Finite size effects on the critical endpoint and conserved charge fluctuations

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In collaboration with:

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EMMI Workshop at Wrocław - Aspects of Criticality II, 2024. 07. 02-04.

What are the typical sizes?

- Typical size of the fireball in **heavy ion collisions** is a few fm.
- Neutron stars and **compact stars** built up from strongly interacting matter (with extra structure) with a size $\sim$ 10 km.
- Several **models with finite** (different) size.
- In field theoretical calculations (LSM, NJL, DS, etc): **infinite size**.

Why does it matter?

- The properties of the system can change significantly.
- Criticality in a finite system?
- The CEP and the first-order region might "disappear".

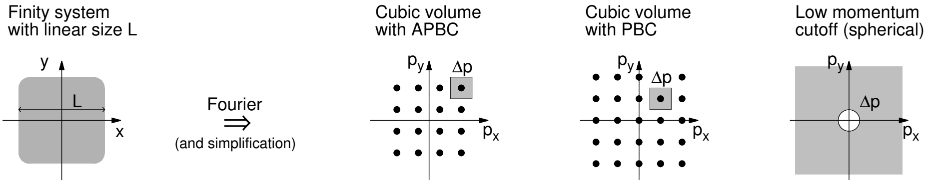
Might be studied in field theoretical models by implementating the finite size effects.

The vicinity of the CEP is accessible with models that are in the thermodynamic limit.

How to consider the finite size effects without losing the advantages of these models?

It is usual to modify the momentum space.

- Discretization: $\int dp \rightarrow \sum_n$
- Low momentum cutoff: $\int_0^\infty dp \rightarrow \int_\lambda^\infty dp$



LSM

Palhares, Fraga and Kodama,
J. Phys. G **38**, 085101 (2011)

NJL

Bhattacharyya, Deb, Ghosh, Ray and Sur,
Phys. Rev. D **87**, no.5, 054009 (2013)

QM model FRG

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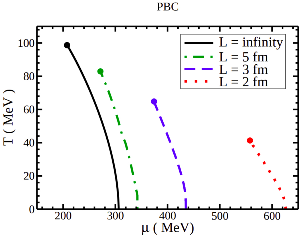
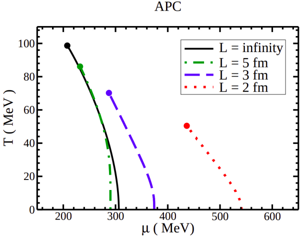
DS approach

Bernhardt, Fischer, Isserstedt and Schaefer,
Phys. Rev. D **104**, no.7, 074035 (2021)



Only zero mode of vacuum part



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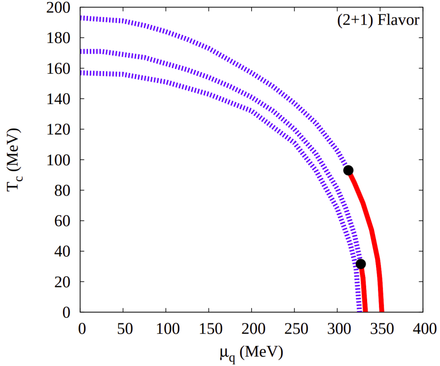

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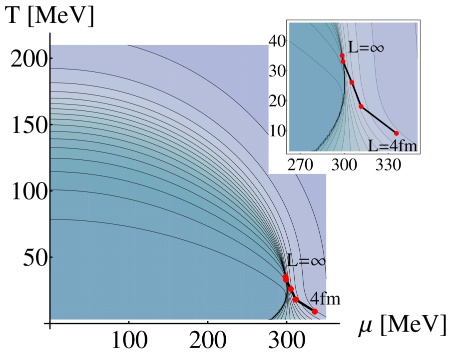

Finite size effects in different models (examples)

LSM

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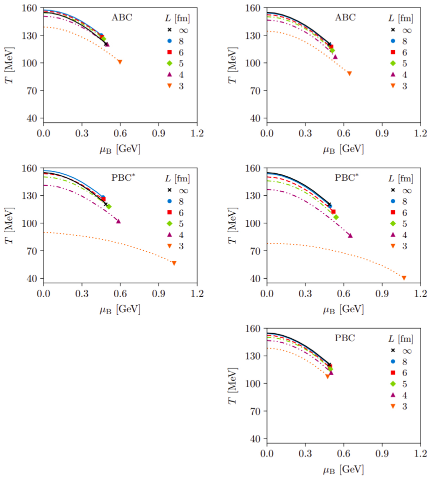

Finite size effects in different models

Various results in (P)NJL, (P)LSM, DS, etc. calculations.
With discretization and low momentum cutoff as well.

	Discretization	**Low cutoff**
LSM	J. Phys. G **38**, 085101 (2011) PoS **FACESQCD**, 017 (2010)	J. Phys. G **44**, no.2, 025101 (2017) Universe **5**, no.4, 94 (2019)
NJL	Mod. Phys. Lett. A **33**, no.39, 1850232 (2018) Phys. Rev. D **101** (2020) 7, 074001 Universe **8**, no.5, 264 (2022)	Phys. Rev. D **87**, no.5, 054009 (2013) Phys. Rev. D **91**, no.5, 051501 (2015) Int. J. Mod. Phys. A **32**, no.13, 1750067 (2017)
FRG	Phys. Rev. D **73**, 074010 (2006) Phys. Rev. D **90**, no.5, 054012 (2014) Phys. Rev. D **95**, no.5, 056015 (2017) Phys. Rept. **707-708**, 1-51 (2017)	
DS	Phys. Rev. D **81**, 094005 (2010) Phys. Rev. D **102**, 114011 (2020) Phys. Rev. D **104**, no.7, 074035 (2021) Phys. Lett. B **841**, 137908 (2023)	Nucl. Phys. B **938**, 298-306 (2019)

Various results in (P)NJL, (P)LSM, DS, etc. calculations.
With discretization and low momentum cutoff as well.

Vacuum part		**Discretization**	**Low cutoff**
NO	**LSM**	J. Phys. G **38**, 085101 (2011) PoS **FACESQCD**, 017 (2010)	J. Phys. G **44**, no.2, 025101 (2017) Universe **5**, no.4, 94 (2019)
Regularized	**NJL**	Mod. Phys. Lett. A **33**, no.39, 1850232 (2018) Phys. Rev. D **101** (2020) 7, 074001 Universe **8**, no.5, 264 (2022)	Phys. Rev. D **87**, no.5, 054009 (2013) Phys. Rev. D **91**, no.5, 051501 (2015) Int. J. Mod. Phys. A **32**, no.13, 1750067 (2017)
Separated Renorm.	**FRG**	Phys. Rev. D **73**, 074010 (2006) Phys. Rev. D **90**, no.5, 054012 (2014) Phys. Rev. D **95**, no.5, 056015 (2017) Phys. Rept. **707-708**, 1-51 (2017)	
Renorm.	**DS**	Phys. Rev. D **81**, 094005 (2010) Phys. Rev. D **102**, 114011 (2020) Phys. Rev. D **104**, no.7, 074035 (2021) Phys. Lett. B **841**, 137908 (2023)	Nucl. Phys. B **938**, 298-306 (2019)

Effective model to study the phase diagram of strongly interacting matter at finite T and μ.

- Quark-Meson model: "simple" linear sigma model with quarks and mesons
  $O(N)$ model, $N_f = 2,\ 2+1$ LSM's
- Based on chiral symmetry

$$SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A \to SU(2)_I \times U(1)_V$$

  broken explicitly (and spontaneously) and with the axial anomaly taken into account
- Can be extended with vector and axial vector nonets (eLSM) **Phys. Rev. D 93, no. 11, 114014 (2016)**
  Isospin symmetric case: 16 mesonic degrees of freedom.
- Polyakov might be added for "statistical confinement":
  Polyakov loop variables give 2 order parameters $\Phi,\ \bar{\Phi}$.

- Mesonic Lagrangian contains the dynamical, the symmetry breaking, and the meson-meson interaction terms.
  - Each meson-meson terms upto 4th order that are allowed by the chiral symmetry.
  - $U(1)_A$ anomaly and explicit breaking of the chiral symmetry.
- Constituent quarks in Yukawa Lagrangian

$$\mathcal{L}_Y = \bar{\psi}\left(i\gamma^\mu \partial_\mu - g_F(S - i\gamma_5 P) - g_V\gamma^\mu(V_\mu + \gamma_5 A_\mu)\right)\psi \tag{1}$$

In mean-field, $g_V \neq 0$ not affects the thermodynamics. **Phys. Rev. D 104, 056013 (2021)**

- SSB with nonzero vev. for scalar-isoscalar sector $\phi_N,\ \phi_S$.

$\Rightarrow m_{u,d} = \dfrac{g_F}{2}\phi_N,\ m_s = \dfrac{g_F}{\sqrt{2}}\phi_S$ fermion masses in $\mathcal{L}_Y$.

Thermodynamics: **Mean field level** effective potential:

- Classical potential.
- Fermionic one-loop correction with vanishing fluctuating mesonic fields.

$$\bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - \mathrm{diag}(m_u, m_d, m_s)\right)\psi$$

  Functional integration over the fermionic fields.
  The momentum integrals are renormalized.

- Polyakov loop potential.

$$\Omega(T, \mu_q) = U_{Cl} + \mathrm{tr}\int_K \log\left(iS_0^{-1}\right) + U(\Phi, \bar{\Phi}) \tag{2}$$

Field equations (FE):

$$\frac{\partial\Omega}{\partial\phi_N} = \frac{\partial\Omega}{\partial\phi_S} = \frac{\partial\Omega}{\partial\bar{\Phi}} = \frac{\partial\Omega}{\partial\Phi} = 0 \tag{3}$$

Curvature meson masses:

$$M^2_{ab} = \left.\frac{\partial^2\Omega}{\partial\varphi_a\partial\varphi_b}\right|_{\{\varphi_i\}=0} \tag{4}$$

Thermodynamics: **Mean field level** effective potential:

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  Functional integration over the fermionic fields.
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- Polyakov loop potential.

Only this term is modified

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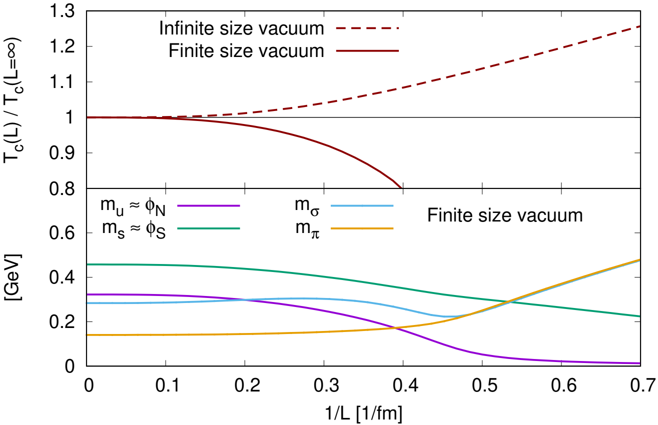
Phys. Rev. D 108 no.7, 076010 (2023)

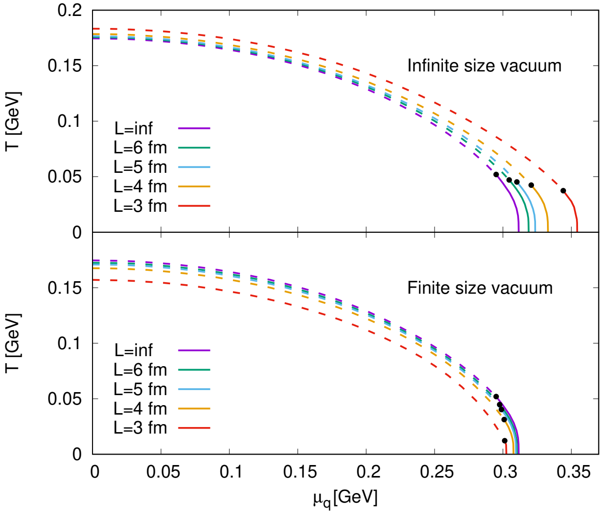
- Fermionic vacuum and matter contribution:

$$\Omega_{\bar{q}q}(T,\mu_q) = \Omega_{\bar{q}q}^{\text{vac}} + \Omega_{\bar{q}q}^{\text{mat}}(T,\mu_q)$$

- The size dependence of $\Omega_{\bar{q}q}^{\text{vac}}$ pushes the system towards chiral restoration
- At $T = 0$ and $\mu_q = 0$ the physical quantities also show the restoration



Low momentum cutoff: Phase diagram and CEP

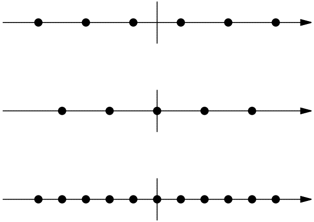


- With $L = \infty$ vacuum part the chirally broken phase expands and the CEP is present even for very small sizes
- With L dependent vacuum part the chirally broken phase will be reduced.

  The CEP disappears at $L = 2.5$ fm, as well as the broken phase at $L = 2$ fm.

What is the **boundary condition**?

- APBC: $\frac{(2\pi)^2}{L^2}\sum_n \left(n+\frac{1}{2}\right)^2$
- PBC: $\frac{(2\pi)^2}{L^2}\sum_n n^2$
- Something else
  e.g. SWC: $\frac{(\pi)^2}{L^2}\sum_n n^2$



Shape of the finite volume: usually **cubic**, sometimes **spherical**

Momentum discretization

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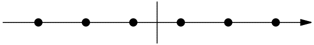
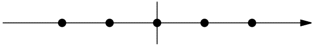


- PBC: $\dfrac{(2\pi)^2}{L^2}\sum_n n^2$



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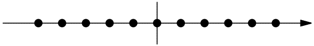
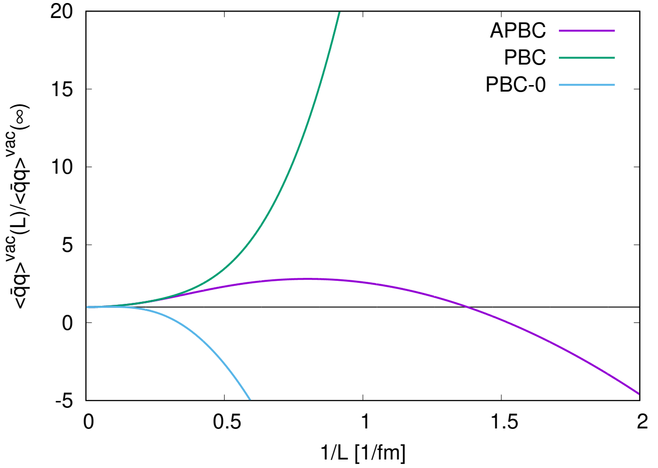
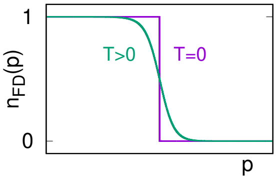


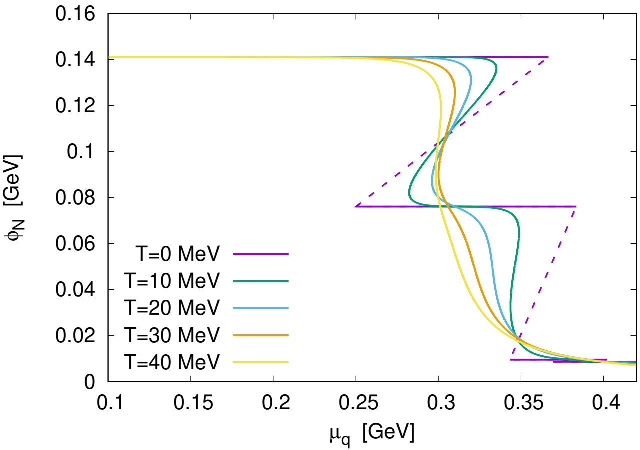
Shape of the finite volume: usually **cubic**, sometimes **spherical**

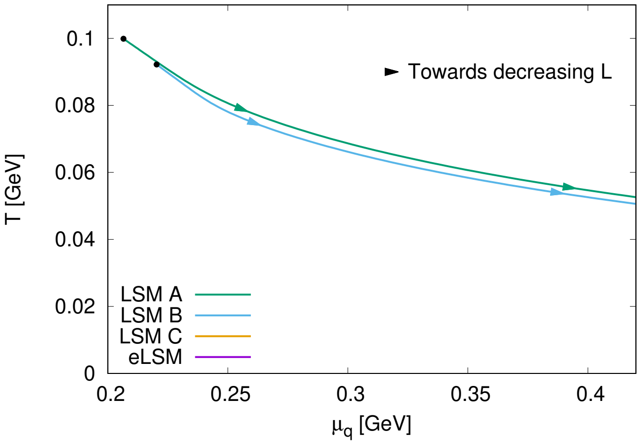
- **Fermionic vacuum contribution**: The solution is not stable for large ϕ_N and ϕ_S.
- With size-dependent vacuum contribution the condensates will increase for APBC (if $L > 1$ fm) and for PBC
- No common solution for the field equation for $L \lesssim 4.5$ fm.



- **Fermionic matter contribution**: Fermi-Dirac statistics to Fermi surface at $T = 0$
- Discrete modes with sharp Fermi surface: staircase-like transition.
- **Appears for QM and NJL models**





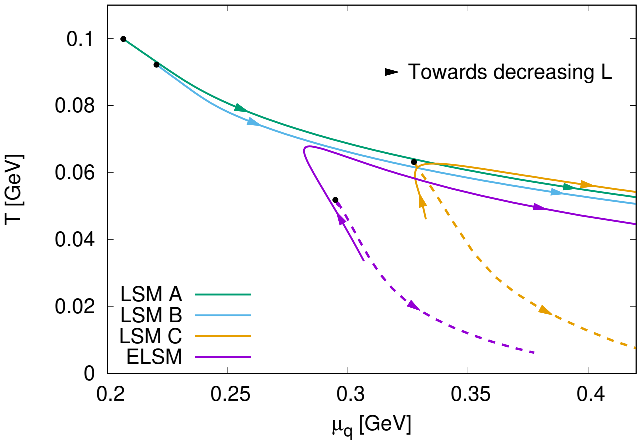


LSM A: J. Phys. G **38**, 085101 (2011)

LSM B, C: Phys. Rev. D **79**, 014018 (2009)

eLSM: Phys. Rev. D **93**, 114014 (2016)

- For "higher lying" models there is a continuous path
- For the "low lying" models the $L \to \infty$ CEP and the $L \to 0$ CEP is not continuously connected
- At $L < 2$ fm the CEP is governed by the first mode entering below the Fermi surface



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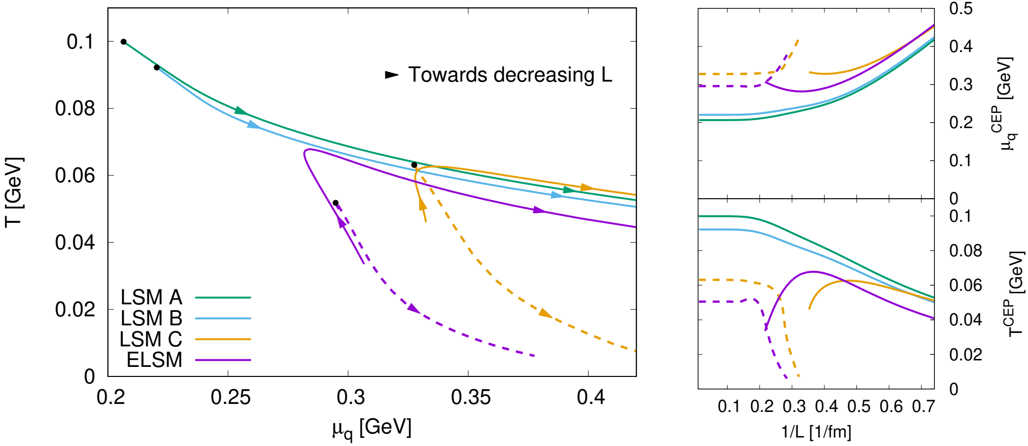
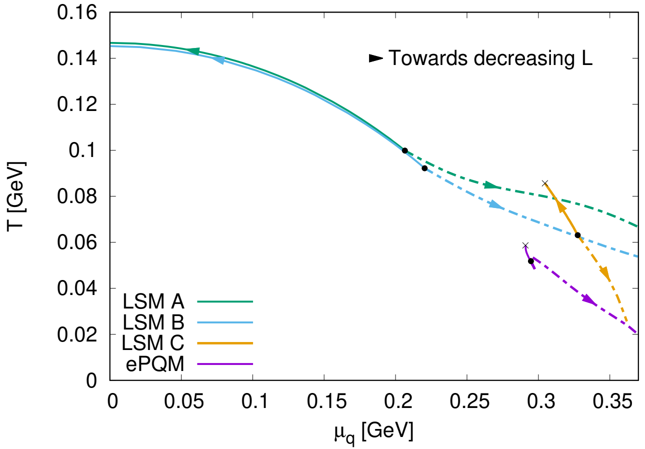


T [GeV]
0.1
0.08
0.06
0.04
0.02
0
0.2
0.25
0.3
0.35
0.4
μ_q [GeV]
► Towards decreasing L
LSM A
LSM B
LSM C
ELSM
μ_q^{CEP} [GeV]
0.5
0.4
0.3
0.2
0.1
0
T^{CEP} [GeV]
0.1
0.08
0.06
0.04
0.02
0
0.1 0.2 0.3 0.4 0.5 0.6 0.7
1/L [1/fm]

CEP with PBC (and the vacuum zero mode)



- With vacuum part of $L = \infty$ new unphysical first-order transition below $L \approx 5.5$ fm
- With (partially included) size-dependent vacuum part the trend is reversed.
- LSM A with zero mode added: J. Phys. G **38**, 085101 (2011)

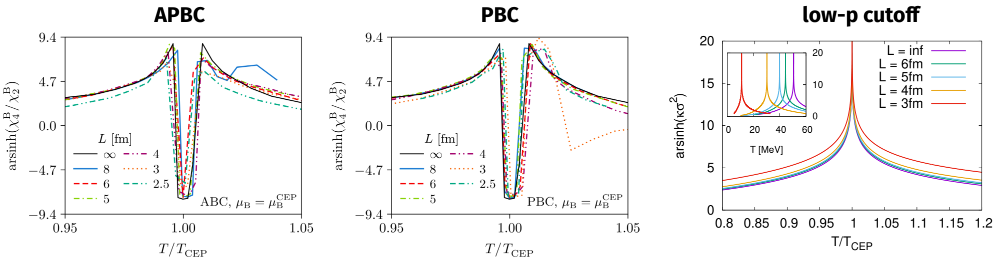
- Identify criticality in experiments with finite size, critical slow down, etc.: Quantities depending on a high power of the correlation length
- Conserved charge fluctuations $\rightarrow$ **Barion fluctuations**
- Ratio of generalized susceptibilities: no explicit size dependence

$$\chi_n = \left.\frac{\partial^n p/T^4}{\partial(\mu_q/T)^n}\right|_T, \qquad S\sigma = \frac{\chi_3}{\chi_2}, \quad \kappa\sigma^2 = \frac{\chi_4}{\chi_2} \tag{5}$$

skewness, kurtosis

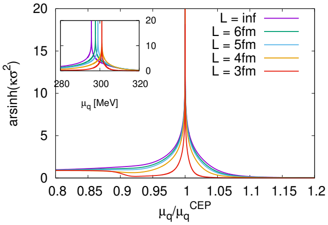
- Can be calculated and measured (not exactly the same quantities)

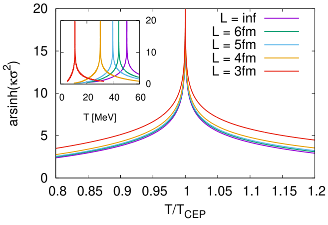
DS Phys.Rev.D **104** (2021) 7, 074035 eLSM Phys.Rev.D **108** (2023) 7, 076010



- No / moderate modification, only at very small sizes.
- Location of CEP might be important. Determines the modification in an absolute location.
- Note: divergence at the CEP if the resolution is good enough.

- The CEP is already very close to the μ_q axis.
- Inset: decreasing trend.





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- Location of CEP might be important. Determines the modification in an absolute location.
- Note: divergence at the CEP if the resolution is good enough.

Summary

- The CEP (most probably) moves to lower T and higher μ_q with the decreasing size.
- **The details of the finite size effects depend on the used momentum space constraint, the boundary condition, and the treatment of the vacuum part.**
- With low momentum cutoff, the size-dependent vacuum term leads to the reduction of the chirally broken phase, and the disappearance of the CEP when the size decreases, contrary to the case with unmodified vacuum contribution.
- With discretization the CEP will be determined by the modes entering below the Fermi surface. The location of the $L \to \infty$ CEP strongly affects its trajectory with the decreasing size. The vacuum part has an especially strong effect in the case of PBC.
- **Small effect on the baryon fluctuations, but the relocation of the CEP is important.**

Thank you!