3\ Q$, hence HTL approximation is justified for heavy quarks with momenta $p = |\mathbf{p}| << m_{bare}$.

- For heavy quarks within HTL approx,

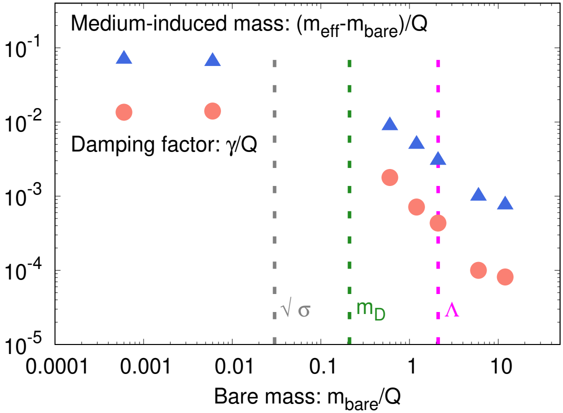
$$\mathrm{Re}\rho_V^0(t) = e^{-\gamma(m_{bare})t}\cos m_{eff}\, t\ ,\ \ \rho_S^0(t) = e^{-\gamma(m_{bare})t}\sin m_{eff}\, t.$$

- For light quarks with zero momenta ($p=0$), expression for the HTL spectral function simplifies to $\mathrm{Re}\rho_V^0(t, p=0) \approx e^{-\gamma(m_{eff}, p=0)t}\cos[\omega(m_{eff}, 0)t]$.

In-Medium effects on the quark quasi-particles

- We obtain effective mass (m_{eff}) via fit to spectral function and subtract the bare mass to obtain medium-induced mass.

[Ref: H. Pandey, S. Schlichting, S.S., Phys. Rev. Lett., 132, 222301 (2024)]

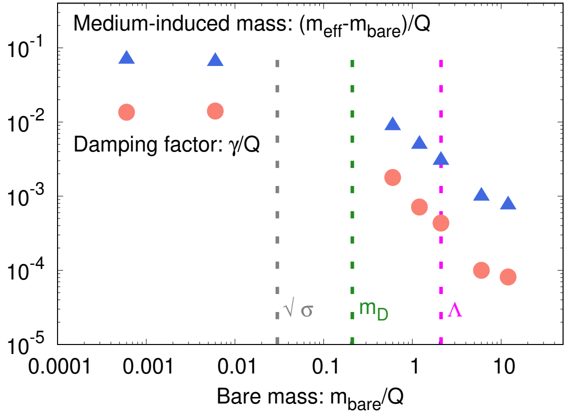


- Width of quasi-particles decreases $> 10\times$ going from light to heavy.

In-Medium effects on the quark quasi-particles

- We obtain effective mass (m_{eff}) via fit to spectral function and subtract the bare mass to obtain medium-induced mass.

[Ref: H. Pandey, S. Schlichting, S.S., Phys. Rev. Lett., 132, 222301 (2024)]



- Width of quasi-particles decreases $> 10\times$ going from light to heavy.

Observations from the analysis of spectral functions

- For $m_{bare}/Q < 0.1$: medium modification is large $\Longrightarrow$ behaves more like a collective excitation than a weakly-interacting quasi-particle.

- For $m_{bare}/Q > 2.0$: in-medium modification is significantly less signifying more stable quasi-particle $\rightarrow$ onset of heavy-quark NR regime. Small width, hence interacts with the medium to get kicks from gluons and diffuse.

- Intermediate $m_{bare}/Q \approx 1.2$, i.e. close to charm mass lies in the transient region $\Longrightarrow$ may diffuse similar to heavier quarks but has to be evolved relativistically to capture the correct dynamics.

Observations from the analysis of spectral functions

- For $m_{bare}/Q < 0.1$: medium modification is large $\Longrightarrow$ behaves more like a collective excitation than a weakly-interacting quasi-particle.
- For $m_{bare}/Q > 2.0$: in-medium modification is significantly less signifying more stable quasi-particle $\rightarrow$ onset of heavy-quark NR regime. Small width, hence interacts with the medium to get kicks from gluons and diffuse.
- Intermediate $m_{bare}/Q \approx 1.2$, i.e. close to charm mass lies in the transient region $\Longrightarrow$ may diffuse similar to heavier quarks but has to be evolved relativistically to capture the correct dynamics.

Observations from the analysis of spectral functions

- For $m_{bare}/Q < 0.1$: medium modification is large $\implies$ behaves more like a collective excitation than a weakly-interacting quasi-particle.
- For $m_{bare}/Q > 2.0$: in-medium modification is significantly less signifying more stable quasi-particle $\rightarrow$ onset of heavy-quark NR regime. Small width, hence interacts with the medium to get kicks from gluons and diffuse.
- Intermediate $m_{bare}/Q \approx 1.2$, i.e. close to charm mass lies in the transient region $\implies$ may diffuse similar to heavier quarks but has to be evolved relativistically to capture the correct dynamics.

Momentum Broadening: How do we calculate it?

- We start with a single quark in a fixed momentum ($\mathbf{P}$) and spin polarization (s) mode with initial conditions

$$\langle b^\dagger_\lambda(t=0,\mathbf{p}) b_{\lambda'}(t=0,\mathbf{p}')\rangle = \delta_{\lambda\lambda'}\ \delta_{\mathbf{pp}'}\ \delta_{\lambda s}\ \delta_{\mathbf{pP}}$$

$$\langle d^\dagger_\lambda(t=0,\mathbf{p}) d_{\lambda'}(t=0,\mathbf{p}')\rangle = 0$$

- Evolving the quark fields, which at time t' is represented as,

$$\Psi(t',\mathbf{x}) = \frac{1}{\sqrt{N^3}}\sum_{\lambda,\mathbf{p}}\left[\phi^u_{\lambda,\mathbf{p}}(t',\mathbf{x}) b_\lambda(t'=0,\mathbf{p}) + \phi^v_{\lambda,\mathbf{p}}(t',\mathbf{x}) d^\dagger_\lambda(t'=0,\mathbf{p})\right]$$

- Momentum mode occupancy is then calculated as,

$$\frac{dN}{d^3\mathbf{q}} = \sum_{\lambda'} |u^\dagger_{\lambda'}(\mathbf{q})\tilde{\phi}^u_s(t',\mathbf{P})|^2$$

Momentum Broadening: How do we calculate it?

- We start with a single quark in a fixed momentum ($\mathbf{P}$) and spin polarization (s) mode with initial conditions

$$\langle b^\dagger_\lambda(t=0,\mathbf{p}) b_{\lambda'}(t=0,\mathbf{p}')\rangle = \delta_{\lambda\lambda'}\ \delta_{\mathbf{p}\mathbf{p}'}\ \delta_{\lambda s}\ \delta_{\mathbf{p}\mathbf{P}}$$
$$\langle d^\dagger_\lambda(t=0,\mathbf{p}) d_{\lambda'}(t=0,\mathbf{p}')\rangle = 0$$

- Evolving the quark fields, which at time t' is represented as,

$$\Psi(t',\mathbf{x}) = \frac{1}{\sqrt{N^3}} \sum_{\lambda,\mathbf{p}} \left[\phi^u_{\lambda,\mathbf{p}}(t',\mathbf{x}) b_\lambda(t'=0,\mathbf{p}) + \phi^v_{\lambda,\mathbf{p}}(t',\mathbf{x}) d^\dagger_\lambda(t'=0,\mathbf{p})\right]$$

- Momentum mode occupancy is then calculated as,

$$\frac{dN}{d^3\mathbf{q}} = \sum_{\lambda'} |u^\dagger_{\lambda'}(\mathbf{q})\tilde{\phi}^u_s(t',\mathbf{P})|^2$$

Momentum Broadening: How do we calculate it?

- We start with a single quark in a fixed momentum ($\mathbf{P}$) and spin polarization (s) mode with initial conditions

$$\langle b^{\dagger}_{\lambda}(t=0,\mathbf{p}) b_{\lambda'}(t=0,\mathbf{p}')\rangle = \delta_{\lambda\lambda'}\ \delta_{\mathbf{p}\mathbf{p}'}\ \delta_{\lambda s}\ \delta_{\mathbf{p}\mathbf{P}}$$

$$\langle d^{\dagger}_{\lambda}(t=0,\mathbf{p}) d_{\lambda'}(t=0,\mathbf{p}')\rangle = 0$$

- Evolving the quark fields, which at time t' is represented as,

$$\Psi(t',\mathbf{x}) = \frac{1}{\sqrt{N^3}} \sum_{\lambda,\mathbf{p}} \left[\phi^{u}_{\lambda,\mathbf{p}}(t',\mathbf{x}) b_{\lambda}(t'=0,\mathbf{p}) + \phi^{v}_{\lambda,\mathbf{p}}(t',\mathbf{x}) d^{\dagger}_{\lambda}(t'=0,\mathbf{p}) \right]$$

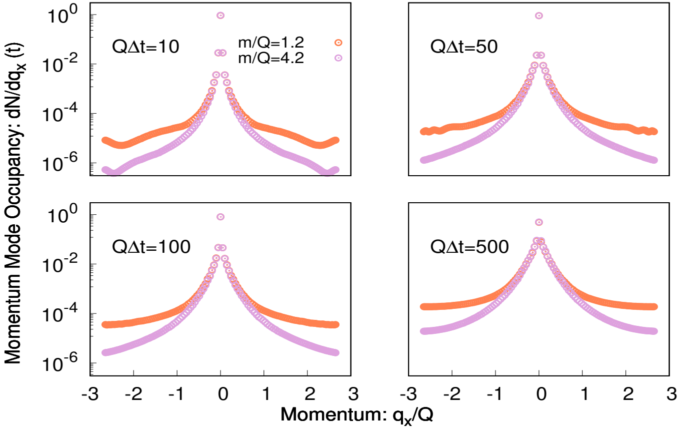
- Momentum mode occupancy is then calculated as,

$$\frac{dN}{d^3\mathbf{q}} = \sum_{\lambda'} |u^{\dagger}_{\lambda'}(\mathbf{q}) \tilde{\phi}^{u}_{s}(t',\mathbf{P})|^2$$

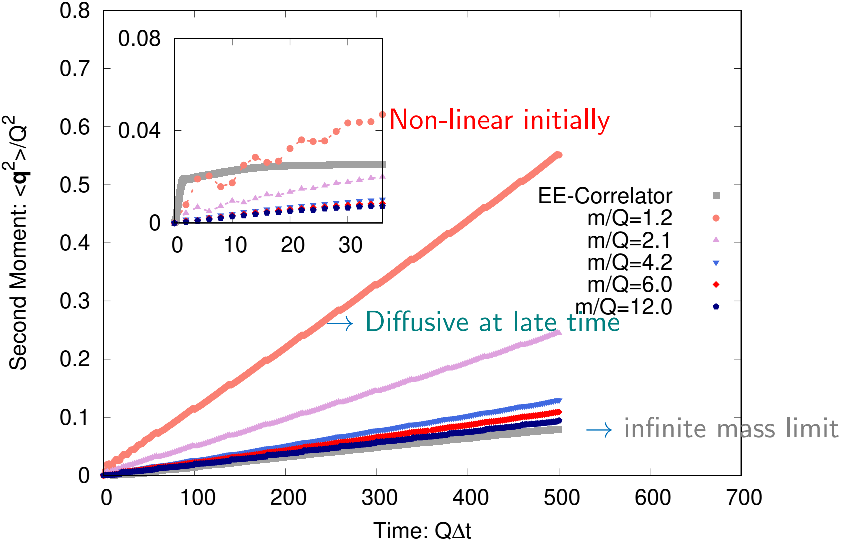
Momentum Broadening of heavy quarks

- Starting with zero initial momenta, the momentum distribution broadens due to kicks it receives from the gluon plasma.

[Ref: H. Pandey, S. Schlichting, S.S., Phys. Rev. Lett., 132, 222301 (2024)]

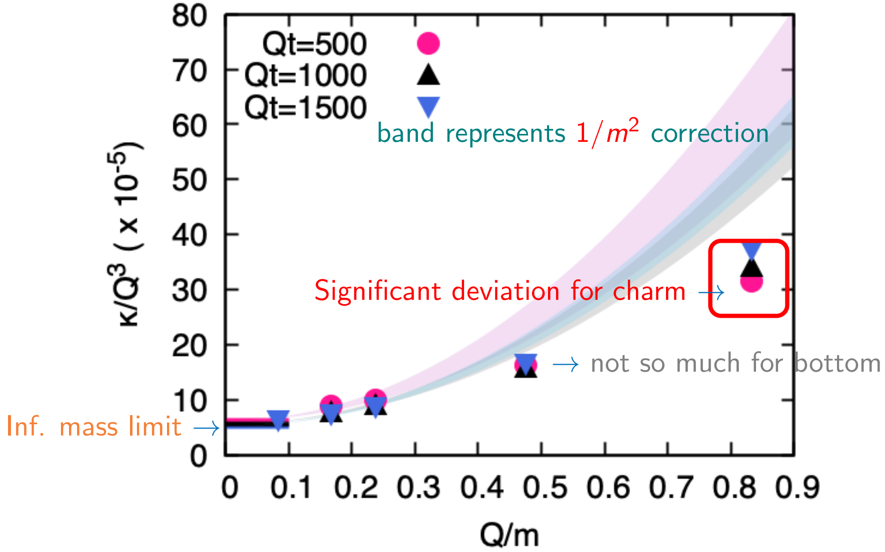


Quantifying broadening through second moment of the momentum distribution [Ref: H. Pandey, S. Schlichting, S.S., Phys. Rev. Lett., 132, 222301 (2024).]



Extracting momentum diffusion coefficient at late times

[Ref: H. Pandey, S. Schlichting, S.S., Phys. Rev. Lett., 132, 222301 (2024).]



Summary and Outlook

- We have set up a formalism to study heavy quark momentum broadening and extraction of heavy quark momentum diffusion coefficient.
- We find that there are large corrections to momentum broadening of charm quarks in full relativistic treatment without resorting to expansion in $1/m^2$ about infinite mass limit.
- The kinetic equilibration time $\tau T \sim \frac{2(m/T)}{\kappa/Q^3}\frac{T^3}{Q^3}$ which is of the order $\tau \sim 1.5$ fm/c assuming an initial temp. of QGP to be around 500 MeV.
- We are studying the broadening phenomena for finite initial momenta for quarks and expanding geometries relevant for HIC.

Summary and Outlook

- We have set up a formalism to study heavy quark momentum broadening and extraction of heavy quark momentum diffusion coefficient.
- We find that there are large corrections to momentum broadening of charm quarks in full relativistic treatment without resorting to expansion in $1/m^2$ about infinite mass limit.
- The kinetic equilibration time $\tau T \sim \frac{2(m/T)}{\kappa/Q^3}\frac{T^3}{Q^3}$ which is of the order $\tau \sim 1.5$ fm/c assuming an initial temp. of QGP to be around 500 MeV.
- We are studying the broadening phenomena for finite initial momenta for quarks and expanding geometries relevant for HIC.

Summary and Outlook

- We have set up a formalism to study heavy quark momentum broadening and extraction of heavy quark momentum diffusion coefficient.
- We find that there are large corrections to momentum broadening of charm quarks in full relativistic treatment without resorting to expansion in $1/m^2$ about infinite mass limit.
- The kinetic equilibration time $\tau T \sim \frac{2(m/T)}{\kappa/Q^3} \frac{T^3}{Q^3}$ which is of the order $\tau \sim 1.5$ fm/c assuming an initial temp. of QGP to be around 500 MeV.
- We are studying the broadening phenomena for finite initial momenta for quarks and expanding geometries relevant for HIC.

Summary and Outlook

- We have set up a formalism to study heavy quark momentum broadening and extraction of heavy quark momentum diffusion coefficient.
- We find that there are large corrections to momentum broadening of charm quarks in full relativistic treatment without resorting to expansion in $1/m^2$ about infinite mass limit.
- The kinetic equilibration time $\tau T \sim \frac{2(m/T)}{\kappa/Q^3}\frac{T^3}{Q^3}$ which is of the order $\tau \sim 1.5$ fm/c assuming an initial temp. of QGP to be around 500 MeV.
- We are studying the broadening phenomena for finite initial momenta for quarks and expanding geometries relevant for HIC.