Proton cumulants from hydrodynamics in light of new STAR data

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EMMI Workshop at the University of Wroclaw – Aspects of Criticality II

July 4, 2024

EMMI Workshop
at the
University of Wrocław
July 2 - 4, 2024, Wrocław, Poland
Aspects of
Criticality II

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QCD under extreme conditions

What we know

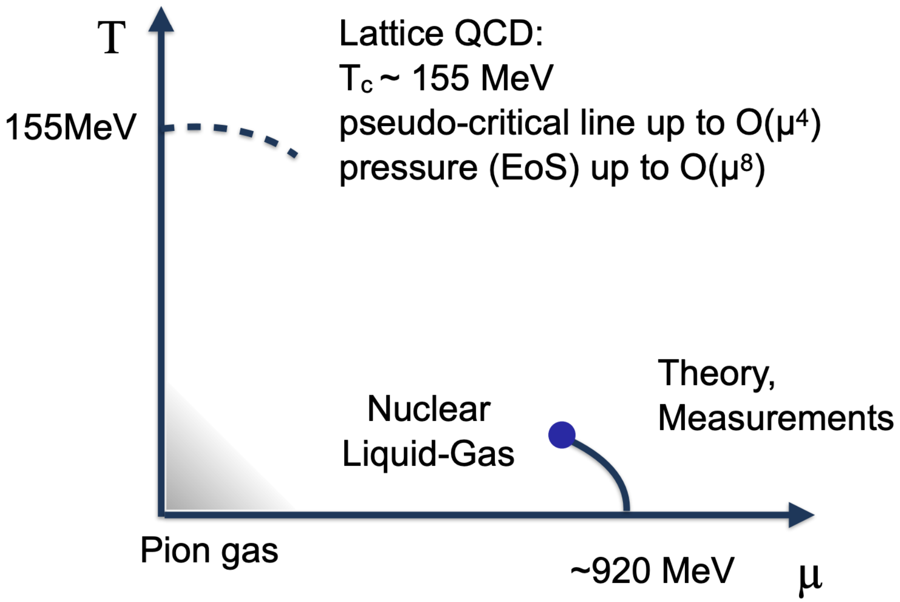

Figure courtesy of V. Koch

- Dilute hadron gas at low T & μ_{B} due to confinement, quark-gluon plasma high T & μ_{B}
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured
- Chiral crossover at $\mu_B = 0$

QCD under extreme conditions

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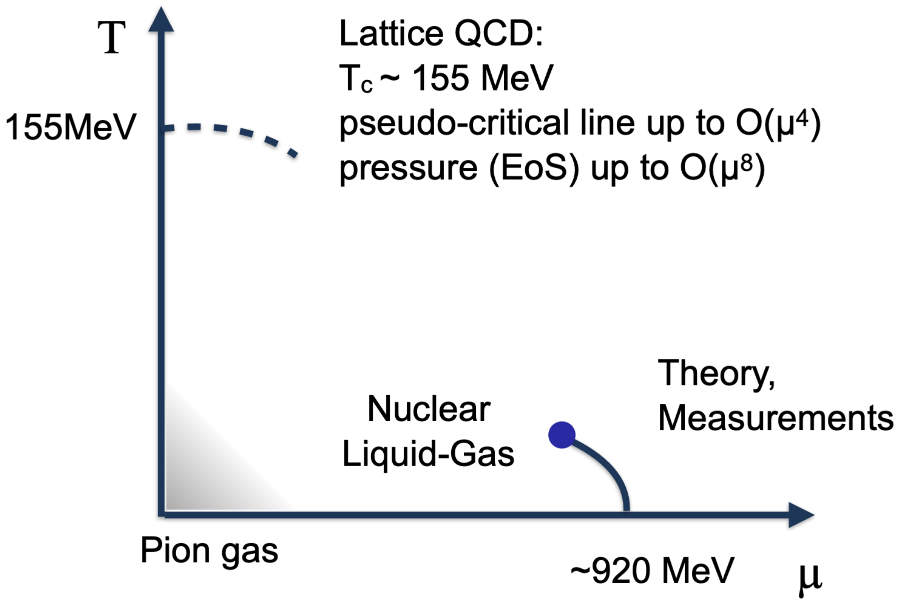

Lattice QCD:
T_c ~ 155 MeV
pseudo-critical line up to $O(\mu^4)$
pressure (EoS) up to $O(\mu^8)$

Figure courtesy of V. Koch

What we hope to know

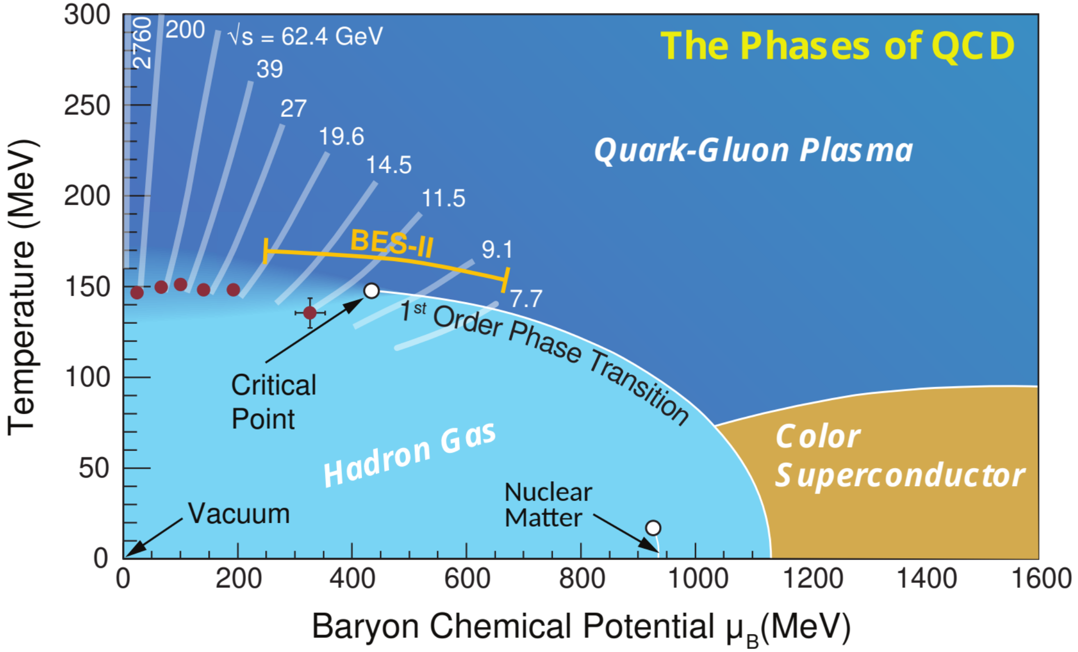

Figure from Bzdak et al., Phys. Rept. '20 & 2015 Long Range Plan

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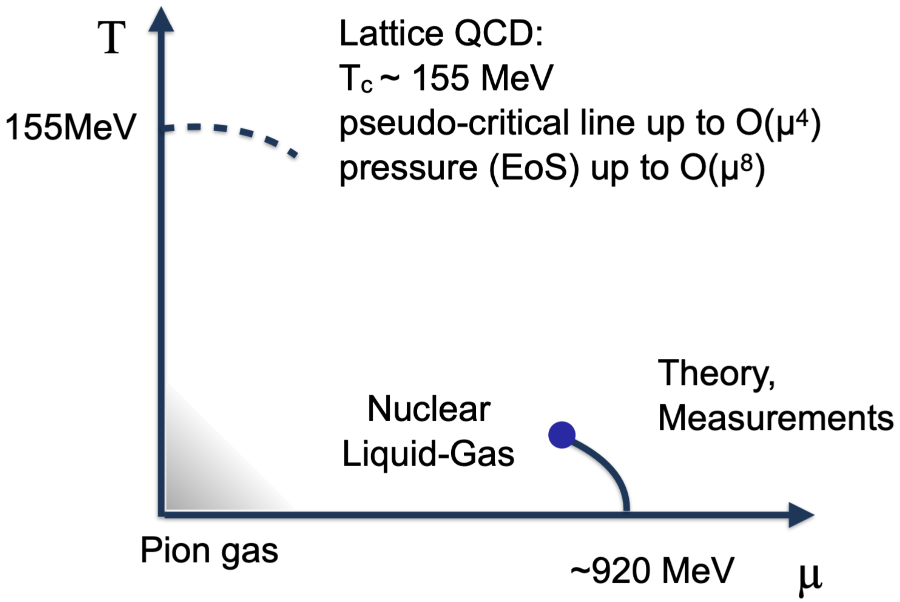

Lattice QCD:
$T_c \sim 155$ MeV
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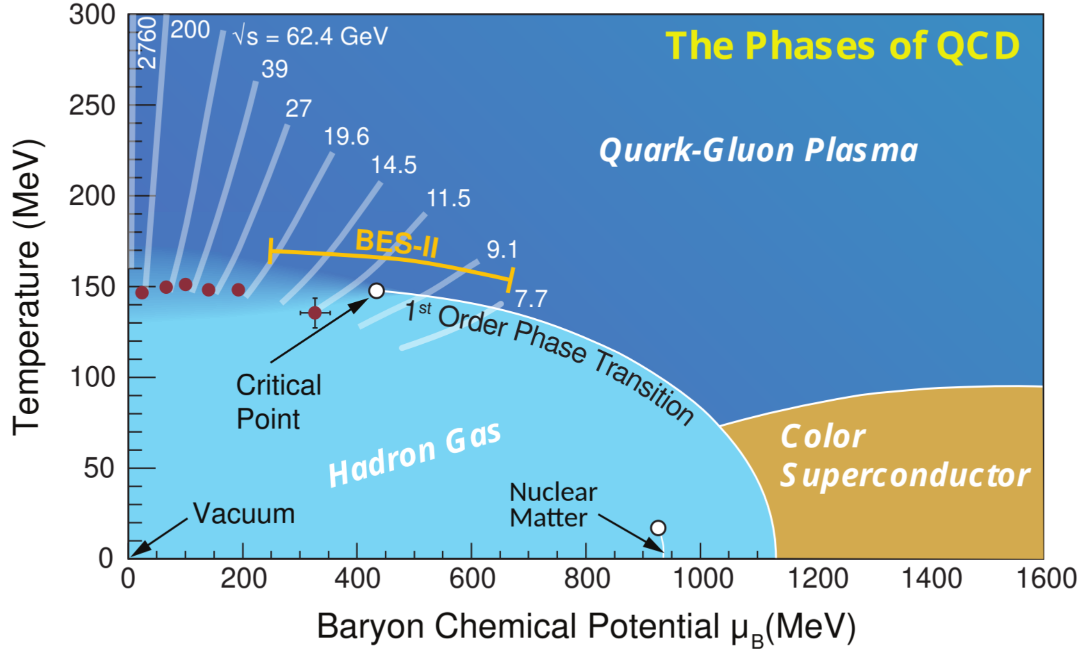
What we hope to know



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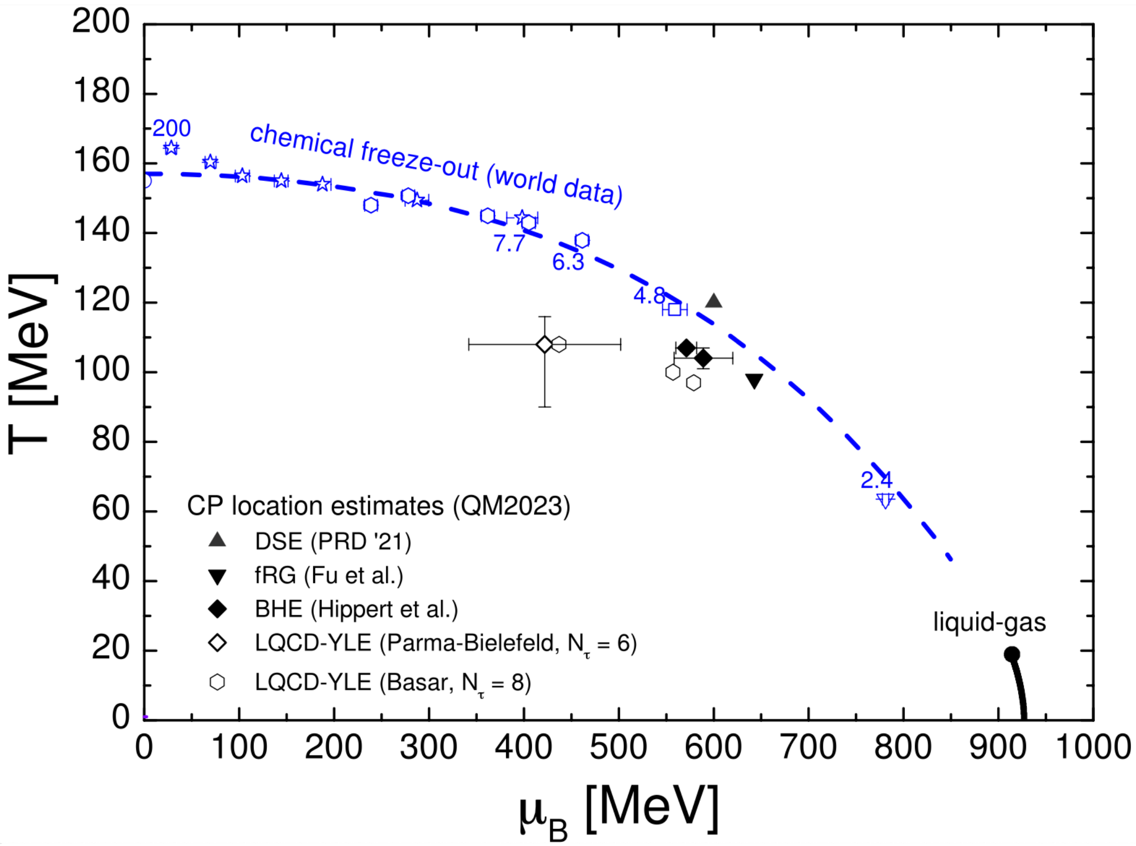
Key question: *Is there a QCD critical point and how to find it?*

Search for critical point with heavy-ion collisions

Control parameters

- Collision energy $\sqrt{s_{NN}} = 2.4 - 5020$ GeV
  - Scan the QCD phase diagram
- Size of the collision region
  - Expect stronger signal in larger systems



A. Lysenko, Poberezhnyuk, Gorenstein, VV, in preparation

Measurements

- Final hadron abundances and momentum distributions **event-by-event**

Chemical freeze-out curve and CP

- Sets lower bound on the temperature of the CP
- **Caveats:** strangeness neutrality ($\mu_S \neq 0$), uncertainty in the freeze-out curve

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

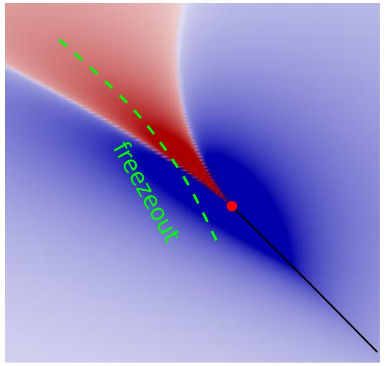
Grand partition function

$$\ln Z^{\text{gce}}(T,V,\mu) = \ln\left[\sum_N e^{\mu N/T} Z^{\text{ce}}(T,V,N)\right]$$

$$\boxed{\kappa_n \propto \frac{\partial^n(\ln Z^{\text{gce}})}{\partial\mu^n}}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- **(QCD) critical point:** large correlation length and fluctuations



M. Stephanov, PRL '09, '11
Energy scans at RHIC (STAR)
and CERN-SPS (NA61/SHINE)

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

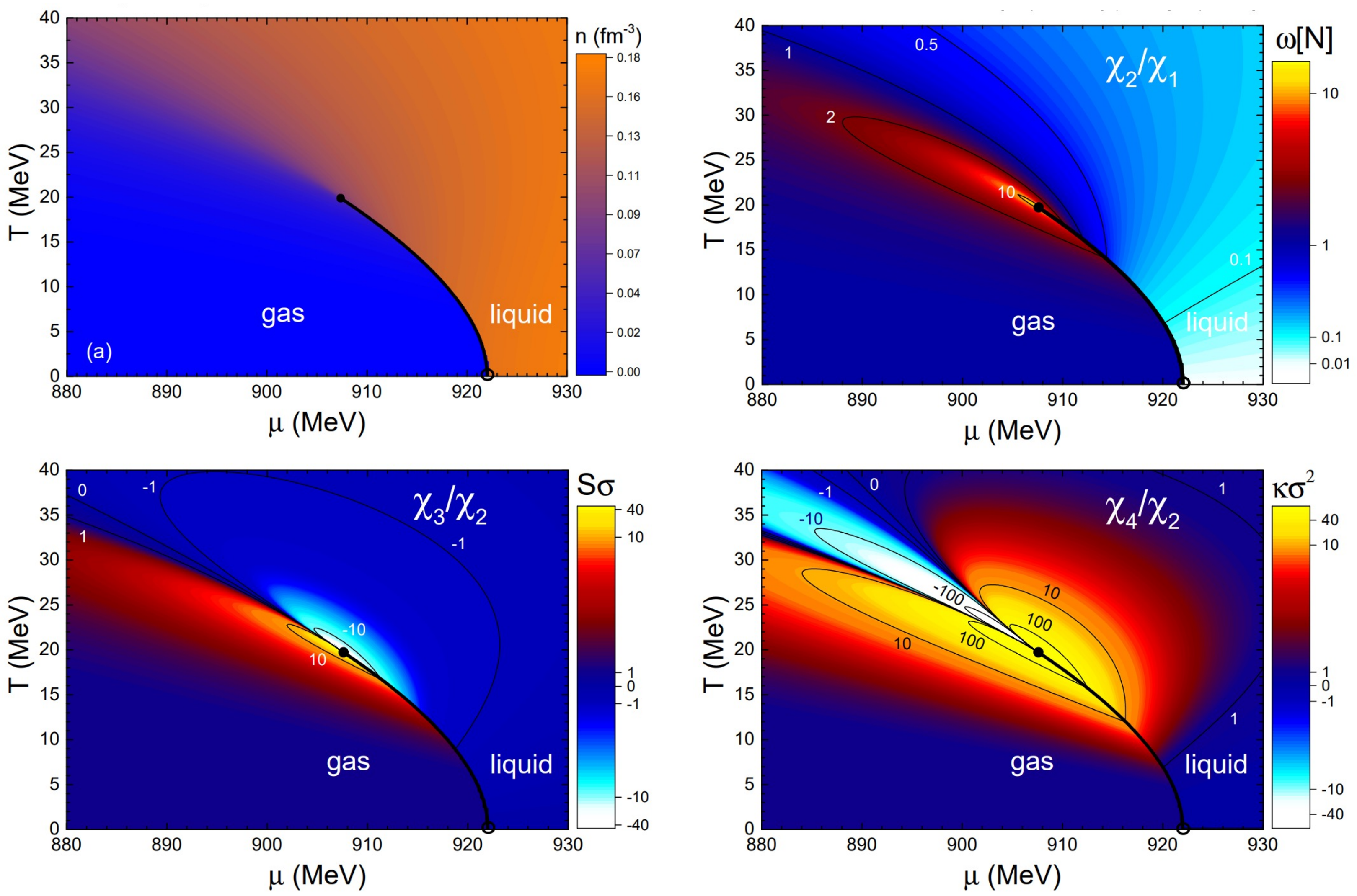
$$\xi \to \infty$$

Looking for enhanced fluctuations and non-monotonicities

Other uses of cumulants:

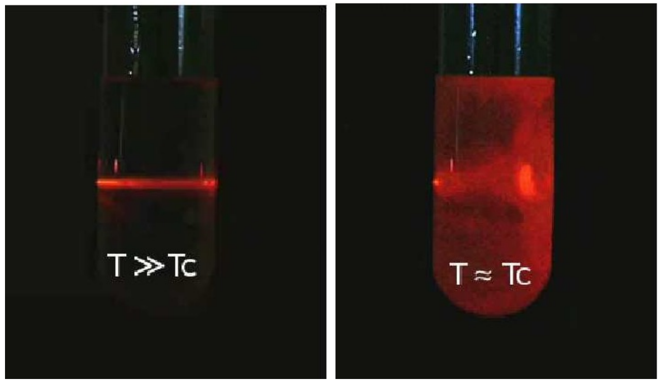
- QCD degrees of freedom
  Jeon, Koch, PRL 85, 2076 (2000)
  Asakawa, Heinz, Muller, PRL 85, 2072 (2000)
- Extracting the speed of sound
  A. Sorensen et al., PRL 127, 042303 (2021)
- Conservation volume V_C
  VV, Donigus, Stoecker, PRC 100, 054906 (2019)

Example: (Nuclear) Liquid-gas transition



VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Critical opalescence



$$\langle N^2\rangle - \langle N\rangle^2 \sim \langle N\rangle \sim 10^{23}$$

in equilibrium

Example: Critical fluctuations in a microscopic simulation

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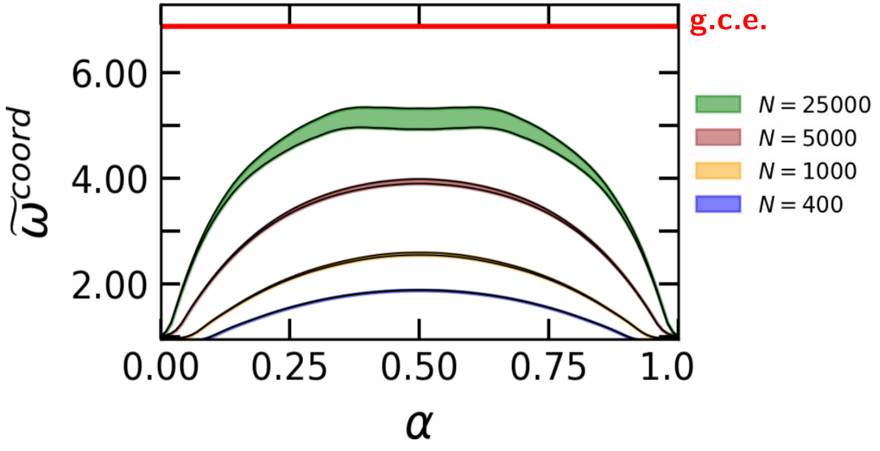
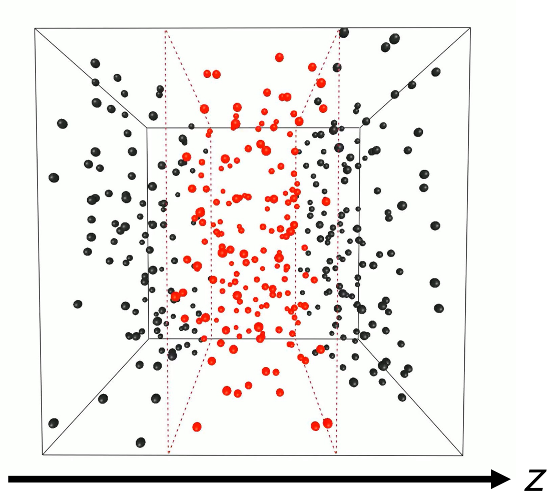
V. Kuznietsov et al., Phys. Rev. C 105, 044903 (2022)

Classical molecular dynamics simulations of the **Lennard-Jones fluid** near Z(2) critical point ($T \approx 1.06 T_c$, $n \approx n_c$) of the liquid-gas transition

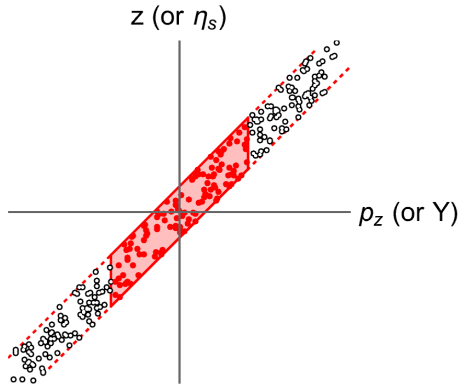
Scaled variance in coordinate space acceptance $|z| < z^{max}$

$$\boxed{\tilde{\omega}^{\text{coord}} = \frac{1}{1-\alpha} \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}}$$





Heavy-ion collisions:
flow correlates p_z and z cuts



- Large fluctuations survive despite strong finite-size effects
- Need coordinate space cuts (collective flow helps)
- Here no finite-time effects

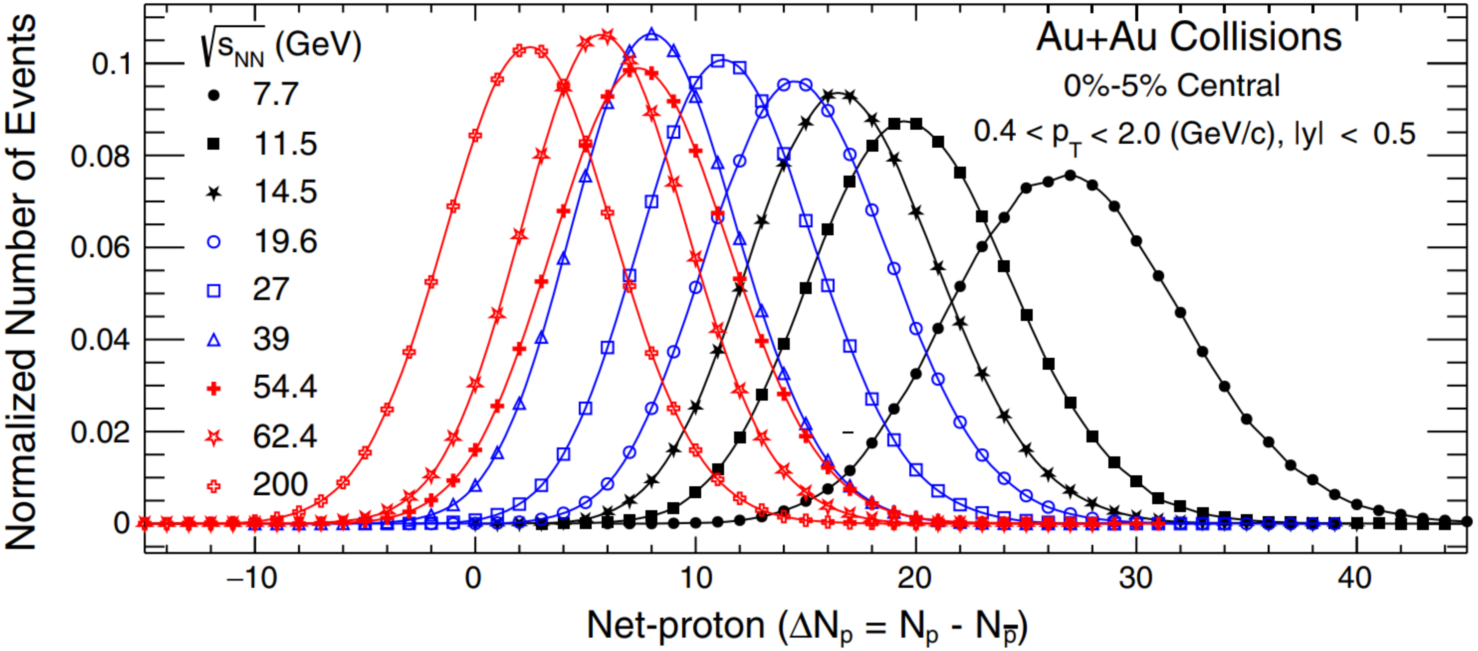
Collective flow and finite-time effects explored in V. Kuznietsov et al., arXiv:2404.00476

Measuring cumulants in heavy-ion collisions

Count the number of events with given number of e.g. (net) protons

$$P(\Delta N_p) \sim \frac{N_{\text{events}}(\Delta N_p)}{N_{events}^{total}}$$

STAR Collaboration, Phys. Rev. Lett. 126, 092301 (2021)



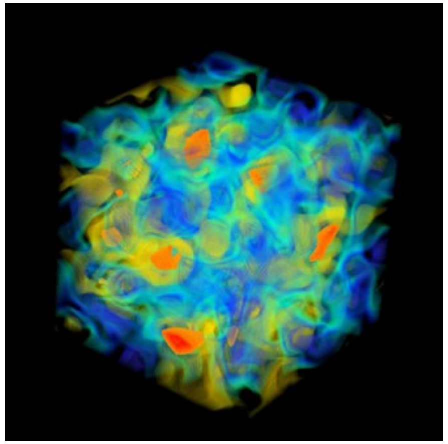
Cumulants are extensive, $\kappa_n \sim V$, use ratios to cancel out the volume

$$\frac{\kappa_2}{\langle N \rangle}, \qquad \frac{\kappa_3}{\kappa_2}, \qquad \frac{\kappa_4}{\kappa_2}$$

Look for subtle critical point signals (tails of the distribution)

Theory vs experiment: Challenges for fluctuations

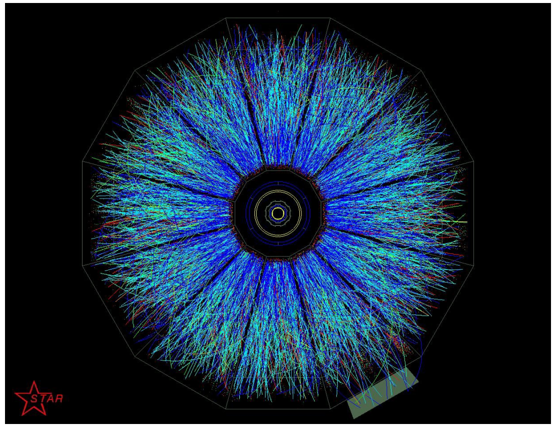
Theory



© Lattice QCD@BNL

- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

Experiment



STAR event display

- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

Need dynamical description

Coordinate vs Momentum space

V. Kuznietsov et al., arXiv:2404.00476

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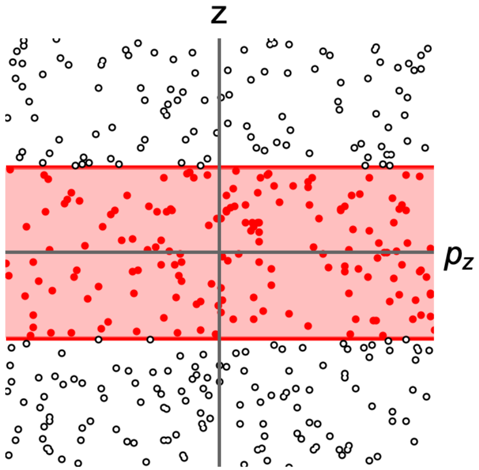
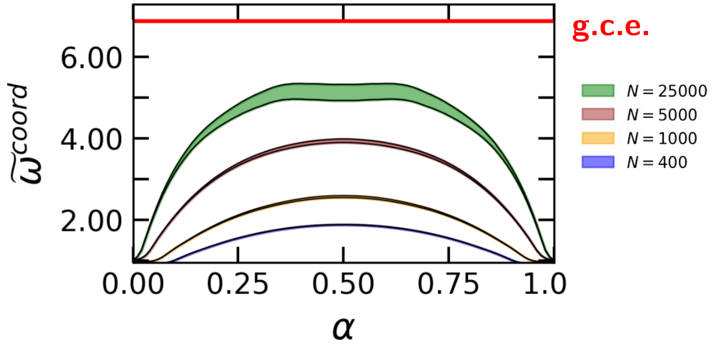
Box setup: Coordinates and momenta are uncorrelated

HICs: Flow (e.g. Bjorken)

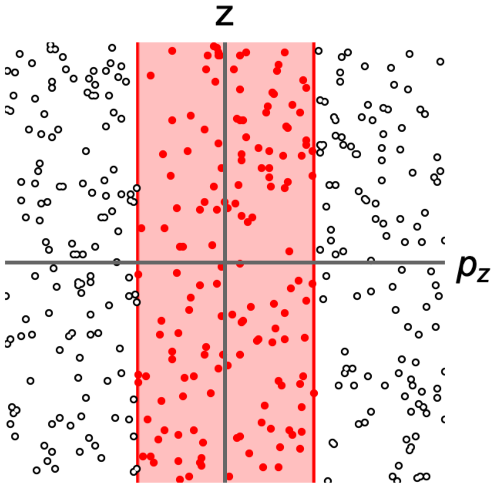
Coordinate space cut

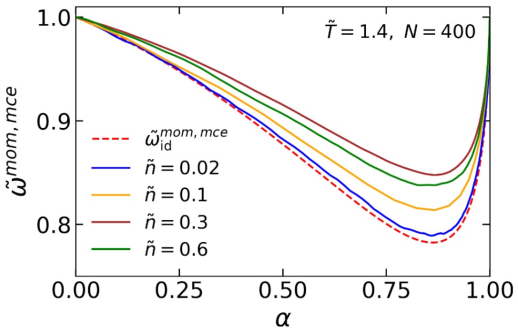




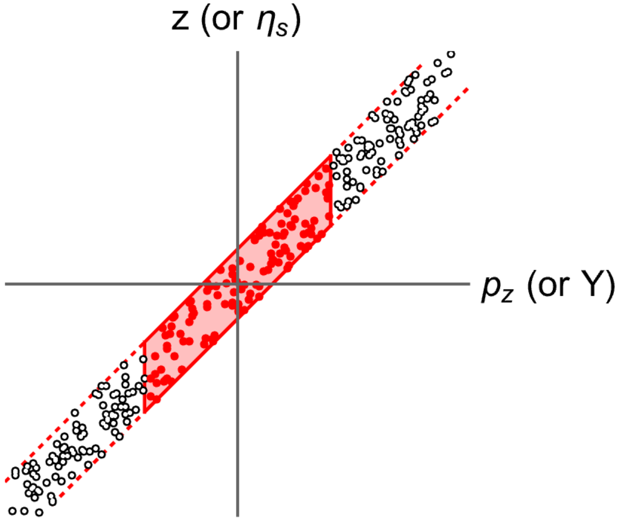
Large correlations

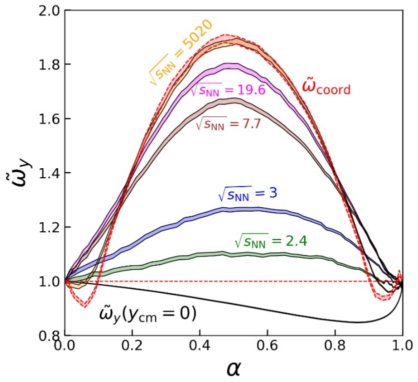
Momentum space cut





Nothing left





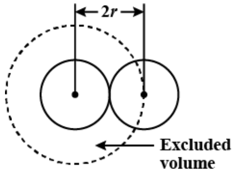
momentum cut $\sim$ coordinate cut + smearing

Dynamical approaches to the QCD critical point search

1. Dynamical model calculations of critical fluctuations [X. An et al., Nucl. Phys. A 1017, 122343 (2022)]
    - Fluctuating hydrodynamics (hydro+) and (non-equilibrium) evolution of fluctuations
    - Equation of state with a tunable critical point [P. Parotto et al, PRC 101, 034901 (2020); J. Karthein et al., EPJ Plus 136, 621 (2021)]
    - Generalized Cooper-Frye particlization [M. Pradeep, et al., PRD 106, 036017 (2022); PRL 130, 162301 (2023)]

Alternatives at high μ_B: hadronic transport/molecular dynamics with a critical point

[A. Sorensen, V. Koch, PRC 104, 034904 (2021); V. Kuznietsov et al., PRC 105, 044903 (2022)]



2. **Deviations from precision calculations of non-critical fluctuations**
    - Non-critical baseline is not flat [Braun-Munzinger et al., NPA 1008, 122141 (2021)]
    - Include essential non-critical contributions to (net-)proton number cumulants
    - Exact baryon conservation + hadronic interactions (hard core repulsion)
    - Based on realistic hydrodynamic simulations tuned to bulk data

[VV, C. Shen, V. Koch, Phys. Rev. C 105, 014904 (2022)]

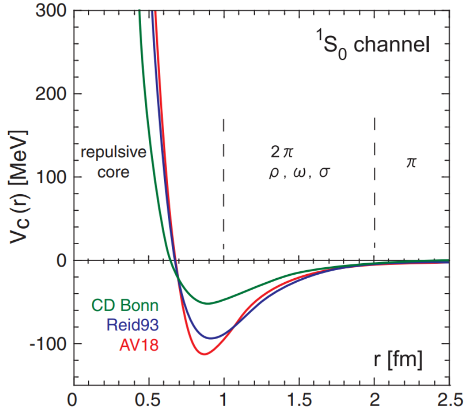
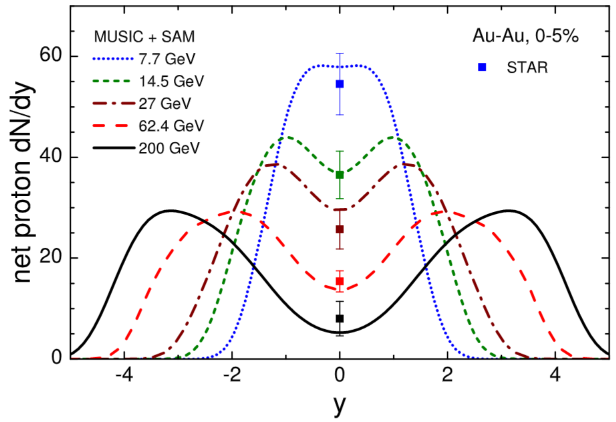


Figure from Ishii et al., PRL '07

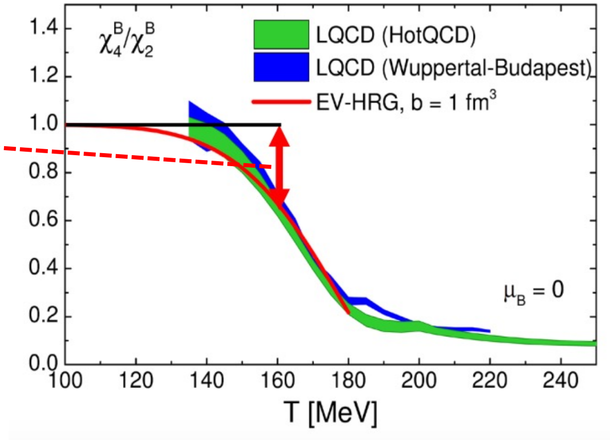
Calculation of non-critical contributions

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

- (3+1)-D viscous hydrodynamics evolution (MUSIC-3.0)
  - Collision geometry-based 3D initial state [Shen, Alzhrani, PRC 102, 014909 (2020)]
  - Crossover equation of state based on lattice QCD [Monnai, Schenke, Shen, Phys. Rev. C 100, 024907 (2019)]
  - Cooper-Frye particlization at $\epsilon_{sw} = 0.26$ GeV/fm^3



- Non-critical contributions are computed at particlization
  - QCD-like baryon number distribution (χ_n^B) via excluded volume b = 1 fm^3 [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]
  - Exact global baryon conservation* (and other charges)
    - Subensemble acceptance method 2.0 (analytic) [VV, Phys. Rev. C 105, 014903 (2022)]
    - or FIST sampler (Monte Carlo) [VV, Phys. Rev. C 106, 064906 (2022)] https://github.com/vlvovch/fist-sampler



- Absent: critical point, local conservation, initial-state/volume fluctuations, hadronic phase

*If baryon conservation is the only effect (no other correlations), non-critical baseline can be computed without hydro
Braun-Munzinger, Friman, Redlich, Rustamov, Stachel, NPA 1008, 122141 (2021)

Cumulants and n-point correlation functions

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Cooper-Frye formula:

$$\omega_p \frac{dN_j}{d^3p} = \int_{\sigma(x)} d\sigma_\mu(x)\, p^\mu\, f_j[u^\mu(x)p_\mu; T(x), \mu_j(x)]$$

To calculate cumulants introduce **n-point density-density correlation function $C_n^B(x)$** (derived from SAM-2.0*)

$$C_1^B(x_1) = \chi_1^B(x_1),$$

$$C_2^B(x_1, x_2) = \underbrace{\chi_2^B(x_1)\,\delta(x_1 - x_2)}_{\text{local correlation}} - \underbrace{\frac{\chi_2^B(x_1)\chi_2^B(x_2)}{\int_{\sigma(x)} d\sigma_\mu(x) u^\mu(x)\, \chi_2^B(x)}}_{\substack{\text{balancing contribution}\\ \text{(baryon conservation)}}},$$

$$\ldots$$

Global baryon conservation:

$$\int d\sigma_\mu(x_i) u^\mu(x_i) C_n^B(x_1, \ldots, x_n) = 0 \quad \text{for} \quad n > 1$$

Generalized Cooper-Frye:

$$\kappa_n^B = \prod_{i=1}^{n} \int_{x_i \in \sigma(x)} d\sigma_\mu(x_i) \int_{|y_i|<0.5,\, 0.4<p_T<2} \frac{d^3p_i}{\omega_{p_i}} p_i^\mu \exp\left[-\frac{p_i^\mu u_\mu(x_i)}{T(x_i)}\right] C_n^B(x_1, \ldots, x_n)$$

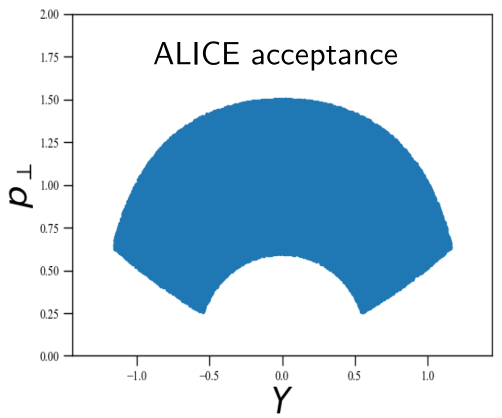
*VV et al., Phys. Rev. C 105, 014903 (2022) & in preparation

Net-particle fluctuations at the LHC (blast-wave model)

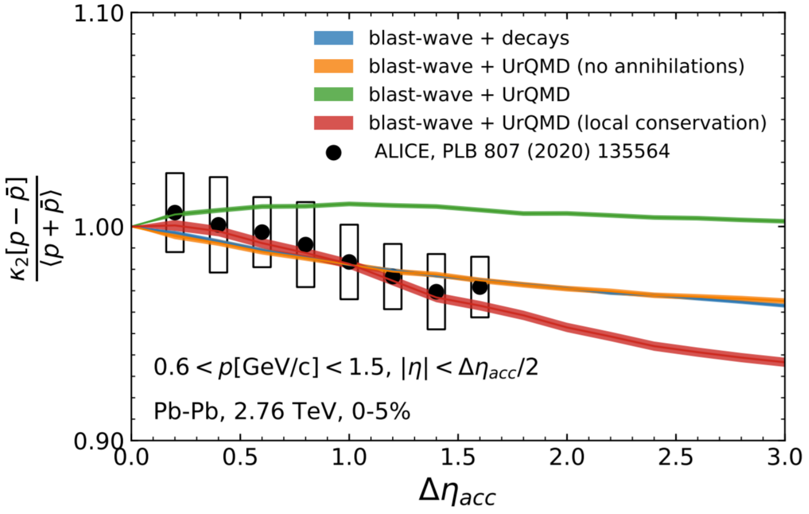
- Net protons described within errors and consistent with either
  - global baryon conservation only, without $B\bar{B}$ annihilations

    ALICE Collaboration, Phys. Lett. B 807, 135564 (2020)
  - or local baryon conservation with $B\bar{B}$ annihilations

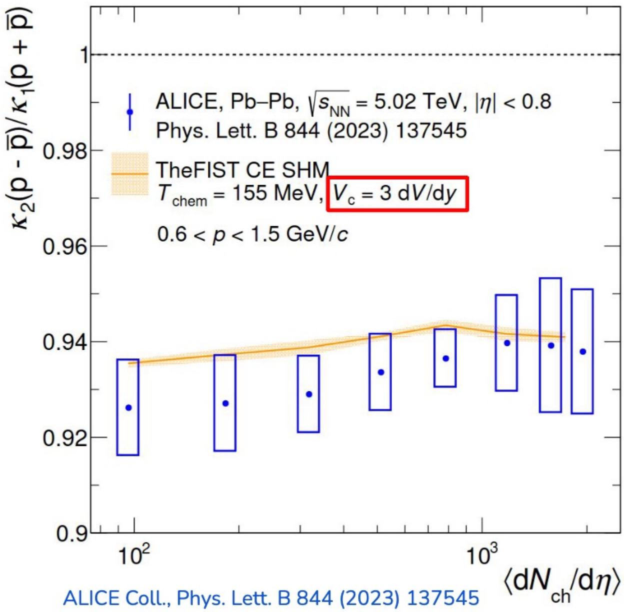
    O. Savchuk et al., Phys. Lett. B 827, 136983 (2022)



$0.6 < p < 1.5$ GeV/c, $\Delta\eta_{acc} = 1.6$



multiplicity dependence



ALICE Coll., Phys. Lett. B 844 (2023) 137545

- Local conservation $V_C \sim 3dV/dy$ preferred by hadron yields* and several other cumulant measurements** (net-Xi, net-Lambda, pQK, …)

  *VV, Donigus, Stoecker, PRC 100, 054906 (2019); **Talks by M. Ciacco & S. Saha (SQM2024)

Local baryon conservation from density correlator

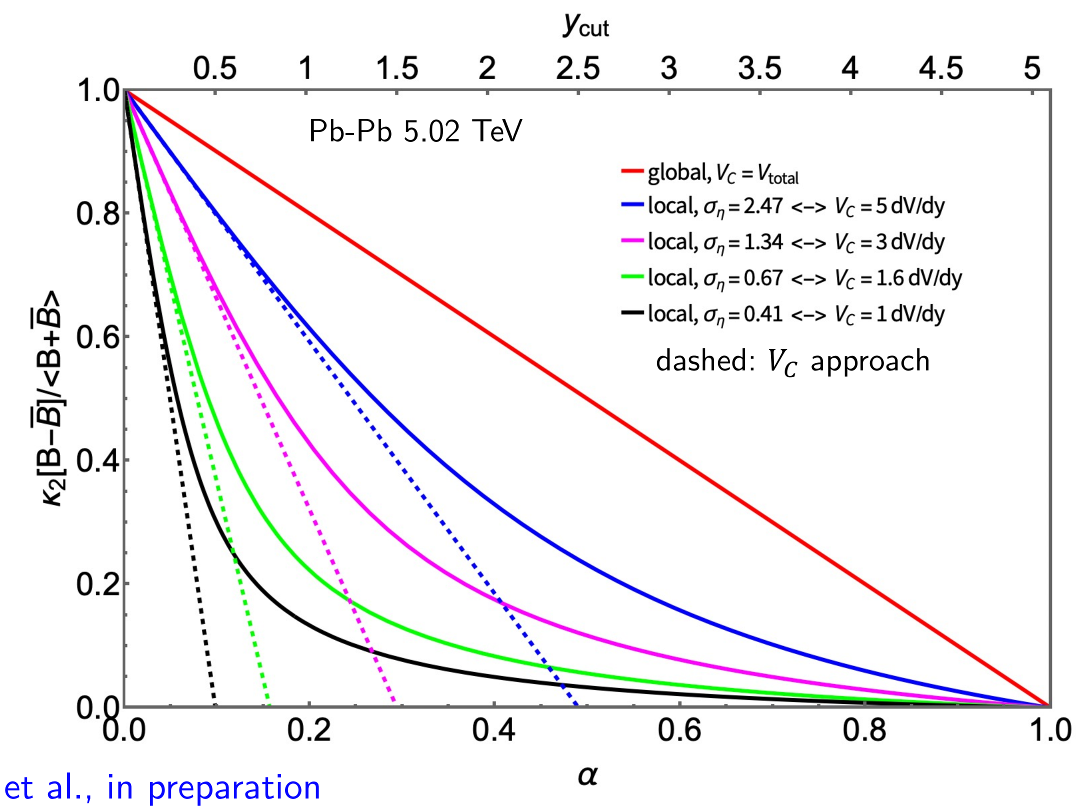
Introduce Gaussian (space-time) rapidity correlation into baryon-conservation balancing term

global conservation

$$C_2^B(\eta_1,\eta_2) = \langle n_B + n_{\bar{B}} \rangle \left[\delta(\eta_1 - \eta_2) - \frac{1}{2\eta_{\max}} \right]$$

+ local conservation

$$C_2^B(\eta_1,\eta_2) = \langle n_B + n_{\bar{B}} \rangle \left[\delta(\eta_1 - \eta_2) - \frac{\tilde{A}\, e^{-\frac{(\eta_1-\eta_2)^2}{2\sigma_\eta^2}}}{2\eta_{\max}} \right]$$



VV et al., in preparation

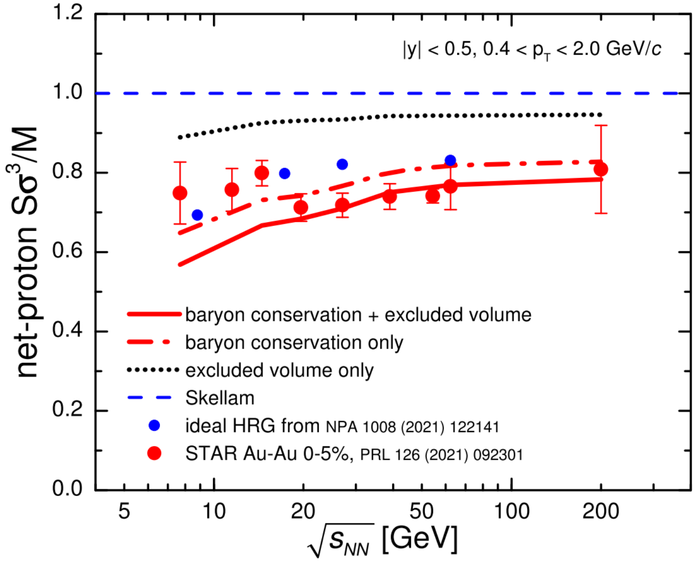
- Extra suppression due to local conservation, similar to studies in the literature

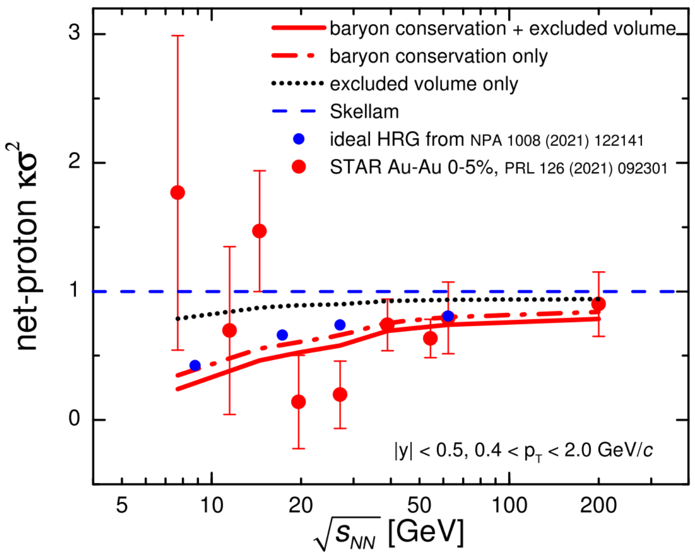
P. Braun-Munzinger et al., arXiv:2312.15534

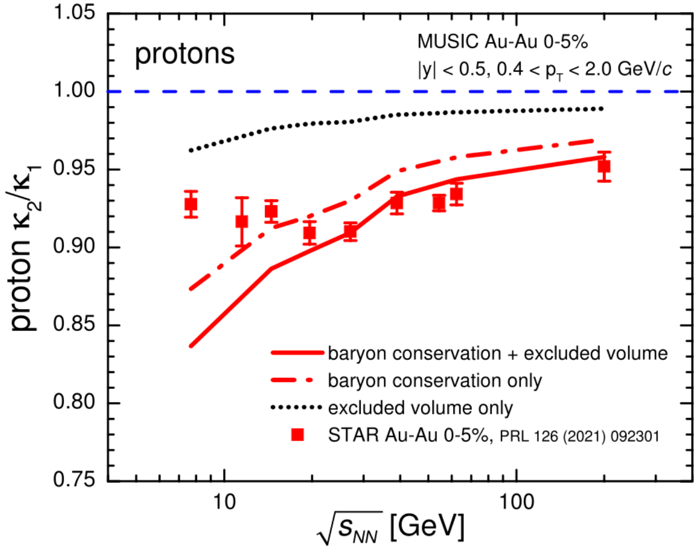
- Linear regime at small alpha establishes connection to the V_C approach
    - $V_C = k\,dV/dy$, $k \approx \sqrt{2\pi}\sigma_\eta$
    - V_C approach breaks down at large y_{cut}, remains accurate around midrapidity

RHIC-BES-I: Net proton cumulant ratios (MUSIC)

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)


net proton C_3/C_1


net proton C_4/C_2

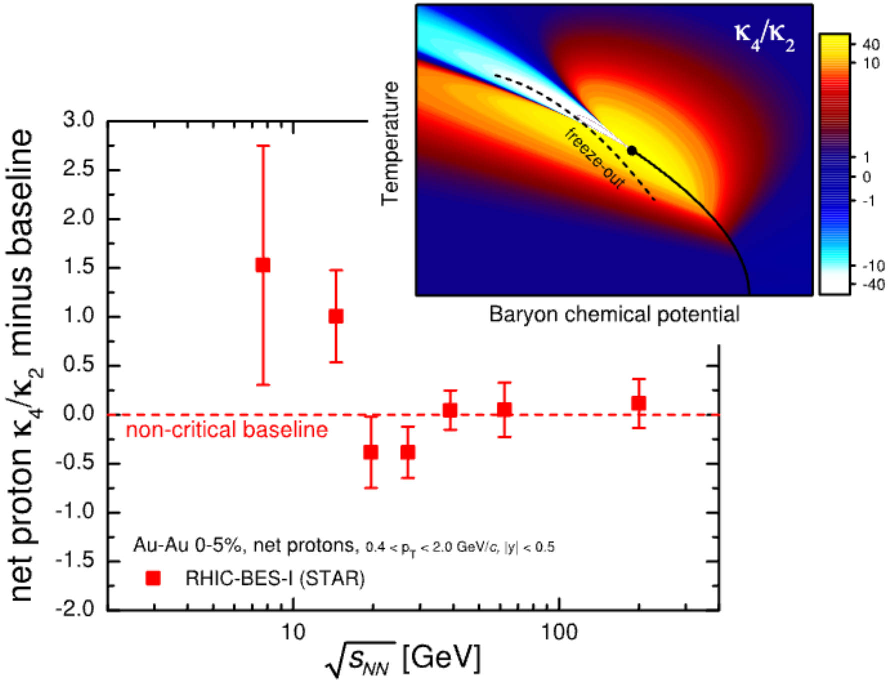

proton C_2/C_1

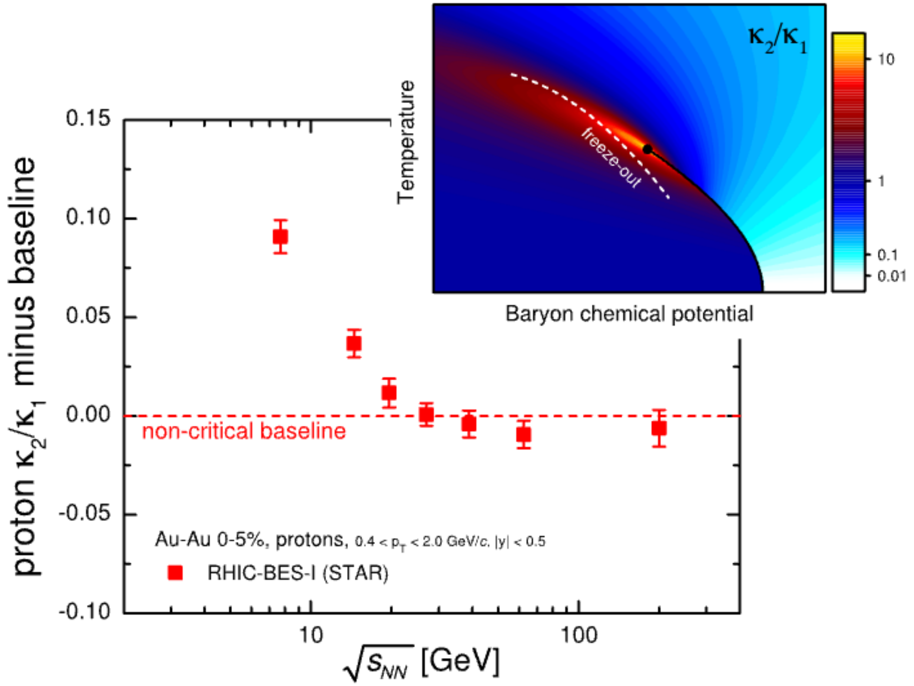
- Data at $\sqrt{s_{NN}} \geq 20$ GeV consistent with non-critical physics (BQS conservation and repulsion)
- Effect from baryon conservation is stronger than repulsion but both are required at $\sqrt{s_{NN}} \geq 20$ GeV
- Deviations from baseline at lower energies?

Hints from RHIC-BES-I

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

Subtracting the hydro baseline

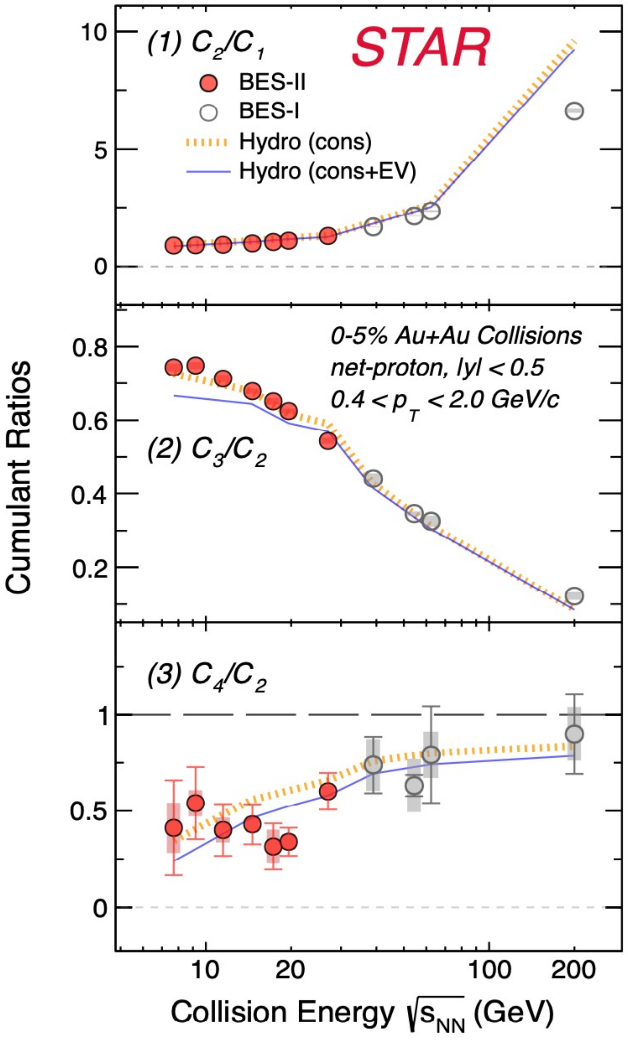




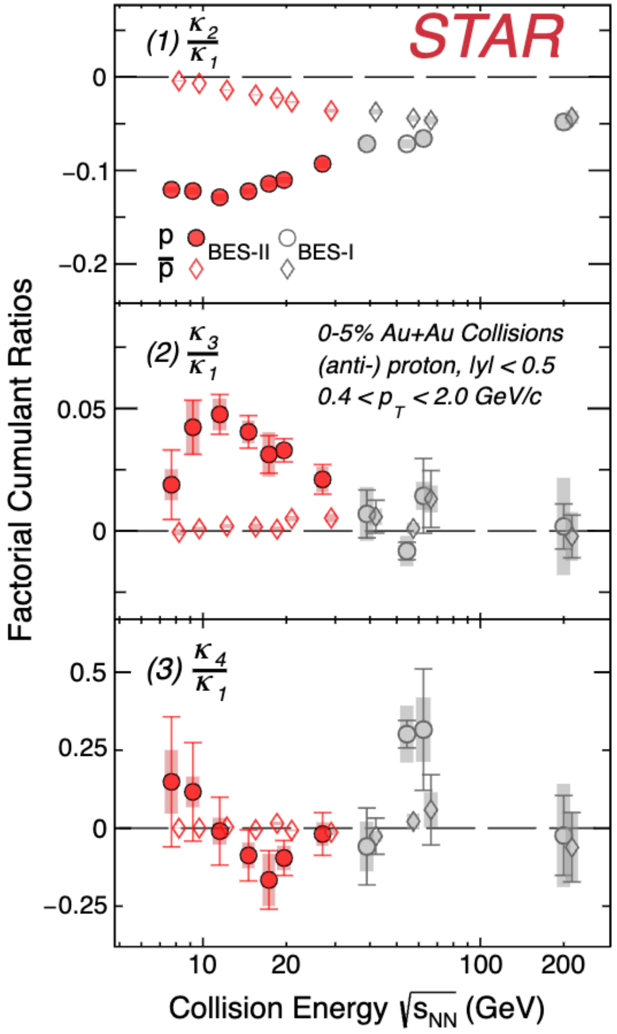
⚠ **Notation:** Here we use κ_n for cumulants and $\hat{C}_n$ for factorial cumulants, STAR uses the opposite

RHIC-BES-II data

A. Pandav, CPOD2024



Net-proton cumulant ratios



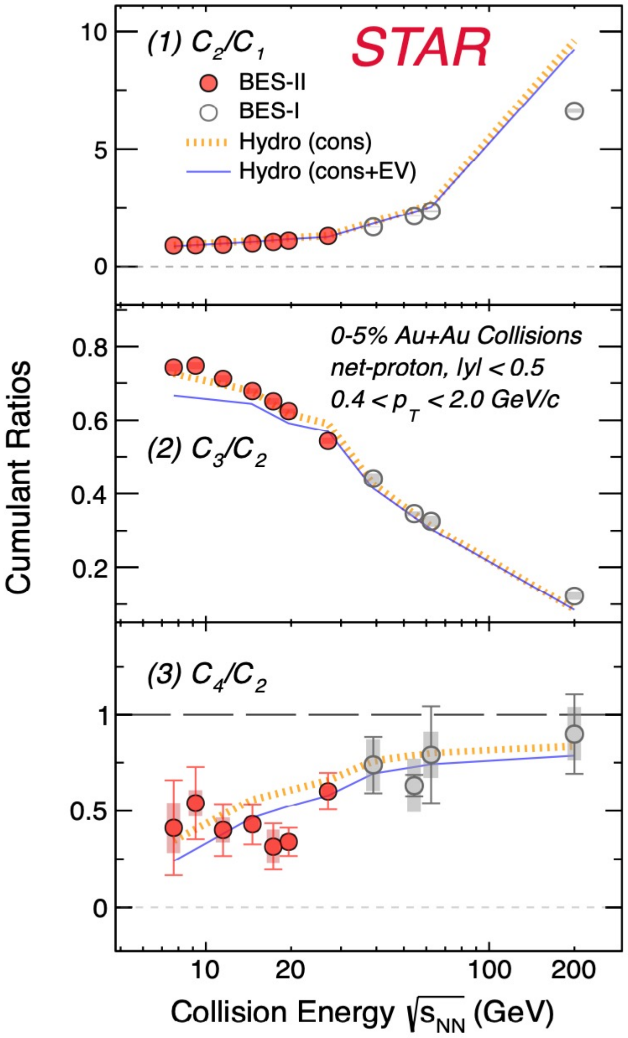
Proton/antiproton factorial cumulant ratios

- No smoking gun signature for CP in ordinary cumulants

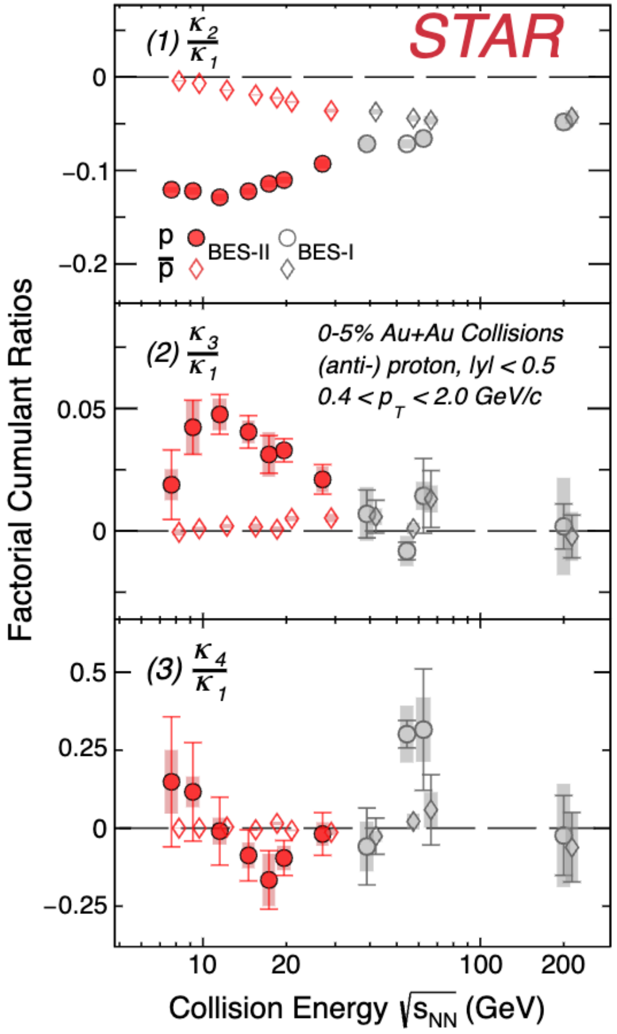
RHIC-BES-II data

A. Pandav, CPOD2024

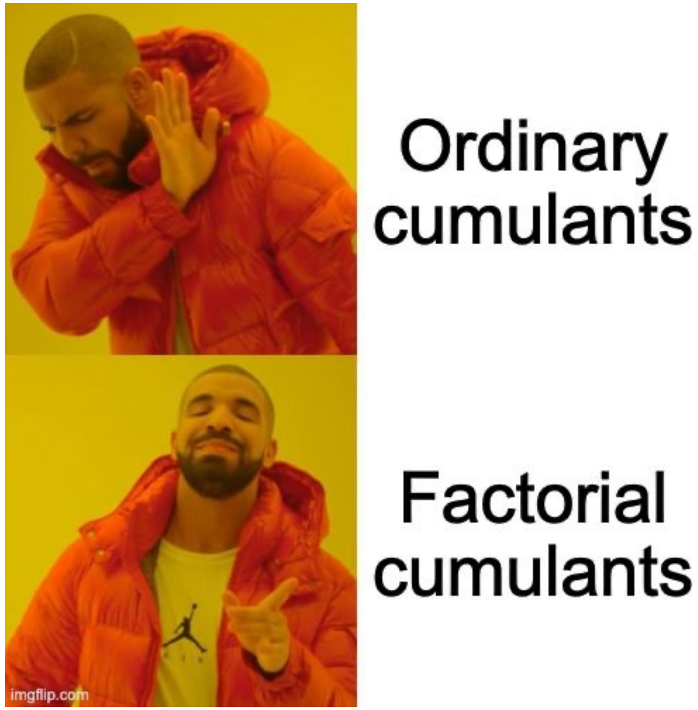
Net-proton cumulant ratios



Proton/antiproton factorial cumulant ratios



- No smoking gun signature for CP in ordinary cumulants
- More structure seen in factorial cumulants



What are factorial cumulants?

Factorial cumulants $\hat{C}_n$ vs ordinary cumulants C_n

Factorial cumulants: ~irreducible n-particle corr.

$$\hat{C}_n \sim \langle N(N-1)(N-2)\ldots\rangle_c$$

$$\hat{C}_1 = C_1$$
$$\hat{C}_2 = C_2 - C_1$$
$$\hat{C}_3 = C_3 - 3C_2 + 2C_1$$
$$\hat{C}_4 = C_4 - 6C_3 + 11C_2 - 6C_1$$

[Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017); Kitazawa, Luo, PRC 96, 024910 (2017)]

Ordinary cumulants: mix corrs. of different orders

$$C_n \sim \langle \delta N^n \rangle_c$$

$$C_1 = \hat{C}_1$$
$$C_2 = \hat{C}_2 + \hat{C}_1$$
$$C_3 = \hat{C}_3 + 3\hat{C}_2 + \hat{C}_1$$
$$C_4 = \hat{C}_4 + 6\hat{C}_3 + 7\hat{C}_2 + \hat{C}_1$$

Factorial cumulants and different physics mechanisms

- Baryon conservation [Bzdak, Koch, Skokov, EPJC '17] — $\hat{C}_n^{\text{cons}} \propto (\hat{C}_1)^n / \langle N_{\text{tot}} \rangle^{n-1}$ *small*
- Excluded volume [VV et al, PLB '17] — $\hat{C}_n^{\text{EV}} \propto b^n$ *small*
- Volume fluctuations [Holzman et al., arXiv:2403.03598] — $\hat{C}_n^{CF} \sim (\hat{C}_1)^n \kappa_n[V]$ *depends on Vfluc*
- Critical point [Ling, Stephanov, PRC '16] — $\hat{C}_2^{CP} \sim \xi^2, \quad \hat{C}_3^{CP} \sim \xi^{4.5}, \quad \hat{C}_4^{CP} \sim \xi^7$ ***large***

- proton vs baryon [Kitazawa, Asakawa, PRC '12] — $\hat{C}_n^B \sim 2^n \times \hat{C}_n^p$ **same sign!**

Factorial cumulants and long-range correlations

Long-range correlations: $\boxed{\frac{\hat{C}_n}{(\hat{C}_1)^n} = \text{const.}}$ at given $\sqrt{s_{NN}}$

- Global (not local) baryon conservation

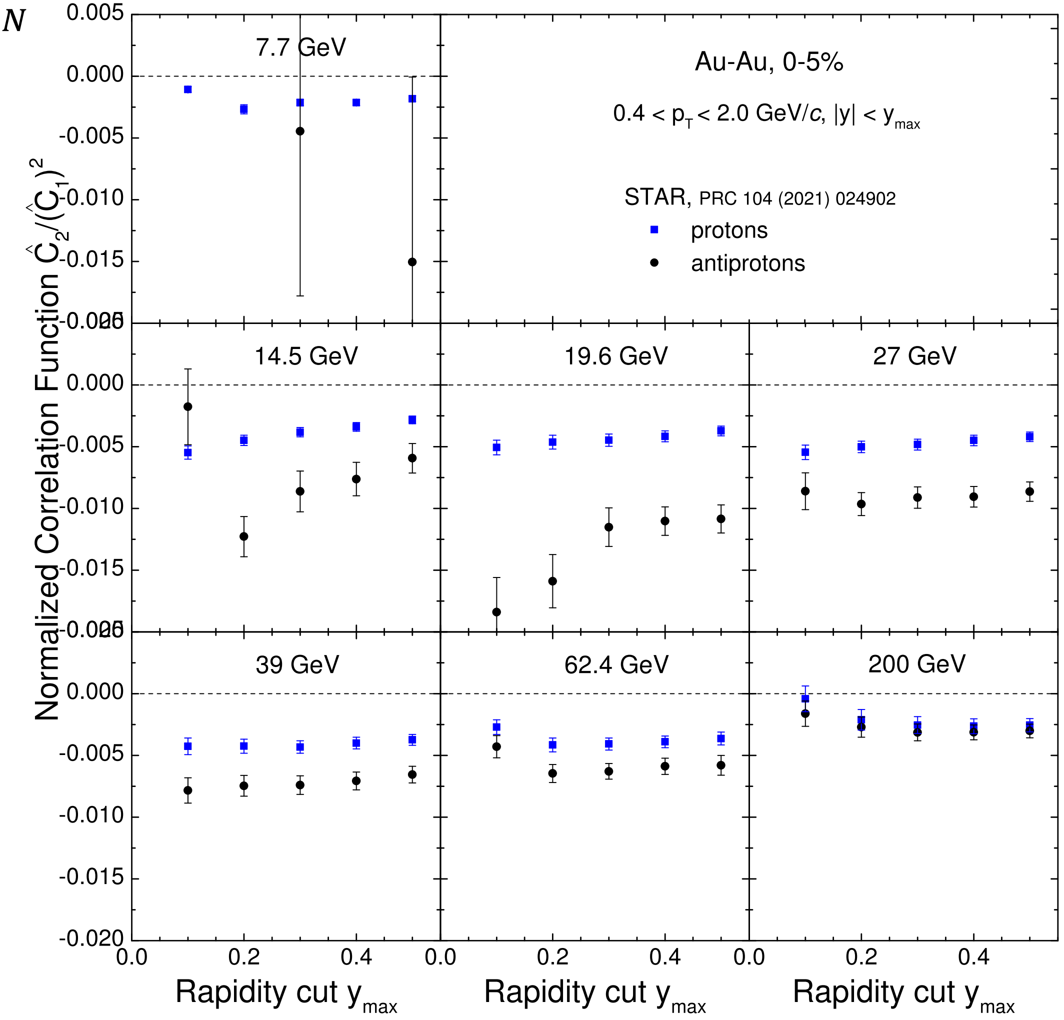
[Bzdak, Koch, Skokov, EPJC 77, 288 (2017)]

- $+$ volume fluctuations

[Holzmann, Koch, Rustamov, Stroth, arXiv:2403.03598]

In particular $\boxed{\frac{\hat{C}_2^p}{(\hat{C}_1^p)^2} \approx \frac{\hat{C}_2^{\bar{p}}}{(\hat{C}_1^{\bar{p}})^2} = \text{const.}}$

BES-I data



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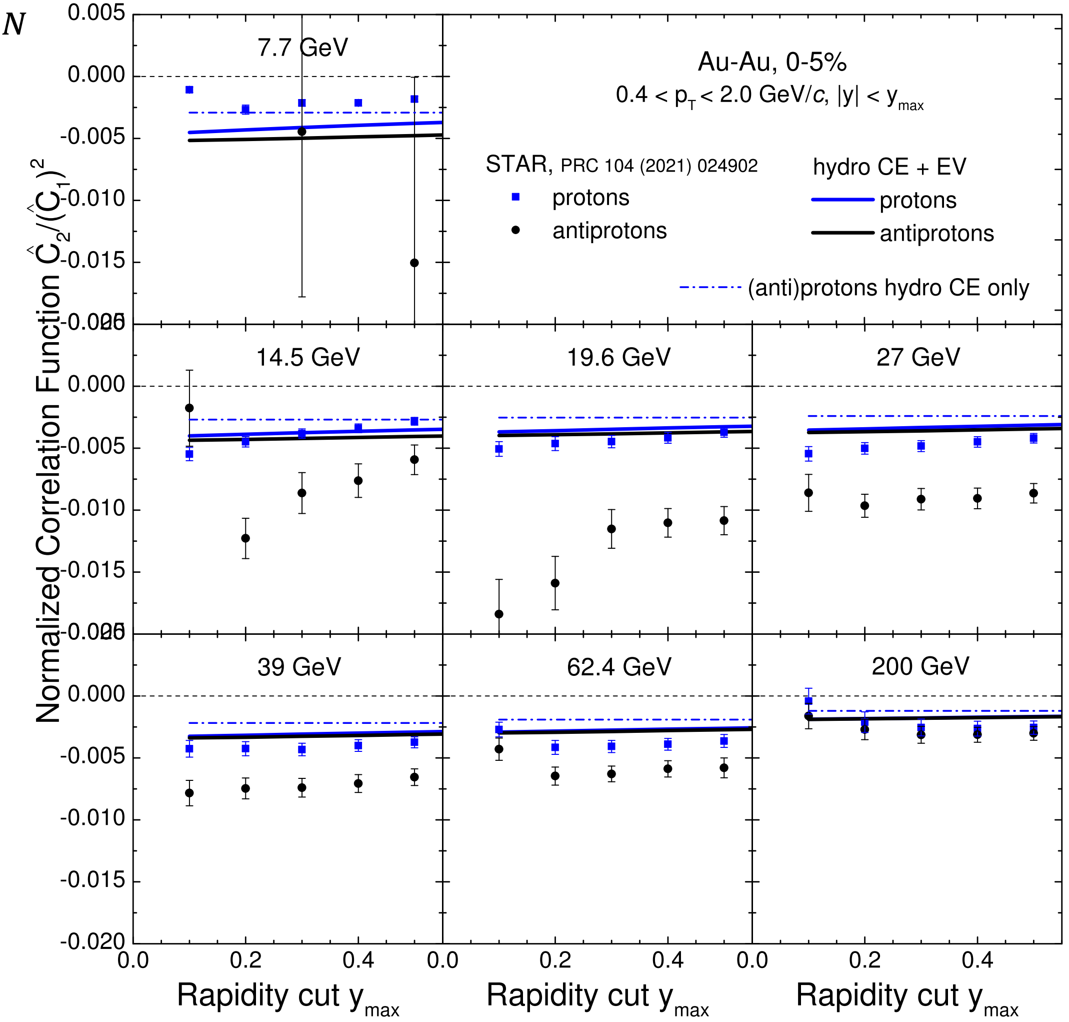
- Significant difference between p and $\bar{p}$ in BES-I
  - Missing baryon annihilation?
- With BES-II one can test the scaling with greater precision and extended coverage in rapidity
  - No need for CBWC

A. Bzdak, V. Koch, VV, in preparation

BES-I data



Factorial cumulants from RHIC-BES-II

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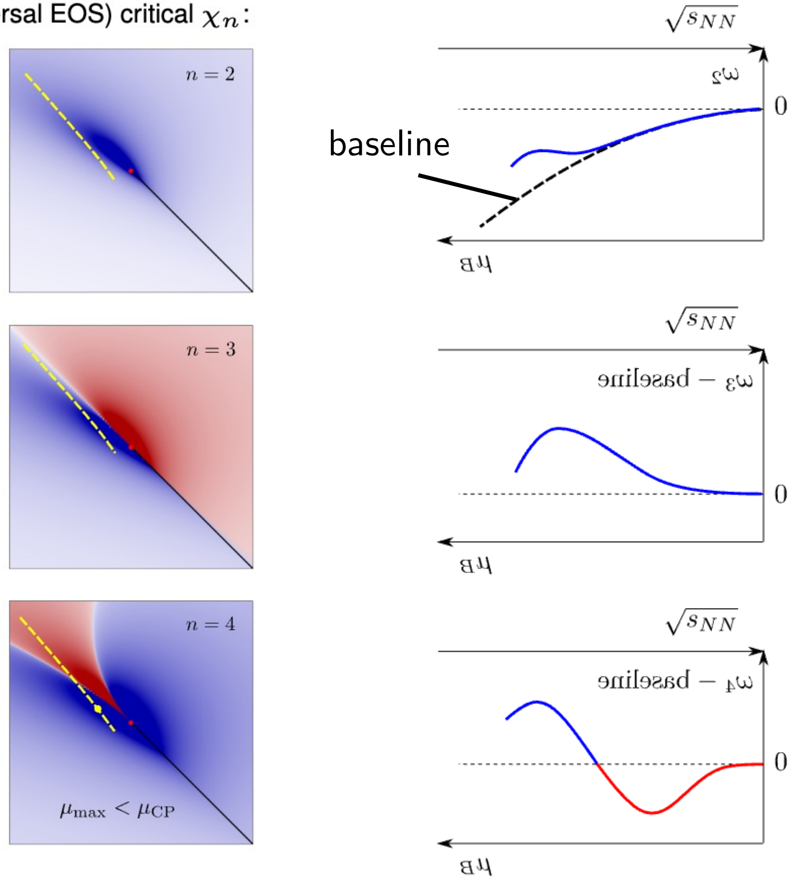
From M. Stephanov (SQM2024):

$$\omega_n = \hat{C}_n/\hat{C}_1$$

(universal EOS) critical χ_n:



Bzdak et al review 1906.00936

Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4 for CP at $\mu_B > 420$ MeV

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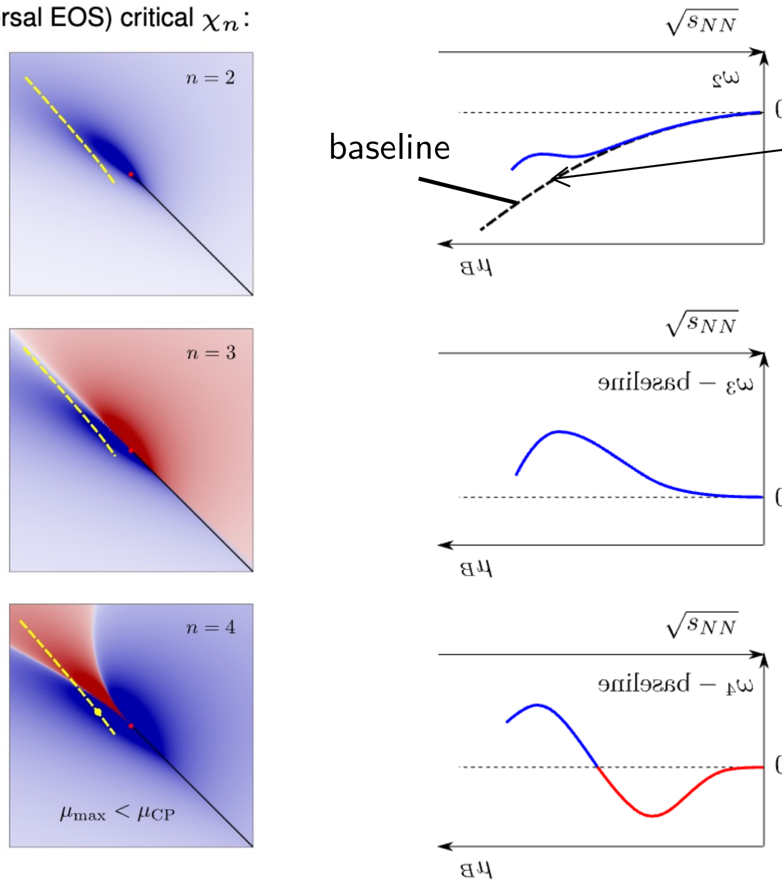
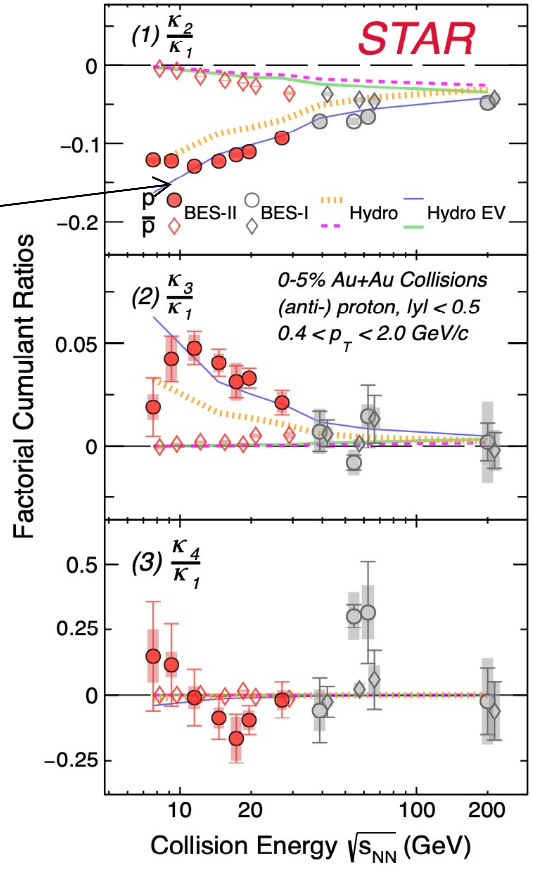


Bzdak et al review 1906.00936

STAR data:



A. Pandav, CPOD2024

baseline (hydro):

VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4 for CP at $\mu_B > 420$ MeV

Factorial cumulants from RHIC-BES-II

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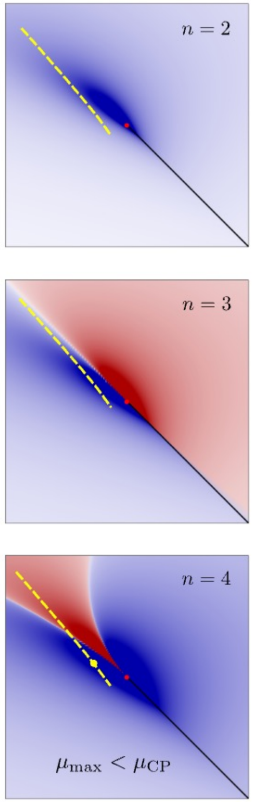
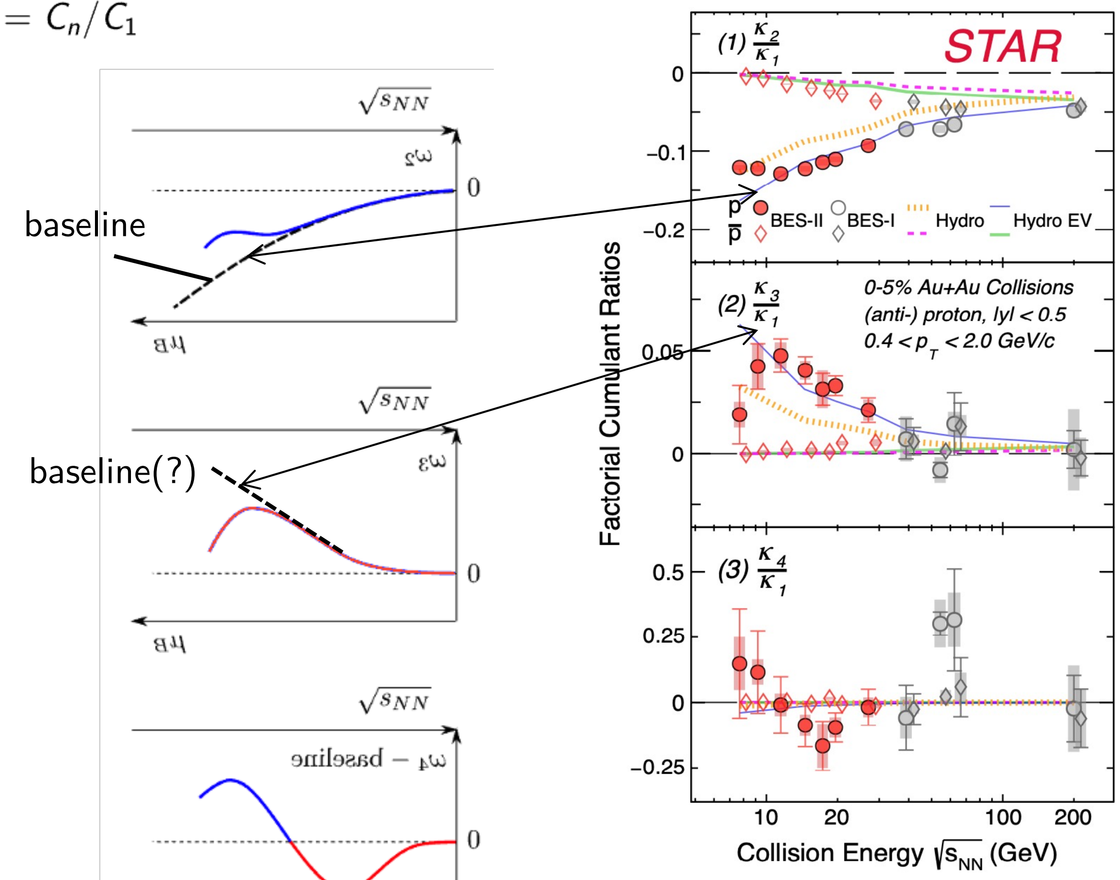


STAR data:



A. Pandav, CPOD2024

Bzdak et al review 1906.00936

Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4 for CP at $\mu_B > 420$ MeV

baseline (hydro):

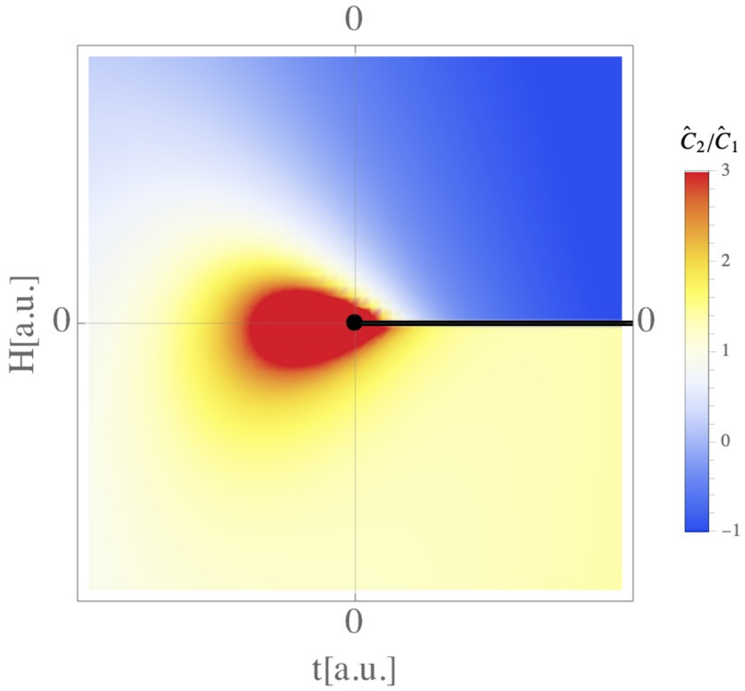
VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

- describes right side of the peak in $\hat{C}_3$
- implies
  - *positive* $\hat{C}_2$ − baseline > 0
  - *negative* $\hat{C}_3$ − baseline < 0

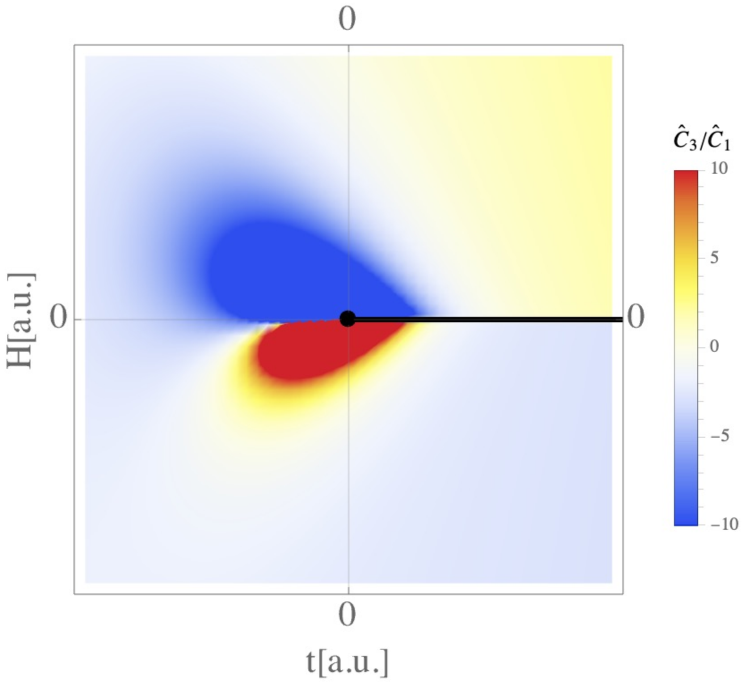
Factorial cumulants from RHIC-BES-II and CP

Factorial cumulants in Ising model

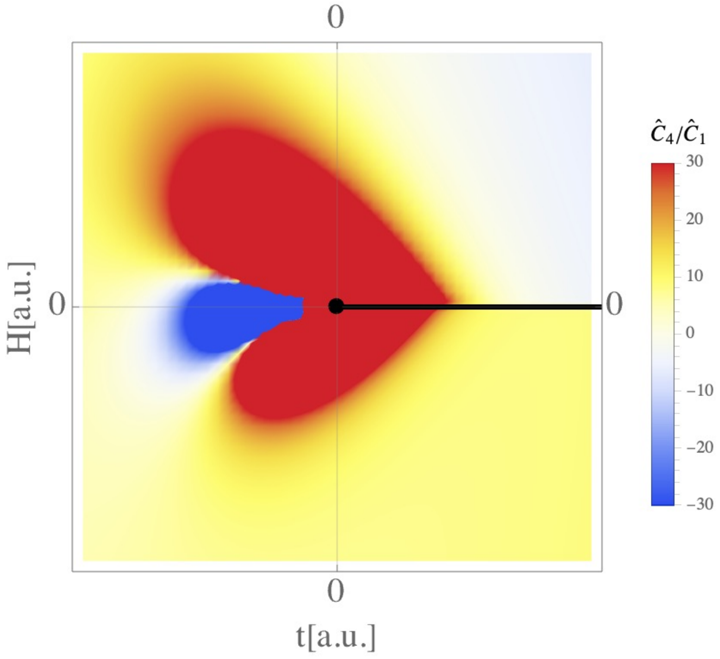
ω_2



ω_3



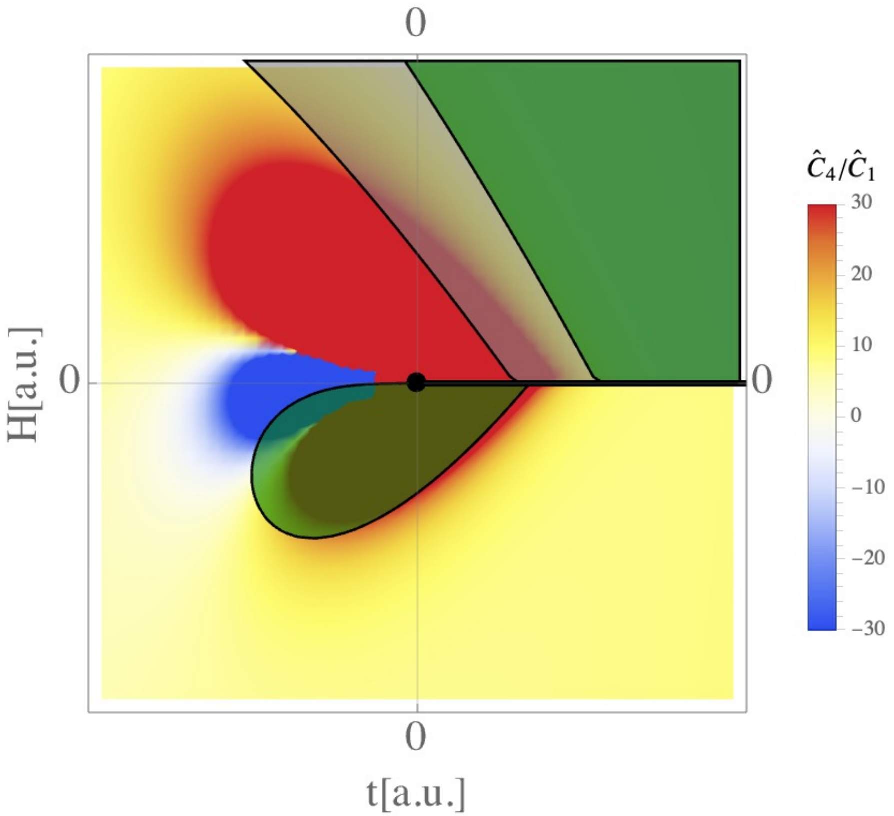
ω_4



Adapted from Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)

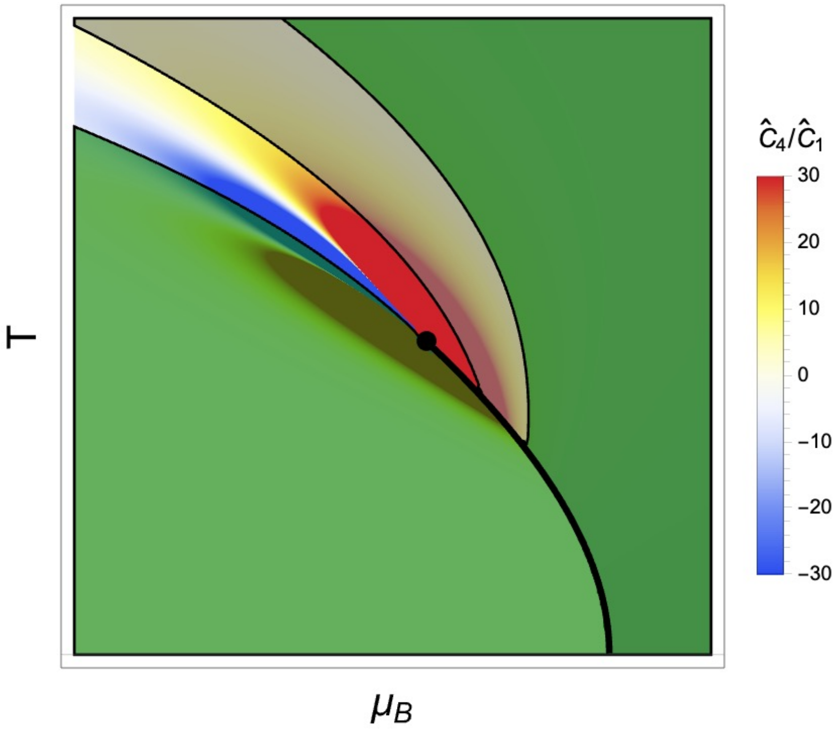
Factorial cumulants from RHIC-BES-II and CP

Exclusion plots



Shaded regions exclude $\hat{C}_2<0$ & $\hat{C}_3>0$

How it may look like in $T-\mu_B$ plane



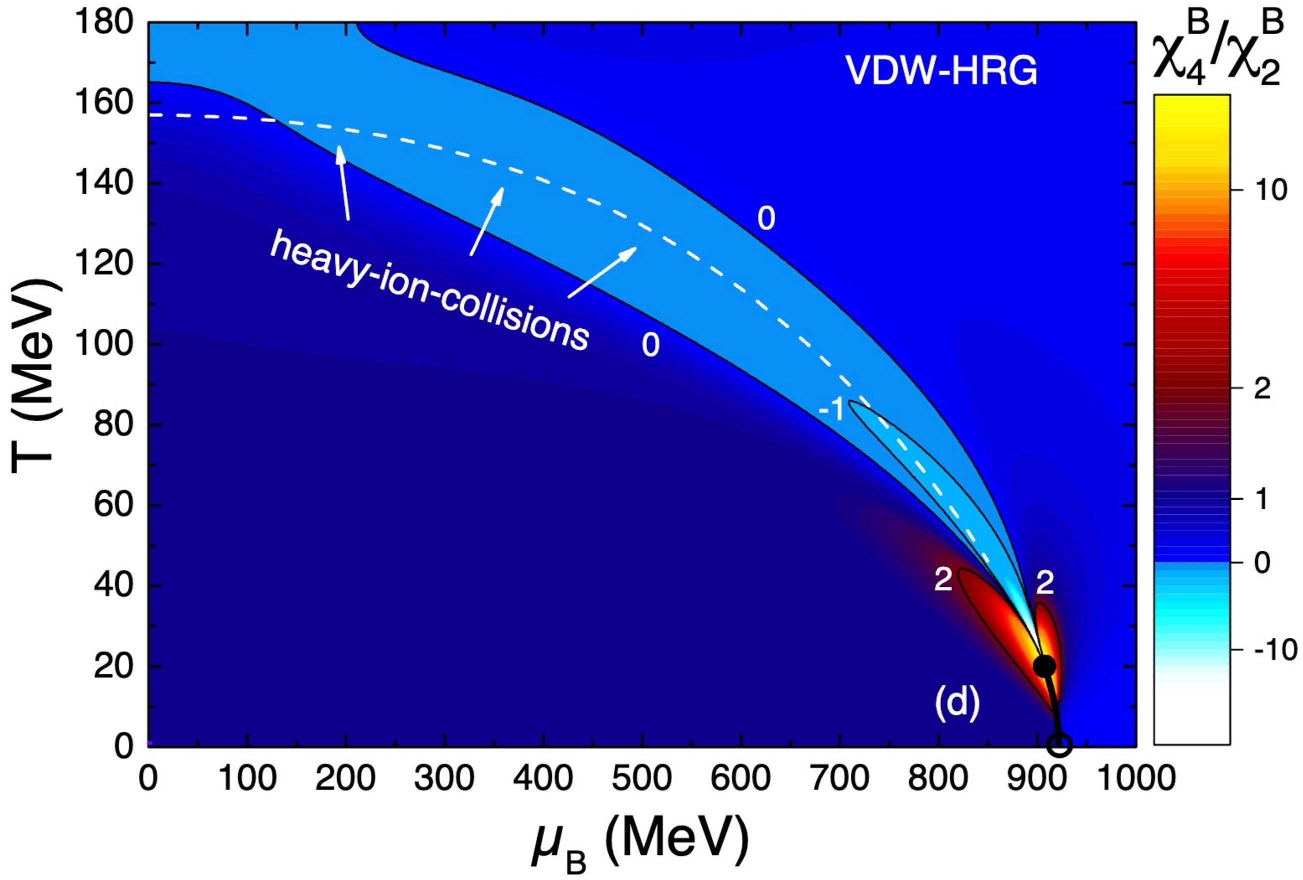
Based on QvdW model of nuclear matter

VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Freeze-out of fluctuations of the QGP side of the crossover?

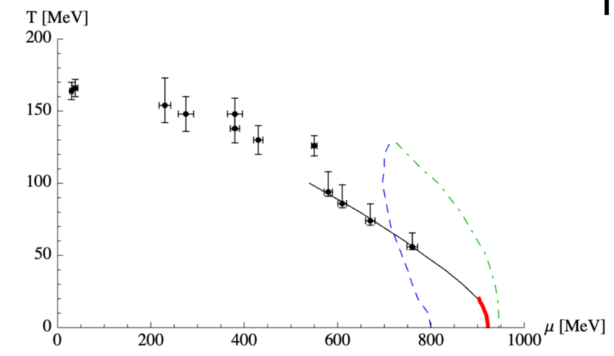
Nuclear liquid-gas transition

HRG with attractive and repulsive interactions among baryons



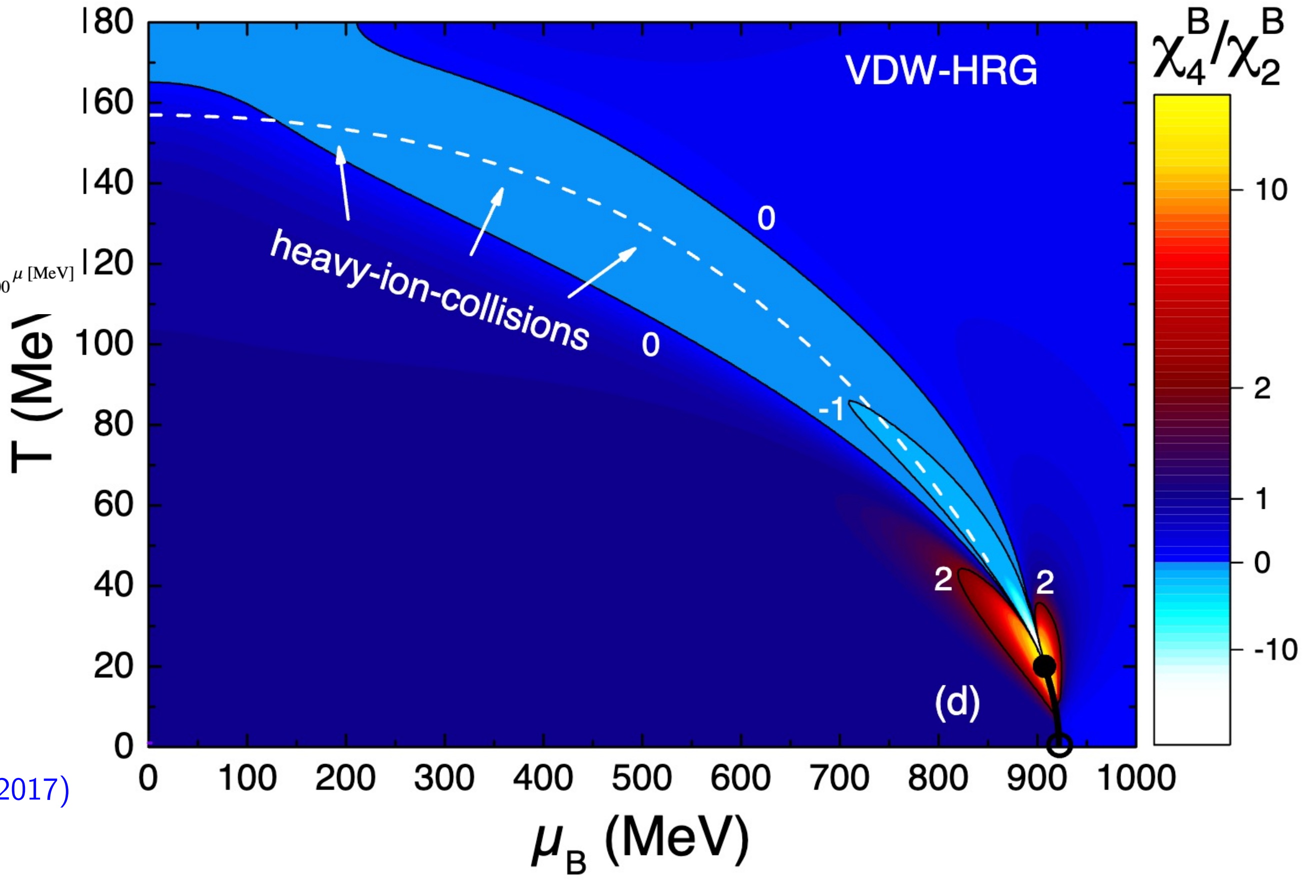
VV, Gorenstein, Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

Nuclear liquid-gas transition

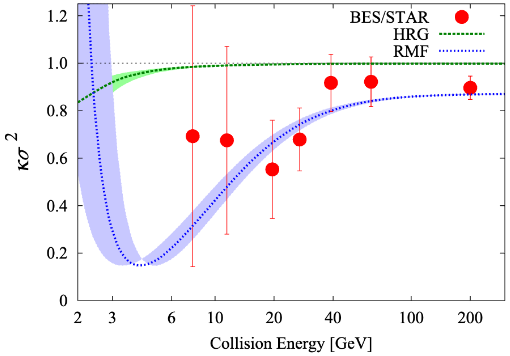


Floerchinger, Wetterich, NPA (2012)

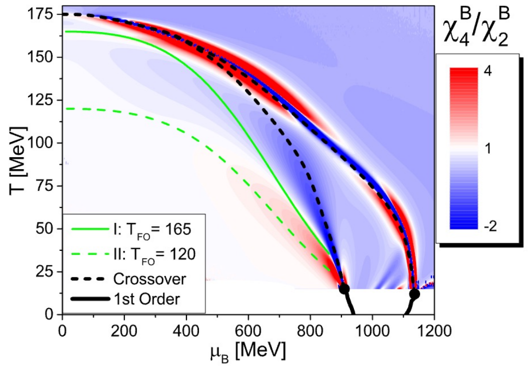
HRG with attractive and repulsive interactions among baryons



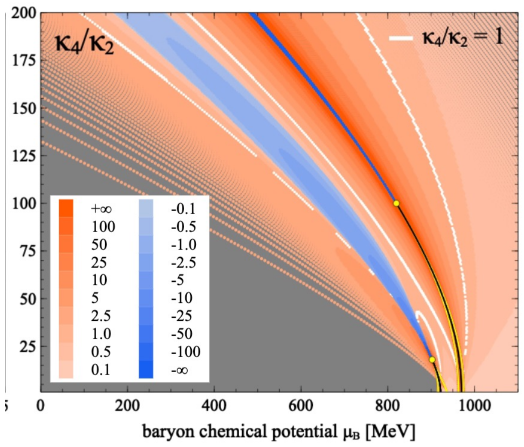
VV, Gorenstein, Stoecker, Phys. Rev. Lett. 118, 182301 (2017)



Fukushima, PRC (2014)



Mukherjee, Steinheimer, Schramm, PRC (2017)

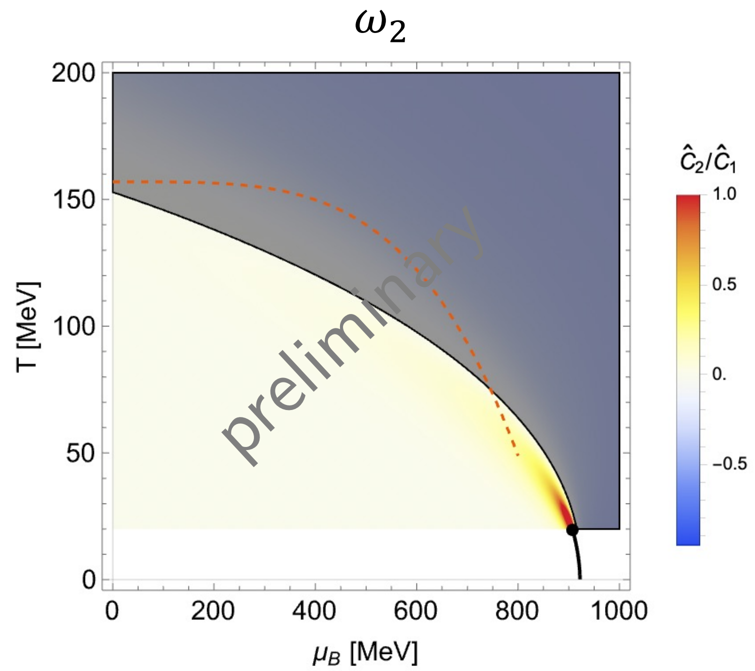


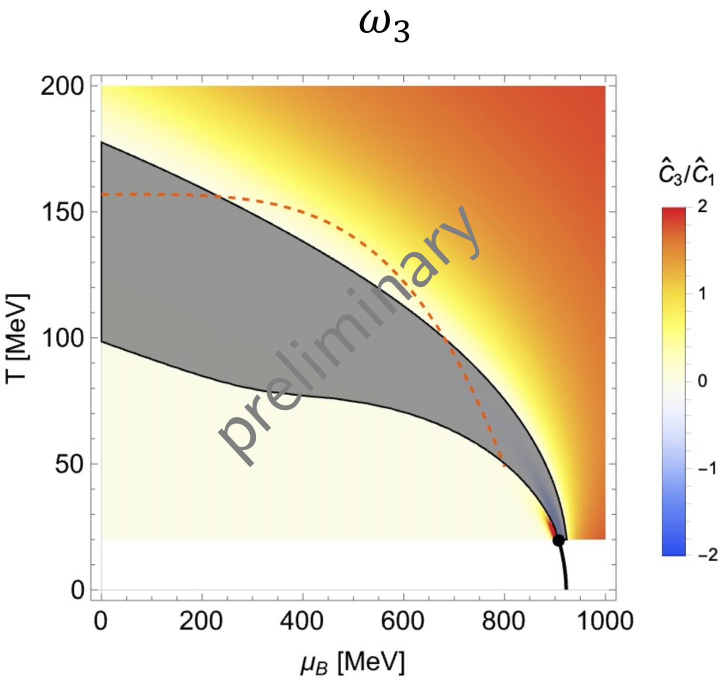
Sorensen, Koch, PRC (2020)

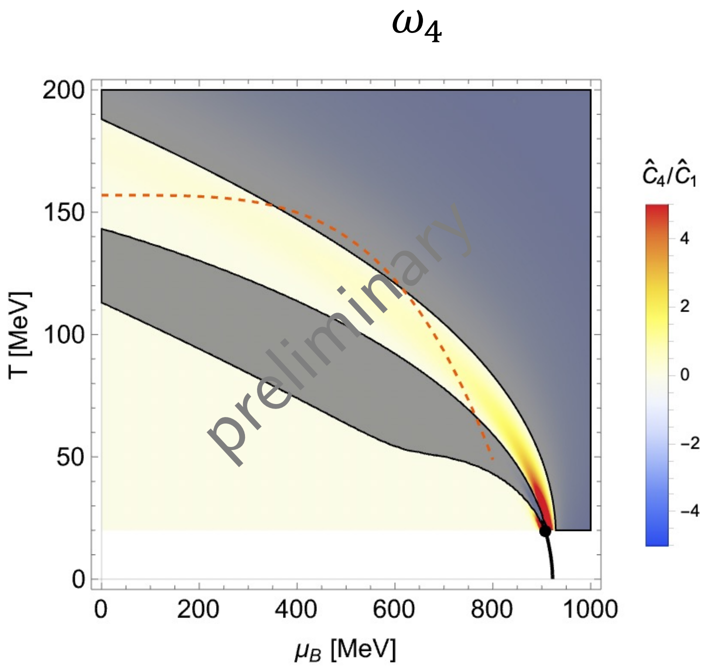
Factorial cumulants and nuclear liquid-gas transition

Calculation in a van der Waals-like HRG model

VV, Gorenstein, Stoecker, EPJA 54, 16 (2018)





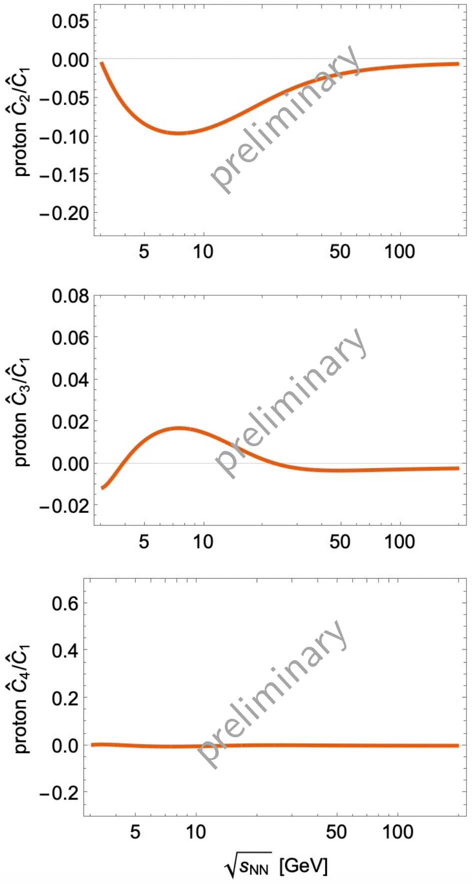


Shaded regions: negative values

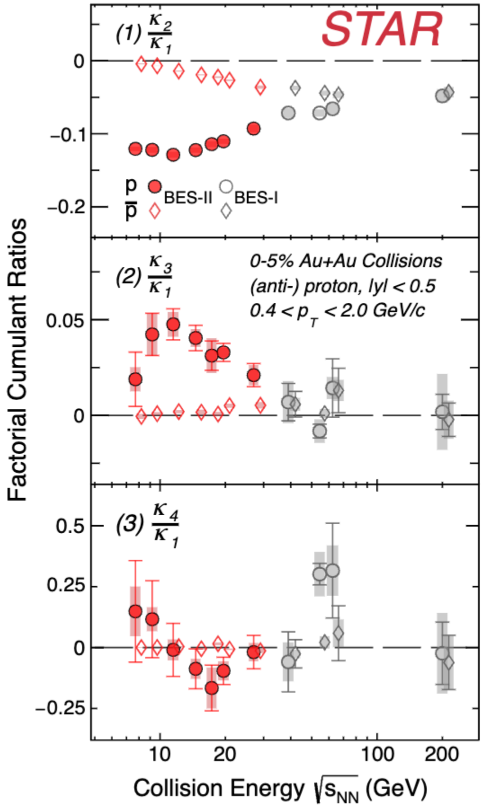
Factorial cumulants and nuclear liquid-gas transition

Calculation in a van der Waals-like HRG model along the freeze-out curve*

VV, Gorenstein, Stoecker, EPJA 54, 16 (2018)



Proton/antiproton factorial cumulant ratios



NB: The calculation is grand-canonical

*Poberezhnyuk et al., PRC 100, 054904 (2019)

Summary

- Proton cumulants are uniquely sensitive to the the CP but challenging to model dynamically
  - factorial cumulants are especially advantageous

- BES-II data
  - Protons are consistent with the *prediction* from non-critical hydro at $\sqrt{s_{NN}} \geq 20$ GeV
  - Non-monotonic structure in factorial cumulants
  - Positive $\hat{C}_2$ and negative $\hat{C}_3$ after subtracting non-critical baseline at $\sqrt{s_{NN}} < 10$ GeV
    - QGP side of the crossover using naïve equilibrium interpretation
    - Nuclear liquid-gas contribution?

Outlook:

- Improved description of non-critical effects, volume fluctuations, and nuclear interactions
- Test global conservation + volume fluctuations baseline through $\hat{C}_n/(\hat{C}_1)^n$ scaling
- Understanding factorial cumulants of antiprotons

Thanks for your attention!

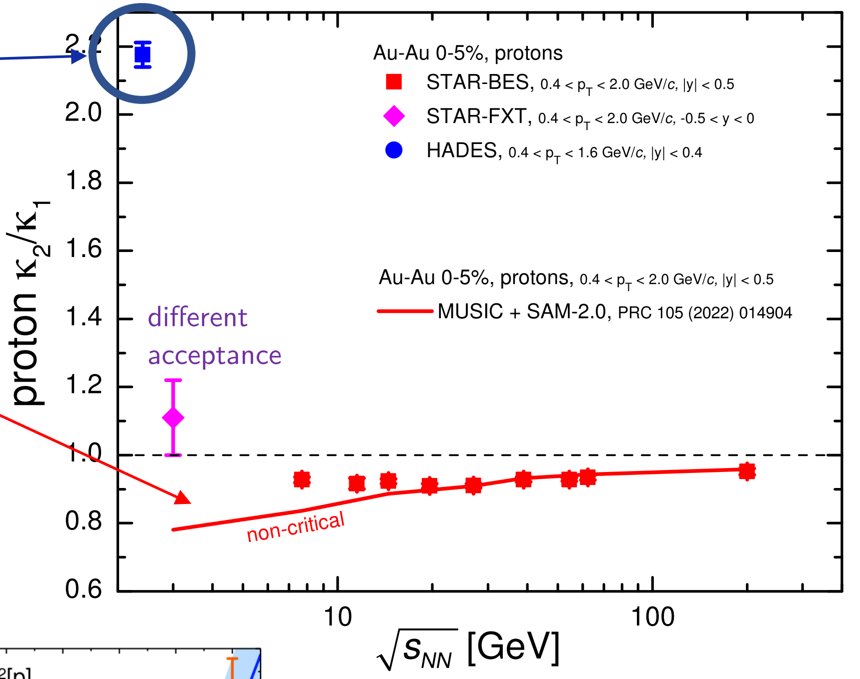
Backup slides

Lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV

- Intriguing hints from HADES@2.4 GeV and STAR-FXT@3GeV: huge excess of two-proton correlations!

[HADES Collaboration, Phys. Rev. C 102, 024914 (2020)]

[STAR Collaboration, Phys. Rev. Lett. 128, 202303 (2022)]



VV, Phys. Rev. C 106, 064906 (2022)

- No change of trend in the non-critical reference
- Additional mechanisms:
  - Nuclear liquid-gas transition (the other QCD critical point)
  - Light nuclei formation/fragmentation
  - Stronger initial state, volume, and baryon stopping fluctuations

Talk by A. Bzdak, Wed 14:20; Poster by A. Rustamov

- Difference in acceptance (-0.5<y<0 vs |y|<0.5)

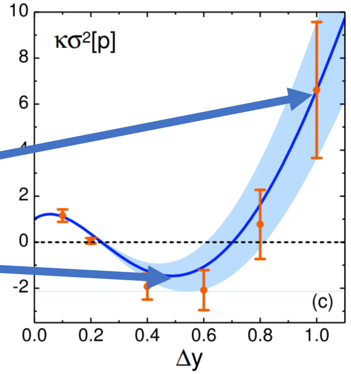
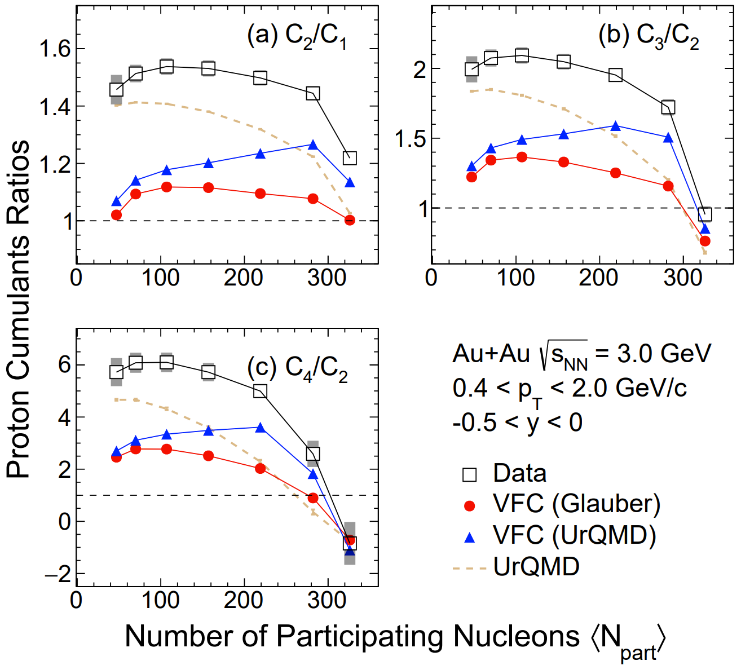


Figure from O. Savchuk et al., PLB 835, 137540 (2022)

- Improved modeling of lower energies required

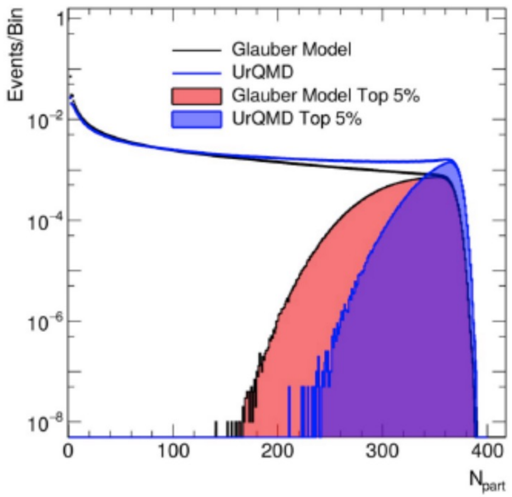
We may want to understand κ_2 first

Lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV



STAR Collaboration, Phys. Rev. Lett. 128 (2022) 202303

STAR-FXT



HADES

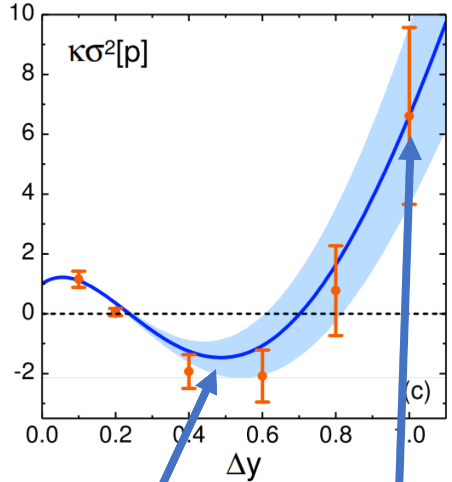


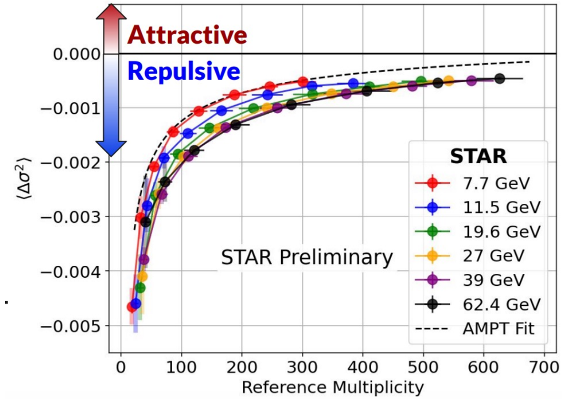
Figure from O. Savchuk et al., PLB 835, 137540 (2022)

- Volume fluctuations/centrality selection appear to play an important role
  - UrQMD is useful for understanding basic systematics associated with it
- Indications for enhanced scaled variance, $\kappa_2/\kappa_1 > 1$
- κ_4/κ_2 negative and described by UrQMD (purely hadronic?), note -0.5<y<0 instead of |y|<0.5

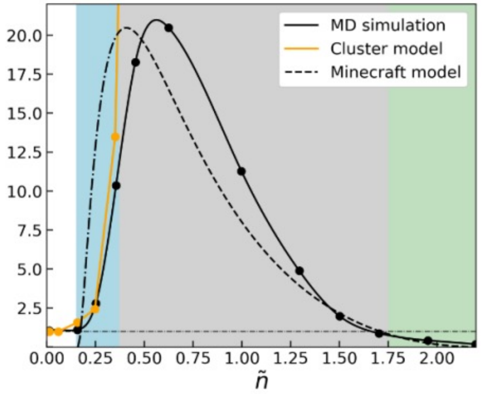
Proper understanding of $\kappa_2/\kappa_1 > 1$ in both HADES and STAR-FXT is missing

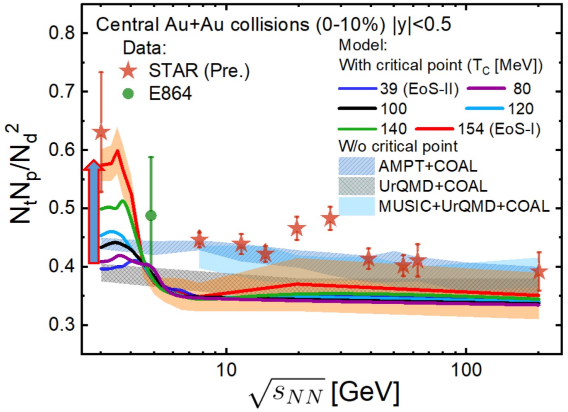
Other observables

- Azimuthal correlations of protons
  - points to repulsion at RHIC-BES



- Light nuclei
  - Spinodal/critical point enhancement of density fluctuations and light nuclei production





- Proton intermittency
  - No structure indicating power-law seen by NA61/SHINE

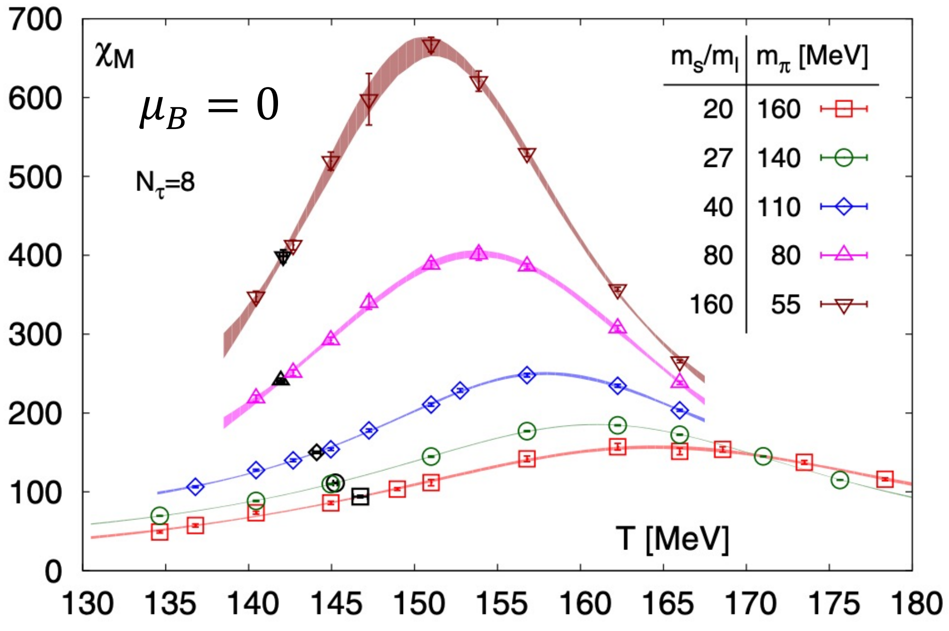
- Directed flow, speed of sound

Consistency in understanding all the observables is required

Hunting for the QCD critical point with lattice QCD

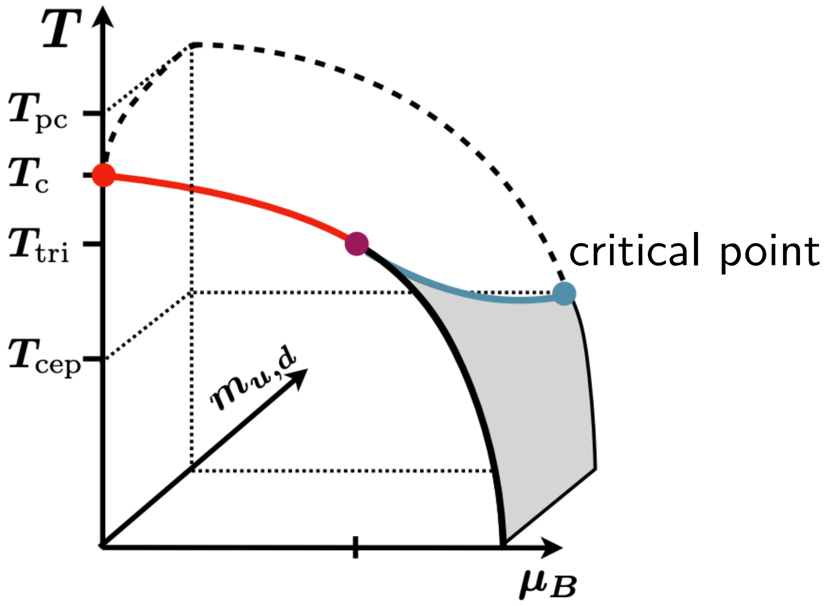
UNIVERSITY OF HOUSTON

Remnants of O(4) chiral criticality at $\mu_B = 0$ quite well established with lattice QCD



HotQCD Collaboration, PRL 123, 062002 (2019)

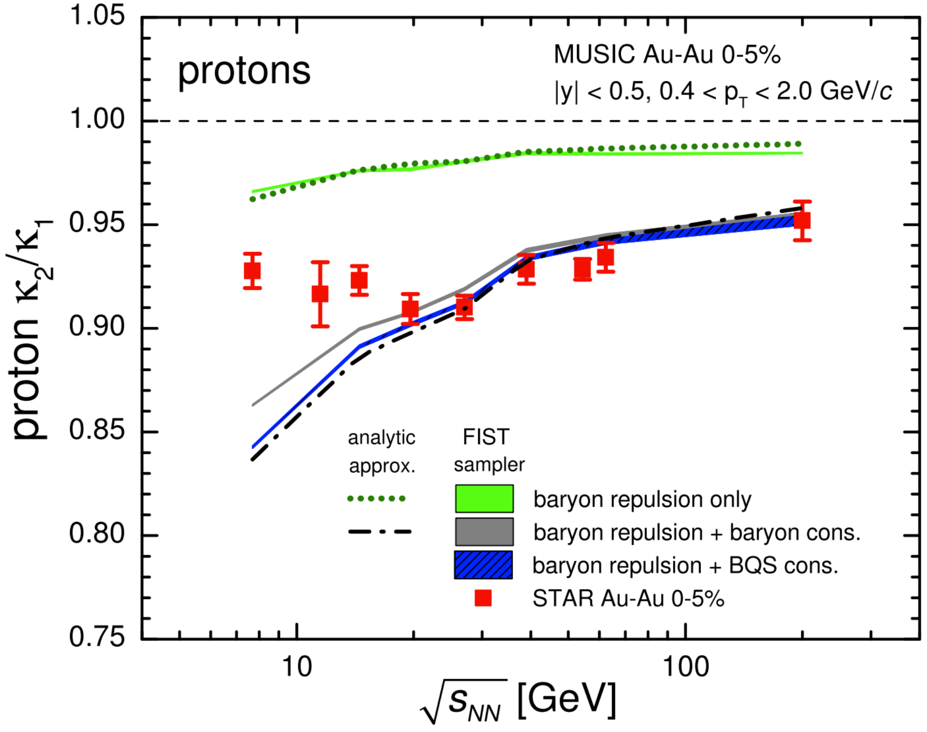
Physical quark masses away the chiral limit: Expect a Z(2) critical point at finite μ_B



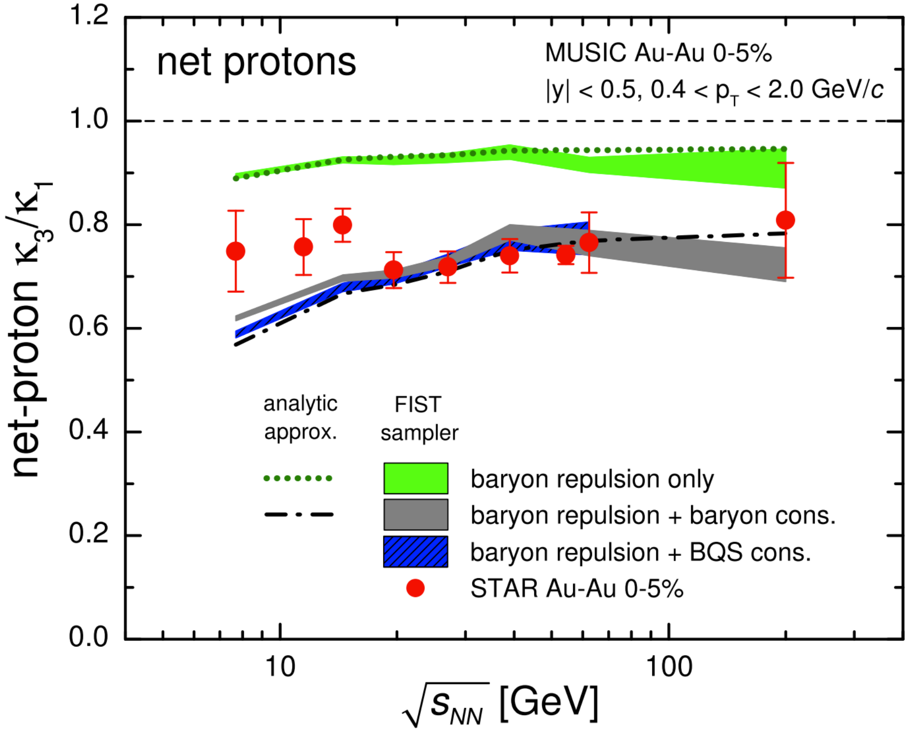
C. Schmidt

Non-critical cumulants: Analytic vs Monte Carlo

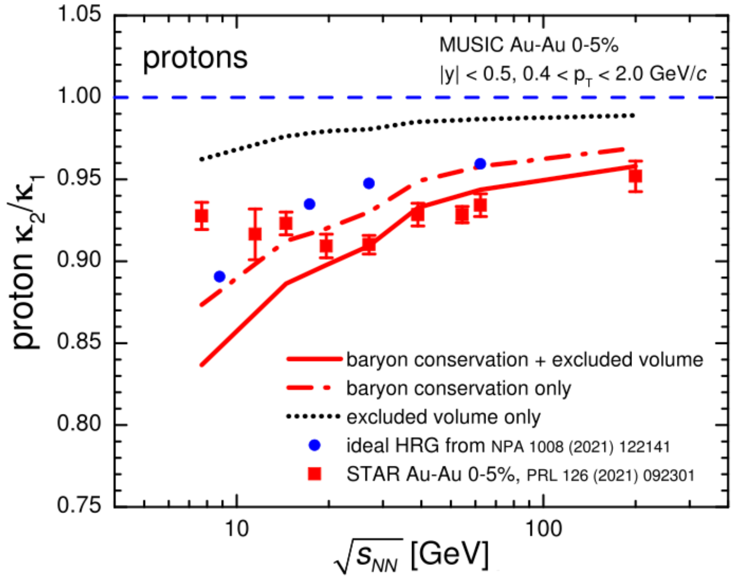
Proton κ_2/κ_1

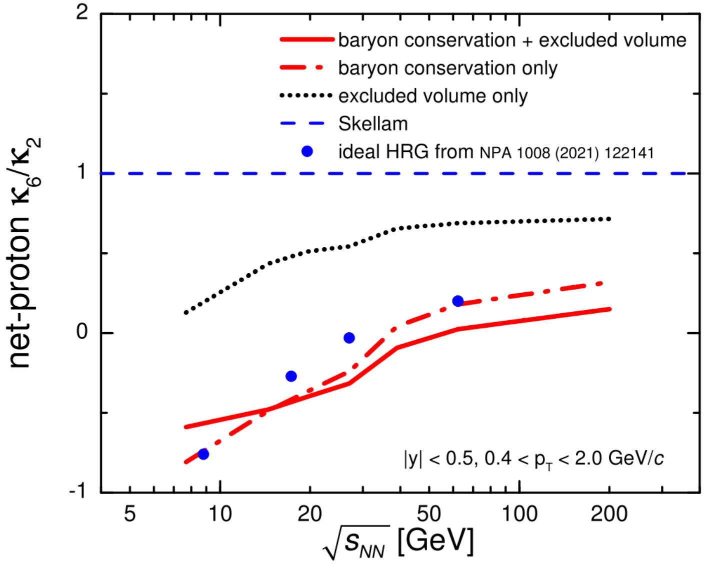


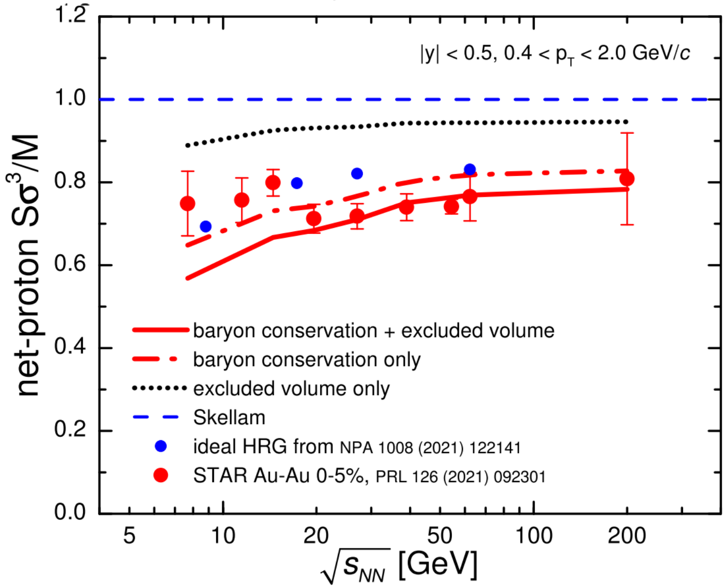
Net-proton κ_3/κ_1

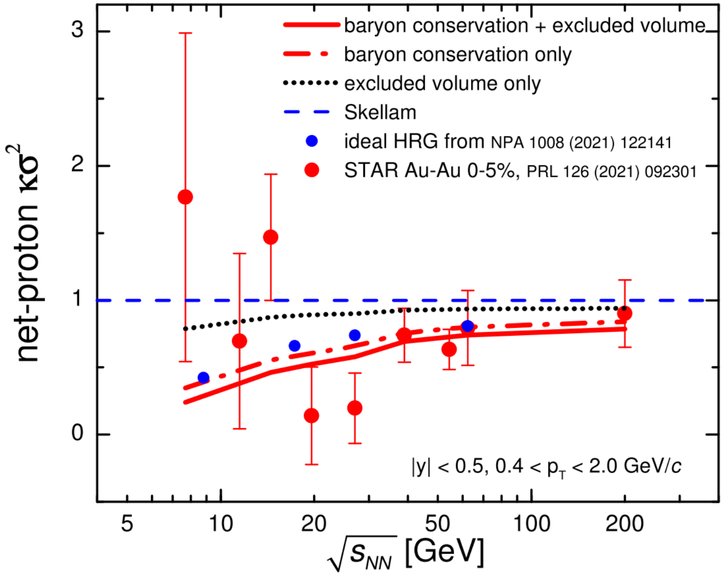


Non-critical cumulants



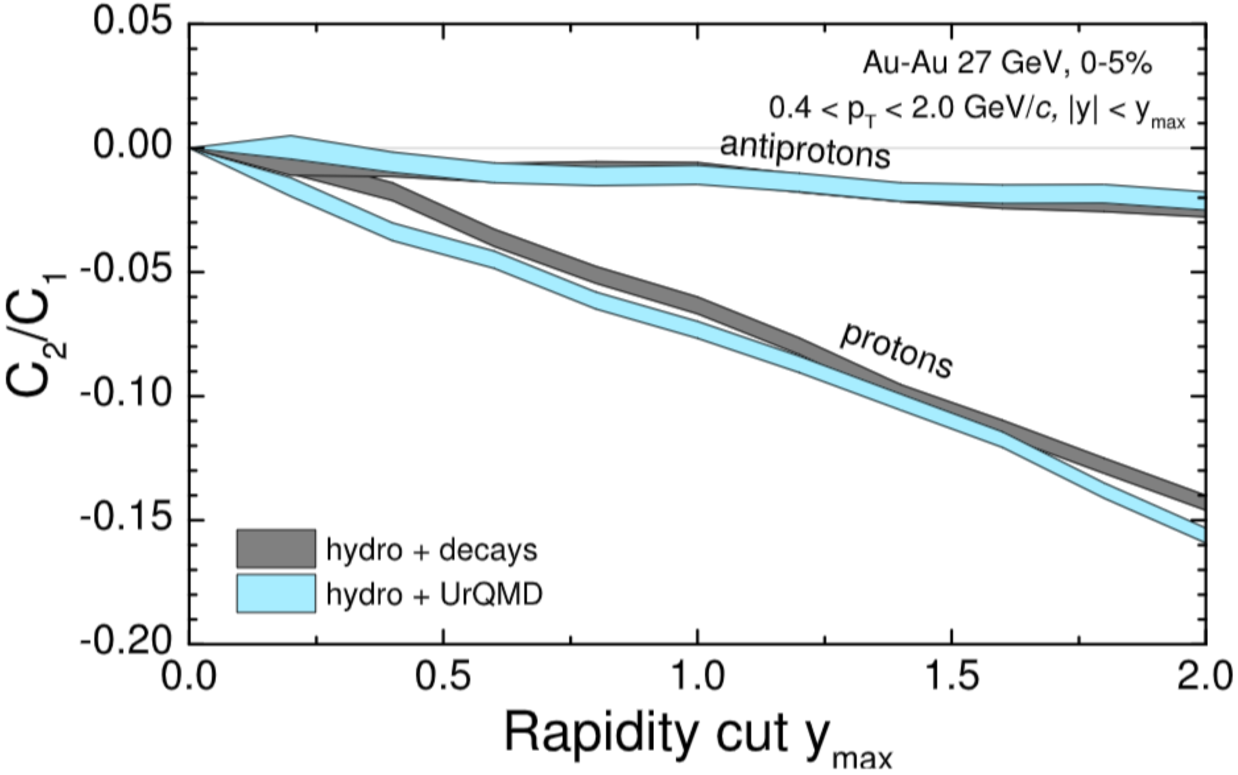




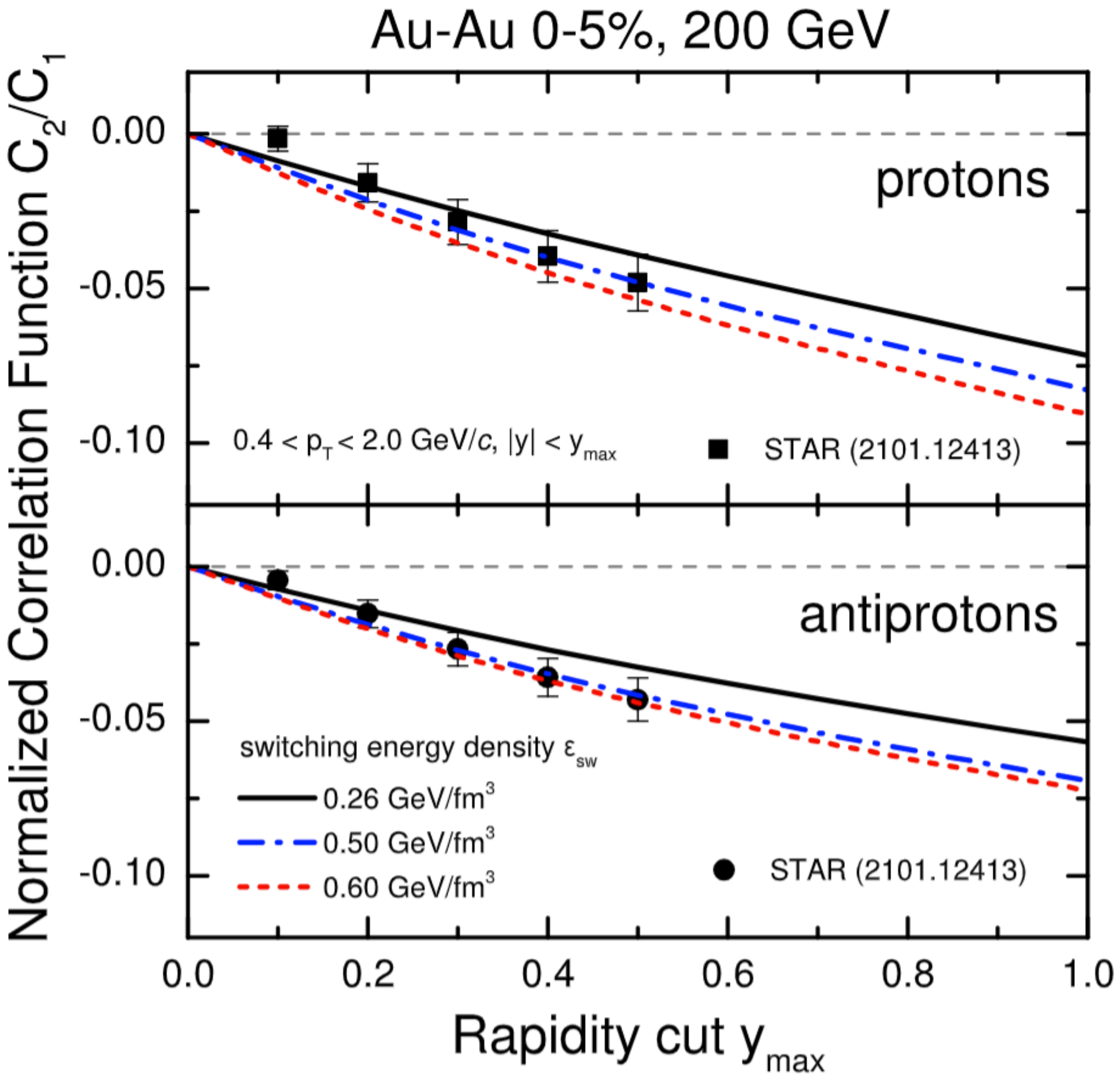


Effect of the hadronic phase

Sample ideal HRG model at particlization with exact conservation of baryon number using Thermal-FIST and run through hadronic afterburner UrQMD

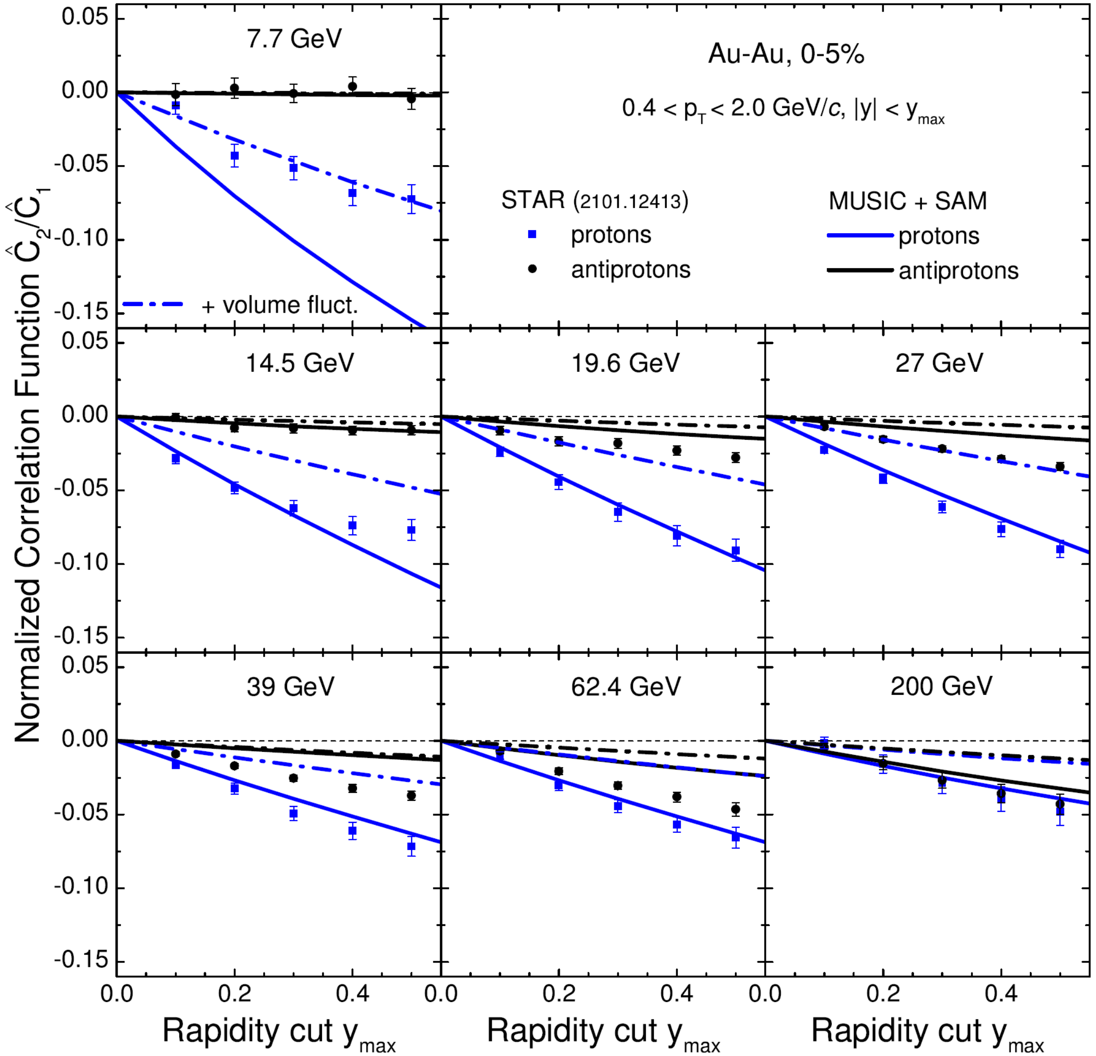


Dependence on the switching energy density



Acceptance dependence of two-particle correlations

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?
- Volume fluctuations? [Skokov, Friman, Redlich, PRC ‘13]
  - $C_2/C_1 \mathrel{+}= C_1 * \Delta v^2$
  - Can improve low energies but spoil high energies?
- **Attractive interactions?**
  - Could work if baryon repulsion turns into attraction in the high-μ_B regime
  - **Critical point?**



Net baryon fluctuations at LHC

- Global baryon conservation distorts the cumulant ratios already for one unit of rapidity acceptance

"lattice QCD" *experiment*

e.g. $$\left.\frac{\chi_4^B}{\chi_2^B}\right|_{T=160MeV}^{\text{GCE}} \simeq {\color{blue}0.67} \; {\color{red}\neq} \; \left.\frac{\chi_4^B}{\chi_2^B}\right|_{\Delta Y_{\text{acc}}=1}^{\text{HIC}} \simeq 0.56$$

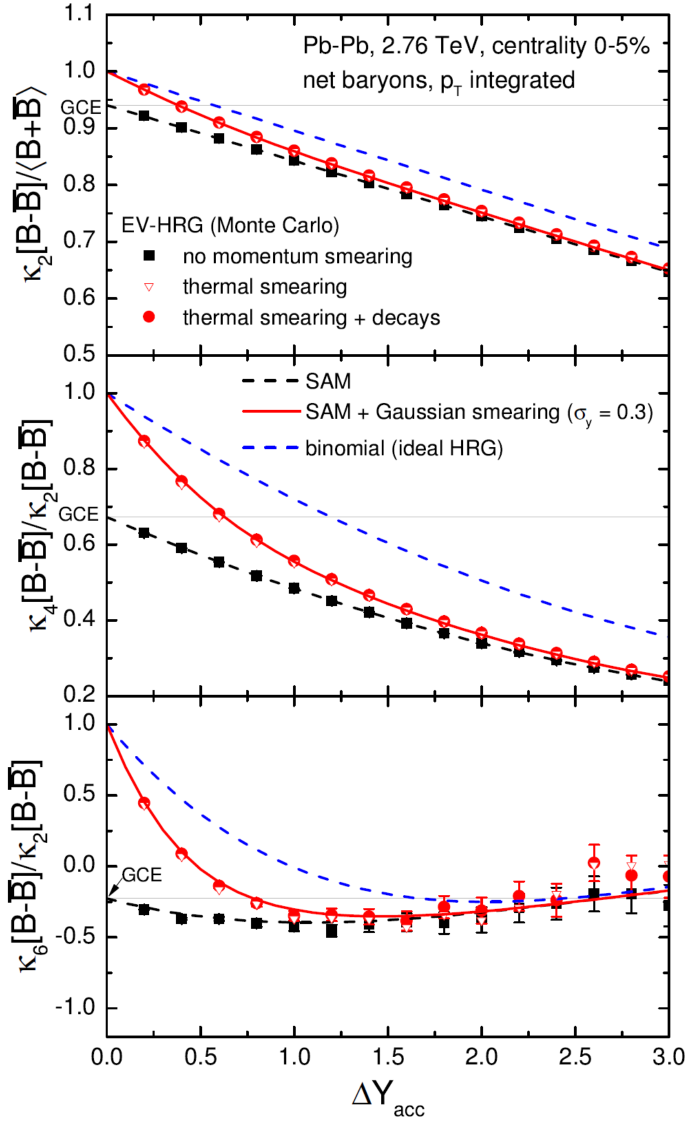
- Neglecting thermal smearing, effects of global conservation can be described analytically via SAM

$$\frac{\kappa_2}{\langle B+\bar{B}\rangle} = (1-\alpha)\frac{\kappa_2^{\text{gce}}}{\langle B+\bar{B}\rangle}, \qquad \alpha = \frac{\Delta Y_{\text{acc}}}{9.6}, \quad \beta \equiv 1-\alpha$$

$$\frac{\kappa_4}{\kappa_2} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B},$$

$$\frac{\kappa_6}{\kappa_2} = [1-5\alpha\beta(1-\alpha\beta)]\,\frac{\chi_6^B}{\chi_2^B} - 10\alpha(1-2\alpha)^2\beta\left(\frac{\chi_4^B}{\chi_2^B}\right)^2$$

- Effect of resonance decays is negligible



VV, Koch, arXiv:2012.09954

Net baryon vs net proton

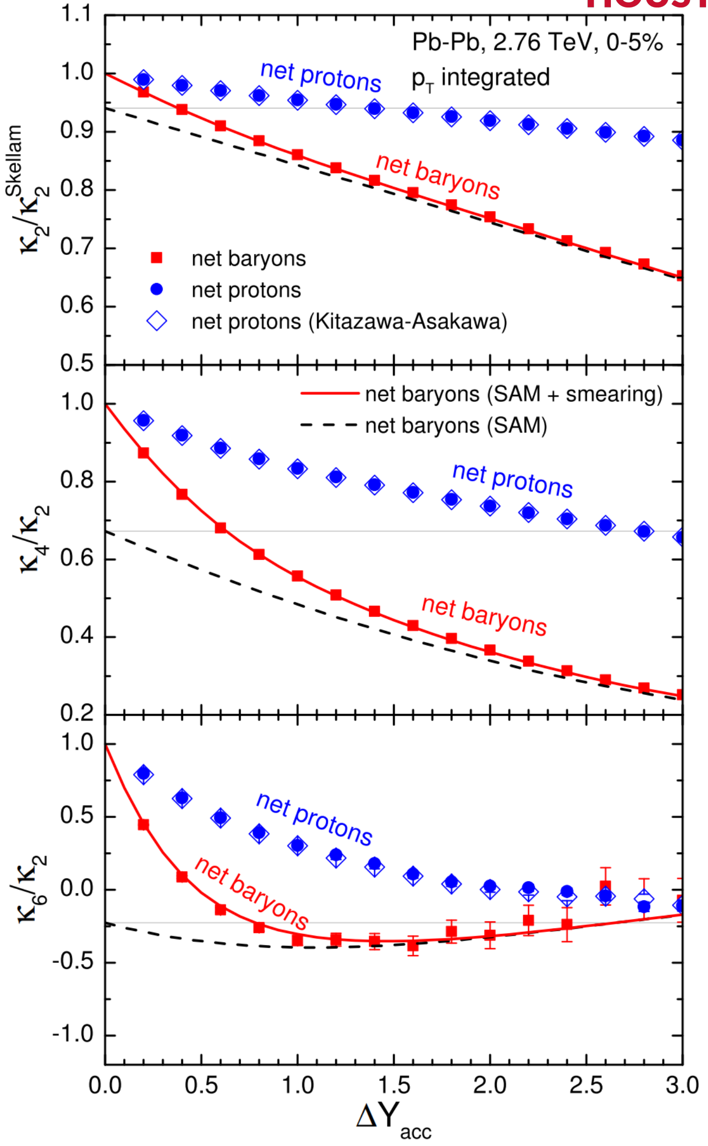
- Thermal smearing distorts the signal at $\Delta Y_{accept} \leq 1$. Net baryons converge to model-independent SAM result at larger ΔY_{accept}

- net baryon ≠ net proton, e.g.

$$\left.\frac{\chi_4^B}{\chi_2^B}\right|^{\mathrm{HIC}}_{\Delta Y_{\mathrm{acc}}=1} \simeq 0.56 \quad \neq \quad \left.\frac{\chi_4^p}{\chi_2^p}\right|^{\mathrm{HIC}}_{\Delta Y_{\mathrm{acc}}=1} \simeq 0.83$$

- Baryon cumulants can be reconstructed from proton cumulants via binomial (un)folding based on isospin randomization [Kitazawa, Asakawa, Phys. Rev. C 85 (2012) 021901]
    - Requires the use of joint factorial moments, only experiment can do it model-independently

unfolding



VV, Koch, arXiv:2012.09954