Equation of state of neutron star matter in a relativistic density functional approach

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TECHNISCHE UNIVERSITÄT DARMSTADT

Brainstorming workshop: Deciphering the equation of state using gravitational waves from astrophysical sources

Institute of Theoretical Physics, University of Warsaw

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Outline

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Introduction

Theory for Nuclei and Nuclear Matter

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- **different approaches**
  - hadronic ’ab-initio’ methods with realistic interactions
    - interactions: potential models, meson exchange, chiral forces, RG evolved, . . .
      (Argonne, Urbana, Tucson-Melbourne, Nijmegen, Paris, Bonn, . . .)
    - many-body methods: BHF/DBHF, SCGF, CBF, VMC, GFMC, AFDMC, . . .
  - QCD-based/inspired descriptions
  - effective field theories (EFT)
  - energy density functionals (EDF)
  - shell models, algebraic models, cluster models, . . .

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  - shell models, algebraic models, cluster models, . . .
- **challenge**:
  description of atomic nuclei and nuclear matter in a unified model
  - methods not always applicable (methodological/technical limitations)

  ⇒ here: **generalized relativistic energy density functional**

Energy Density Functionals for Nuclei and Nuclear Matter

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- **various types**
  - nonrelativistic or relativistic/covariant
  - often derived from mean-field models in different approximations (Hartree, Hartree-Fock, Hartree-Fock-Bogoliubov)
  - nucleons (hyperons, other baryons, clusters, ...) as degrees of freedom

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- **examples**
  - Skyrme Hartree-Fock models
    - zero-range two- and three-body interactions
  - Gogny Hartree-Fock models
    - finite-range two-body interaction, three-body as in Skyrme
  - relativistic models
    - field-theoretical approach, mean-field approximation
    - interaction by meson exchange (σ, ω, ρ, ...)
    - medium effects:
      — nonlinear models (selfcoupling of mesons)
      — density dependent couplings

Generalized Relativistic Energy Density Functional

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- **relativistic mean-field approach**
  with *density dependent minimal meson-nucleon couplings*
  and *meson-nucleon tensor couplings (new!)*

  details: see S. Typel and S. Shlomo, arXiv:2408.00425

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- **degrees of freedom**
  - baryon: nucleons, hyperons (optional)
    $\Rightarrow$ quasiparticles with effective mass $M_i^* = M_i - S_i$
    and effective chemical potential $\mu_i^* = \mu_i - V_i$
  - mesons: σ, ω, ρ $\Rightarrow$ treated as classical fields
  - light clusters (d, t, ^{3}He, α), heavy clusters

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  - light clusters (d, t, ^{3}He, α), heavy clusters
- **effective in-medium interaction**
  - phenomenological approach
    $\Rightarrow$ model parameters to be determined
  - scalar (S_i) and vector (V_i) potentials with rearrangement contributions
    $\Rightarrow$ thermodynamic consistency

Particles, Fields, and Lagrangian Density

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- **particles and fields**
  - nucleons (Ψ_p, Ψ_n) with (vacuum) masses M_p, M_n
  - photons (A_μ) and mesons (σ, ω_μ, $\vec{\rho}_\mu$) with masses M_σ, M_ω, M_ρ
  - field tensors $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $G_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$, $\vec{H}_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu$

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- **Lagrangian density**

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \sum_{\eta=p,n} \overline{\Psi}_\eta \left(\gamma_\mu i\mathcal{D}^\mu_\eta - \sigma_{\mu\nu} \mathcal{T}^{\mu\nu} - \mathcal{M}_\eta \right) \Psi_\eta - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
& + \frac{1}{2} \left(\partial^\mu \sigma \partial_\mu \sigma - M_\sigma^2 \sigma^2 - \frac{1}{2} G_{\mu\nu} G^{\mu\nu} + M_\omega^2 \omega_\mu \omega^\nu - \frac{1}{2} \vec{H}_{\mu\nu} \cdot \vec{H}^{\mu\nu} + M_\rho^2 \vec{\rho}_\mu \vec{\rho}^\nu \right)
\end{aligned}
$$

  - covariant derivative $i\mathcal{D}^\mu_\eta = i\partial^\mu - \Gamma_\omega \omega^\mu - \Gamma_\rho \vec{\rho}^\mu \cdot \vec{\tau} - \Gamma_\gamma A^\mu \frac{1+\tau_{3\eta}}{2}$
  - mass operator $\mathcal{M}_\eta = M_\eta - \Gamma_\sigma \sigma$
  - tensor contribution $\mathcal{T}^{\mu\nu} = \frac{\Gamma_{T\omega}}{2M_p} G^{\mu\nu} + \frac{\Gamma_{T\rho}}{2M_p} \vec{H}^{\mu\nu} \cdot \vec{\tau}$
  - couplings Γ_ω, Γ_ρ, Γ_σ (depend on baryon density n_b) and $\Gamma_{T\omega}$, $\Gamma_{T\rho}$ (constants)

Field Equations

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- use **Euler-Lagrange equations** $\frac{\partial \mathcal{L}}{\partial \phi} = \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}$
  for all fields $\phi = \Psi_\eta$, $\overline{\Psi}_\eta = \Psi_\eta^\dagger \gamma^0$, σ, ω_μ, $\vec{\rho}_\mu$

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- apply **approximations**
  - photon/meson fields ⇒ classical fields
  - Hartree approximation for many-body wave function of nucleons
  - no-sea approximation ⇒ negative-energy states not considered
  - only static solutions (no time dependence), no change of isospin
    ⇒ only one component of vector fields remains

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    $\Rightarrow$ only one component of vector fields remains
- self-consistent **solution** of coupled field equations
  - nuclear matter
    - meson fields: constants, field equations trivial
    - nucleons: modified plane-wave states
  - nuclei
    - meson fields determined using expansion in Riccati-Bessel functions
    - numerical solution of Dirac equation with Lagrange-mesh method
      (see, e.g, S. Typel, Front. Phys. 6 (2018) 73)

Energies

- **energy density** from energy-momentum tensor

$$\varepsilon(\vec{r}) = \langle T^{00} \rangle \qquad T^{\mu\nu} = \sum_{\phi} \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial^\nu \phi - g^{\mu\nu} \mathcal{L}$$

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    by numerical integration over spatial coordinates
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      (cm motion, rotation for non-spherical nuclei)

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    by numerical integration over spatial coordinates
    - correction for breaking of symmetries
      (cm motion, rotation for non-spherical nuclei)
  - nuclear matter: energy per nucleon $E/A = \varepsilon/n_b - M_{\text{nucleon}}$
    with average nucleon mass M_{nucleon}
    - analytic expression for temperature zero
    - particles & antiparticles at finite temperature
    - no contribution of tensor terms

Tensor Couplings

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- previous studies:
  - already suggested in early applications of relativistic mean-field models (see, e.g., M. Rufa et al., Phys. Rev. C 38 (1988) 390), but not explored extensively
  - some initial parameterizations without fine tuning in S. Typel and D. Alvear Terrero, Eur. Phys. J. A 56 (2020) 160
  - recent study of effects, without fully selfconsistent fit of parameters in M. Salinas and J. Piekarewicz, Phys. Rev. C 109 (2024) 045807

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- relevance of tensor couplings:
  - adds new freedom in description of surface properties of nuclei
  - releases strong correlation: effective mass
    $\leftrightarrow$ strength of σ meson field
    $\leftrightarrow$ size of spin-orbit splittings
  - acts only in nuclei but not in homogenous nuclear matter

Model Parameters

Parameterisation of Density Dependence of Couplings

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- general ansatz: $\Gamma_m(n_b) = \Gamma_m(n_{\mathrm{ref}})\, f_m(x)$
  with coupling $\Gamma_m(n_{\mathrm{ref}})$ at reference density $n_{\mathrm{ref}} = n_{\mathrm{sat}}$
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  with coupling $\Gamma_m(n_{\rm ref})$ at reference density $n_{\rm ref} = n_{\rm sat}$
  and function $f_m(x)$ with $x = n_b/n_{\rm ref}$
- functional forms as introduced in
  S. Typel and H.H. Wolter, Nucl. Phys. A 656 (1999) 331
  - isoscalar mesons ($m = \sigma, \omega$) $\Rightarrow$ rational function

$$f_m(x) = a_m \frac{1 + b_m(x + d_m)^2}{1 + c_m(x + d_m)^2} \quad \text{with constraints} \quad f(1) = 1 \quad f''(0) = 0$$

  - isovector meson ($m = \rho$) $\Rightarrow$ exponential function

$$f_m(x) = \exp\left[-a_m\,(x-1)\right]$$

Determination of Model Parameters I

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- fit of parameters to properties of nuclei $\Rightarrow$ experimental observables
  - not to indirectly obtained quantities, e.g., nuclear matter parameters
  - not to constraints from other theories, e.g., χEFT

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- selection of observables $\mathcal{O}_i$ ($i = 1, \ldots, 7$)
  - nuclear binding energies B
  - quantities related to charge form factor
    - charge radius r_c, diffraction radius r_d, surface thickness σ
  - rms radii r_n of single valence neutron above closed shells
  - spin-orbit splittings ΔE_{so}
  - constraint isoscalar monopole giant resonance energies E_{mono}

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- selection of nuclei, mostly (semi-)closed-shell nuclei
  - ^{16}O, ^{17}O, ^{24}O, ^{28}O, ^{34}Si, ^{34}Ca, ^{40}Ca, ^{41}Ca, ^{48}Ca, ^{48}Ni, ^{56}Ni, ^{68}Ni, ^{78}Ni, ^{90}Zr, ^{100}Sn, ^{116}Sn, ^{132}Sn, ^{140}Ce, ^{144}Sm, ^{208}Pb

$\Rightarrow$ 20 nuclei with $N_{\mathrm{data}} = 50$ data points

Determination of Model Parameters II

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- minimisation of objective function

$$\chi^2(\{p_k\}) = \sum_{i=1}^{N_{\text{obs}}} \chi_i^2(\{p_k\}) \qquad \chi_i^2(\{p_k\}) = \sum_{n=1}^{N_i^{(\text{obs})}} \left[\frac{\mathcal{O}_i^{(\text{model})}(n, \{p_k\}) - \mathcal{O}_i^{(\text{exp})}(n)}{\Delta \mathcal{O}_i} \right]^2$$

by variation of parameters $\{p_k\}$

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by variation of parameters $\{p_k\}$

- readjustment of uncertainties $\Delta\mathcal{O}_i$ so that

$$\frac{\chi^2(\{p_k\})}{N_{\rm dof}} = 1 \qquad \frac{\chi_i^2(\{p_k\})}{N_i^{\rm (obs)}} = \frac{\chi^2(\{p_k\})}{N_{data}}$$

with $N_{\rm dof} = N_{\rm data} - N_{\rm par} \Rightarrow$ reasonable model uncertainties

$(\Delta\mathcal{O}_i = \left[(\Delta\mathcal{O}_i^{\rm (exp)})^2 + (\Delta\mathcal{O}_i^{\rm (fit)})^2\right]^{1/2}$ for binding energies)

Determination of Model Parameters III

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- model parameters
  - M_σ (all other masses fixed)
  - $\Gamma_\sigma(n_{\mathrm{ref}})$, $\Gamma_\omega(n_{\mathrm{ref}})$, $\Gamma_\rho(n_{\mathrm{ref}})$, and their density dependence (5 parameters)
  - $\Gamma_{T\omega}$, $\Gamma_{T\rho}$

  $\Rightarrow N_{\mathrm{par}} = 11$ (9) for model with (without) tensor couplings
- no direct fit of all original model parameters
  - use nuclear matter parameters $\Rightarrow$ quasi-analytic conversion

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  - use nuclear matter parameters $\Rightarrow$ quasi-analytic conversion
- technical approach:
  combination of simplex method and diagonalisation
  of second derivative of $\chi^2 \Rightarrow$ direction of χ^2 reduction
- determination of uncertainties (and correlation coefficients) from

$$\mathcal{M}_{ij} = \frac{1}{2} \left. \frac{\partial^2 \chi^2}{\partial p_i \partial p_j} \right|_{\rm min} \qquad \overline{\Delta\mathcal{O}_1 \Delta\mathcal{O}_2} = \sum_{ij} \frac{\partial \mathcal{O}_1}{\partial p_i} (\mathcal{M}^{-1})_{ij} \frac{\partial \mathcal{O}_2}{\partial p_j} \qquad \Delta\mathcal{O} = \sqrt{\overline{\Delta\mathcal{O}\Delta\mathcal{O}}}$$

Parameterisation I

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- **models**
  - DDT: full model with tensor couplings
    $\Rightarrow$ base model, fixes uncertainties $\Delta\mathcal{O}_i$
  - variation DDTC:
    reduction of Coulomb field ($Z \to Z - 1$) to consider exchange term approximately (not discussed here)
  - DD2: previous, often used parameterisation
    (S. Typel et al., Phys. Rev. C 81 (2010) 015803)

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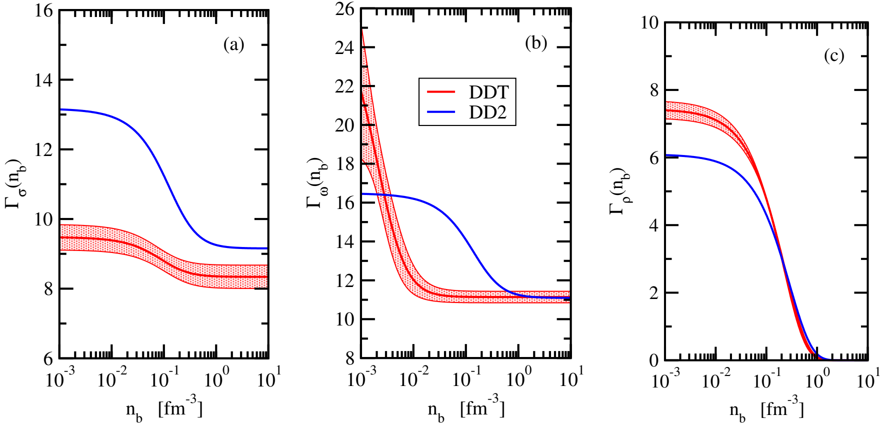
  - DD2: previous, often used parameterisation
    (S. Typel et al., Phys. Rev. C 81 (2010) 015803)
- self-consistent **model uncertainties**

$\mathcal{O}_i$	B [MeV]	r_c [fm]	r_d [fm]	σ [fm]	r_n [fm]	ΔE_{so} [MeV]	E_{mono} [MeV]
$\Delta\mathcal{O}_i^{(\mathrm{fit})}$	0.619311	0.013364	0.017155	0.026851	0.008249	0.240832	0.430714

Parameterisation II

- **density dependence of couplings**



Results

Results
Nuclei I

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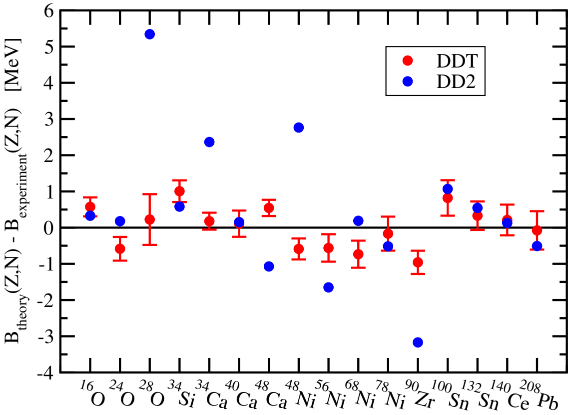
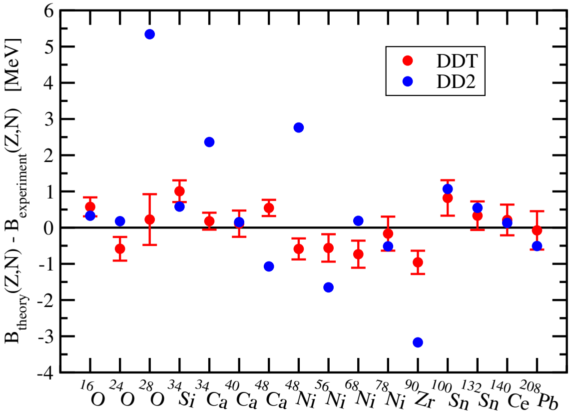
- **binding energies**



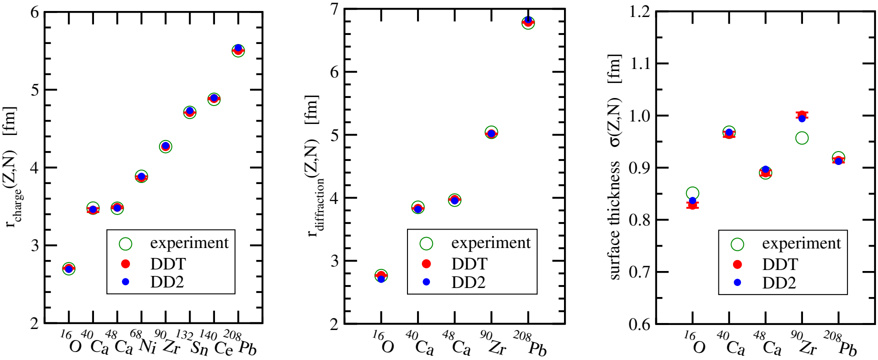
Results
Nuclei I

- **binding energies**
- particular improvements with DDT:
  - ^{28}O less bound than ^{24}O
  - better energy difference of mirror nuclei ^{34}Si — ^{34}Ca ^{48}Ca — ^{48}Ni



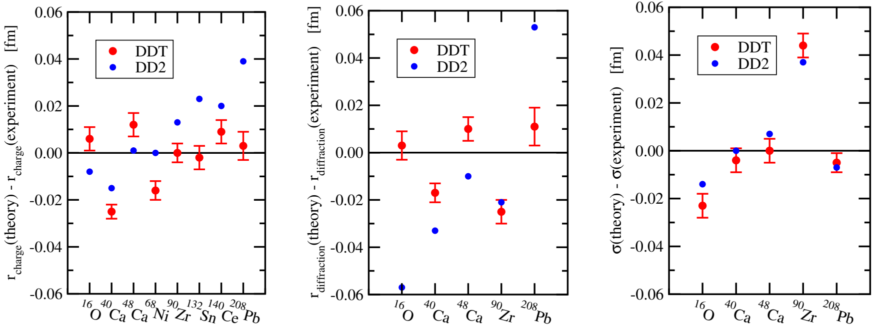
Results
Nuclei II

- **charge radii, diffraction radii, surface thicknesses**



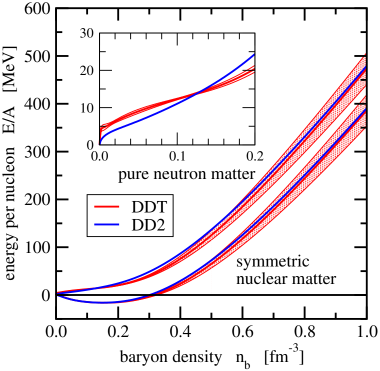
Results
Nuclei III

- **charge radii, diffraction radii, surface thicknesses**



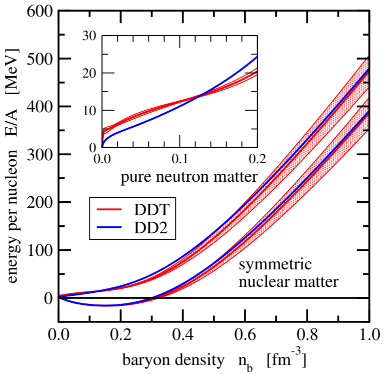
Results
Nuclear Matter I

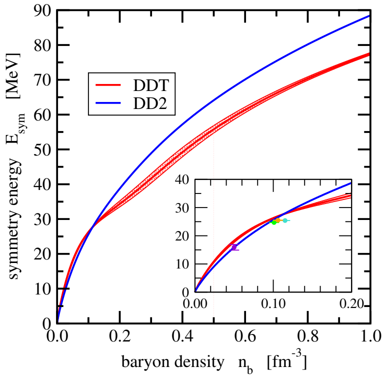
- **equation of state**



Results
Nuclear Matter I

- **equation of state** and **symmetry energy**





Results
Nuclear Matter II

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- **nuclear matter parameters**
  - energy per nucleon

$$E/A(n_b, \alpha) = E_0(n_b) + E_{\mathrm{sym}}(n_b)\alpha^2 + \dots \qquad \text{with} \qquad \alpha = (n_n - n_p)/n_b$$

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  - energy per nucleon of symmetric nuclear matter

$$E_0(n_b) = -B + \frac{1}{2}Kx^2 + \frac{1}{6}Qx^3 + \dots \qquad \text{with} \qquad x = (n_b - n_{\text{sat}})/(3n_{\text{sat}})$$

    B: binding energy at saturation, K: incompressibility coefficient,
    Q: skewness parameter

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    B: binding energy at saturation, K: incompressibility coefficient,
    Q: skewness parameter
  - symmetry energy

$$E_{\rm sym}(n_b) = J + Lx + \frac{1}{2}K_{\rm sym}x^2 + \dots$$

    J: symmetry energy at saturation, L: slope parameter,
    $K_{\rm sym}$: symmerty incompressibility coefficient

Results
Nuclear Matter III

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- **nuclear matter parameters**
  - n_{sat}: saturation density
  - M^{*}_{nuc} (Dirac) effective nucleon mass at saturaton

	B [MeV]	K [MeV]	Q [MeV]	J [MeV]	L [MeV]	K_{sym} [MeV]	n_{sat} [fm^{-3}]	M^{*}_{nuc} [M_{nuc}]
DDT	16.23 $\pm$0.03	229.20 $\pm$7.99	88.57 $\pm$230.06	31.21 $\pm$0.50	34.24 $\pm$4.33	-69.54 $\pm$15.45	0.15493 $\pm$0.00076	0.65729 $\pm$0.00132
DD2	16.03	242.72	168.77	31.67	55.03	-93.22	0.14908	0.56252

- special features of DDT:
  - small values of K and L
  - large values of n_{sat} and M^{*}_{nuc}

Results
Neutron Stars I

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- **properties of non-rotating, spherical neutron stars**
  - solve Tolman-Oppenheimer-Volkoff equation
    (R. Tolman, Phys. Rev. 55 (1939) 364,

    J.R. Oppenheimer and G.M. Volkoff, Phys. Rev. 55 (1939) 374)

$$\frac{dP}{dr} = -G\frac{M(r)\varepsilon(r)}{r^2}\left[1+\frac{P(r)}{\varepsilon(r)}\right]\left[1+\frac{4\pi r^3 P(r)}{M(r)}\right]\left[1-\frac{2GM(r)}{r}\right]^{-1}$$

with mass inside radius R

$$M(r) = 4\pi\int_0^r dr'\,(r')^2\,\varepsilon(r')$$

for given central density $n_{\mathrm{central}} \Rightarrow$ mass-radius relation

Results
Neutron Stars I

TECHNISCHE UNIVERSITÄT DARMSTADT

- **properties of non-rotating, spherical neutron stars**
  - solve Tolman-Oppenheimer-Volkoff equation
    (R. Tolman, Phys. Rev. 55 (1939) 364,
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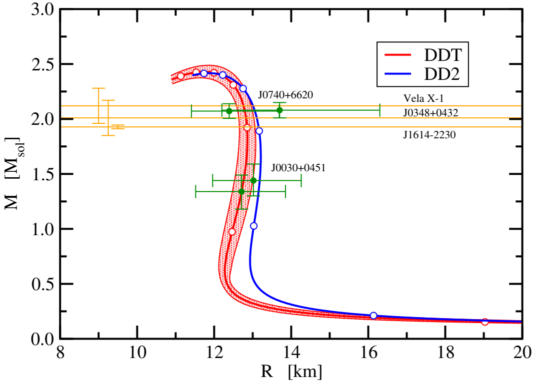
- essential ingredient: equation of state (ε, P, hadrons & leptons) of charge-neutral matter in β equilibrium, proper crust EOS

Results
Neutron Stars II

- **mass-radius relation**

  open symbols: multiples of saturation density



Results
Neutron Stars II

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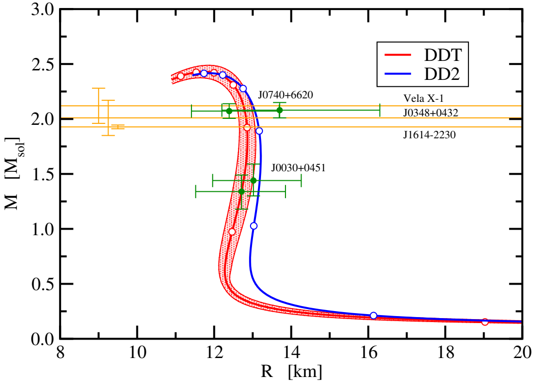
- **mass-radius relation**

  open symbols: multiples of saturation density

	DDT	DD2
$R_{1.4}$ [km]	12.744	13.172
M_{Max} [M_{sol}]	2.430	2.417
R_{max} [km]	11.739	11.869
$n_{\mathrm{central}}^{(\mathrm{max})}$ [fm^{-3}]	0.85785	0.85114



Correlations anc Clusters

Correlations and Composite Particles in Nuclear Matter

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- interacting many-body system $\Rightarrow$ **many-body correlations**
  - at lowest densities: only two-body correlations relevant
  - with increasing density: three-, four-, many-body correlations
    $\Rightarrow$ formation of many-body bound states: **nuclei = clusters**
  - with increasing temperature: competition with **entropy**

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- **composite particles**
  - at high densities: action of **Pauli principle**
    $\Rightarrow$ blocking of states
    $\Rightarrow$ suppression of correlations
    $\Rightarrow$ dissolution of clusters
  - **theoretical description?**

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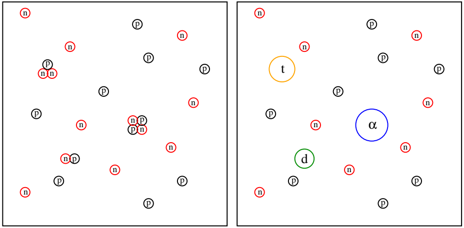
  - **theoretical description?**
- physical versus chemical picture
  ⇒ **degrees of freedom**



Description of Correlations at Low Densities

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- **finite temperature, exact limit $\Rightarrow$ virial equation of state (VEOS)**
  (E. Beth and G. Uhlenbeck, Physica 3(1936) 729, Physica 4 (1937) 915;
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- **extension of VEOS**
  ⇒ **generalized (cluster) Beth-Uhlenbeck approach** ⇒ physical picture
  (G. Röpke, L. Münchow, and H. Schulz, NPA 379 (1982) 536,
  M. Schmidt, G. Röpke, and H. Schulz, Ann. Phys. 202 (1990) 57,
  G. Röpke, N.-U. Bastian et al., NPA 897 (2013) 70)
  ⇒ suppression of cluster formation with increasing density

Cluster Formation and Dissolution

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- example: **deuteron as two-body correlation**
  - n-p-d system, no interactions
  - no deuteron suppression at high densities in NSE or standard VEOS

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  - geometric picture (finite size of particles)
    $\Rightarrow$ **excluded-volume mechanism**
    – applications to compact star matter
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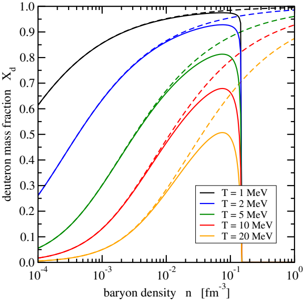
    T. Fischer et al., EPJ A 50 (2014) 46; M. Hempel, PRC 91 (2015) 055897)
    – generalized formulation, different interpretation
    (S. Typel, EPJ A 52 (2016) 16)

symmetric n-p-d matter



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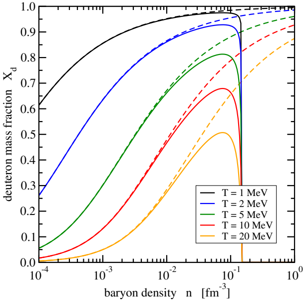
    - generalized formulation, different interpretation (S. Typel, EPJ A 52 (2016) 16)
  - medium modification of cluster properties $\Rightarrow$ **mass shifts**
    - action of Pauli principle $\Rightarrow$ blocking of states
    - density, temperature, momentum dependence

symmetric n-p-d matter



Mass Shifts I

TECHNISCHE UNIVERSITÄT DARMSTADT

- **concept applies to composite particles: clusters**
  - light and heavy nuclei
  - nucleon-nucleon correlations in continuum
    $\Rightarrow$ medium dependent resonances
- **effective change of masses/binding energies**

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- **concept applies to composite particles: clusters**
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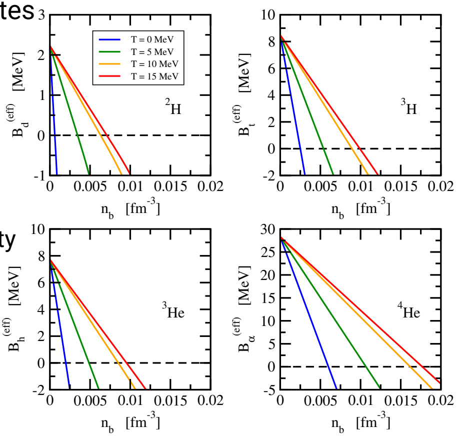
- **effective change of masses/binding energies**
- **two major contributions** $\Delta m_i = \Delta m_i^{\text{strong}} + \Delta m_i^{\text{Coul}}$
  - strong shift $\Delta m_i^{\text{strong}} = \Delta m_i^{\text{meson}} + \Delta m_i^{\text{Pauli}}$
    - effects of strong interaction (coupling to mesons)
    - Pauli exclusion principle: blocking of states in the medium
      $\Rightarrow$ reduction of binding energies
      $\Rightarrow$ cluster dissolution at high densities: Mott effect
      $\Rightarrow$ replaces traditional excluded-volume mechanism
  - electromagnetic shift Δm_i^{Coul} (in stellar matter)
    - electron screening of Coulomb field $\Rightarrow$ increase of binding energies

$\Rightarrow$ rearrangement contribution in density functional

Mass Shifts II

TECHNISCHE UNIVERSITÄT DARMSTADT

- light nuclei and NN scattering states
  - **parametrisation from Gerd Röpke (Rostock)**
    simplified and modified for high densities and temperatures
  - scattering states: mass shifts as for deuteron
  - dependence of $\Delta m_i^{\text{Pauli}}$ on temperature and effective density
    $n_i^{\text{eff}} = \frac{2}{A_i} \left[Z_i Y_q + N_i (1 - Y_q) \right] n_b$
  - Δm_i^{Coul} in Wigner-Seitz approximation
  - full coupling of nucleons in clusters to meson fields



Mass Shifts III

TECHNISCHE UNIVERSITÄT DARMSTADT

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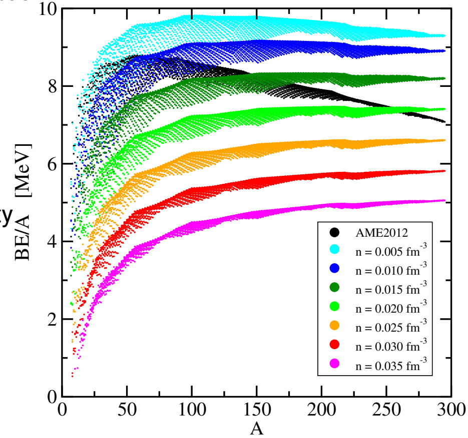
  - $\Delta m_i^{\mathrm{Coul}}$ in Wigner-Seitz approximation
  - full coupling of nucleons in clusters to meson fields
- heavy nuclei
  - heuristic parametrisation



Correlations at High Densities

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- baryon density n above n_{sat}
  $\Rightarrow$ no clusters expected as degrees of freedom
  $\Rightarrow$ only single baryons (nucleons, hyperons, ...)
- **microscopic models** (e.g. Brueckner HF)
  $\Rightarrow$ explicit two-particle correlations

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  - mixture of baryons as quasiparticles
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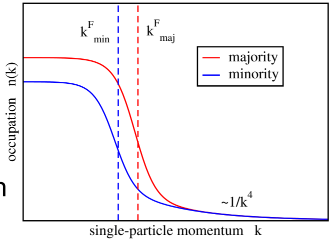


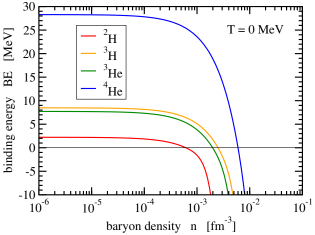
- **experiments**
  nucleon knockout from nuclei in inelastic electron scattering
  (O. Hen et al. (CLAS Collaboration), Science 346 (2014) 614, ...)
  $\Rightarrow$ no sharp cut-off, high-momentum tail

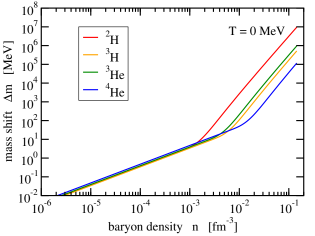
Mass Shifts at High Densities

- **choice of density dependence of cluster mass shifts**
  - low densities: linear in n as given by parameterisation of Gerd Röpke
  - higher densities (above Mott density): steeper function ($\propto n^3$, artificial) to avoid reappearance of clusters

$\Rightarrow$ no clusters above saturation density by construction

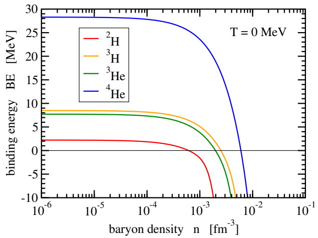
$\Rightarrow$ transition to mixture of nucleons as quasiparticles

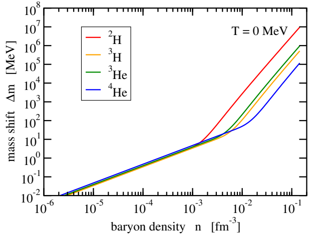




Mass Shifts at High Densities

- **choice of density dependence of cluster mass shifts**
  $\Rightarrow$ no clusters above saturation density by construction
  $\Rightarrow$ transition to mixture of nucleons as quasiparticles
- representation of short-range correlations (SRC) above saturation density in energy density functionals?
  $\Rightarrow$ **quasi-deuterons as surrogate for two-body correlations**





Compact Star Matter

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- number of independent **chemical potentials**
  $=$ number of **conserved charges**
  - baryon number $\rightarrow$ baryon chemical potential μ_B
  - charge number $\rightarrow$ charge chemical potential μ_Q
  - electron/muon lepton number $\rightarrow$ electron/muon lepton potential μ_{L_e}/μ_{L_μ}
  - strangeness number $\rightarrow$ strangeness chemical potential μ_S (usually $\mu_S = 0$)

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- **chemical equilibrium** $\Rightarrow$ relation of chemical potentials
  $\mu_i = B_i\mu_B + Q_i\mu_Q + L_{ei}\mu_{L_e} + L_{\mu i}\mu_{L_\mu} + S_i\mu_S$
  with baryon, charge,... numbers $B_i, Q_i, \ldots$ of particle i

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  with baryon, charge,... numbers $B_i, Q_i, \ldots$ of particle i
- condition of **charge neutrality** fixes μ_Q
- condition of β **equilibrium** (compact stars) fixes $\mu_{L_e} = 0$ ($\mu_{L_\mu} = 0$)
  $\Rightarrow$ only one independent chemical potential (μ_B)

Global EoS for Astrophysical Applications I

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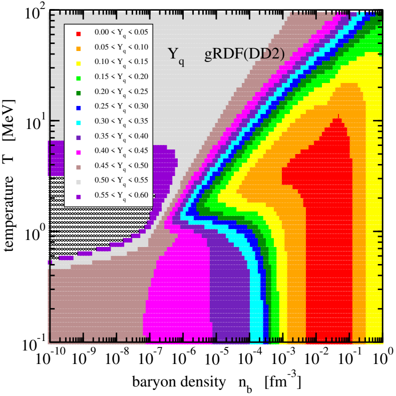
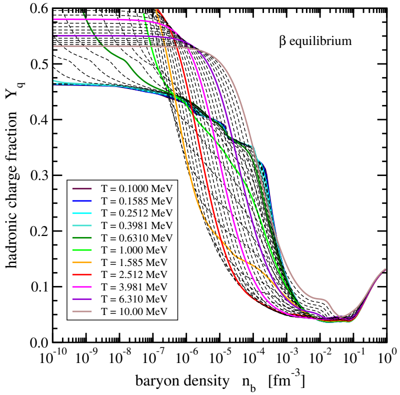
- **hadronic charge fraction $Y_q = \sum_i Q_i n_i / n_b$** (without leptons)
  $\Rightarrow$ neutronisation with increasing baryon density





Global EoS for Astrophysical Applications II

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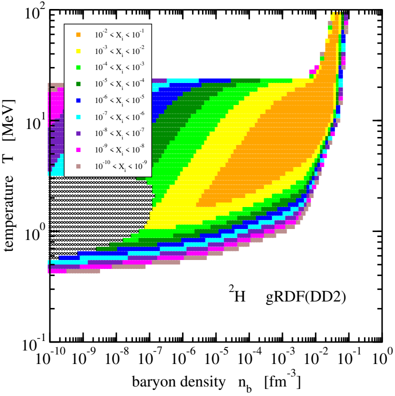
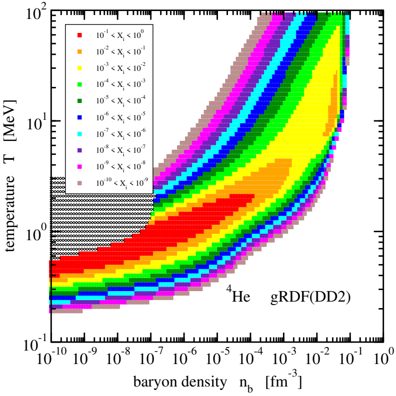
- mass fractions $X_i = A_i n_i / n_b$ of ^{2}H and ^{4}He





Global EoS for Astrophysical Applications III

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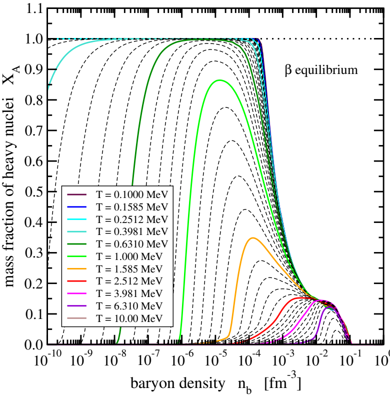
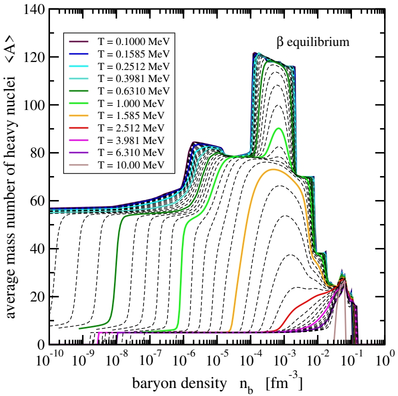
- **mass fraction X_{heavy} and average mass number $\langle A \rangle$ of heavy nuclei**





Global EoS for Astrophysical Applications IV

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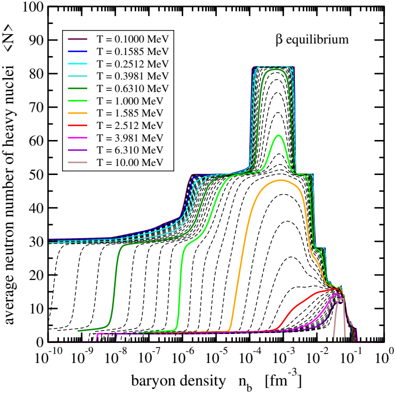
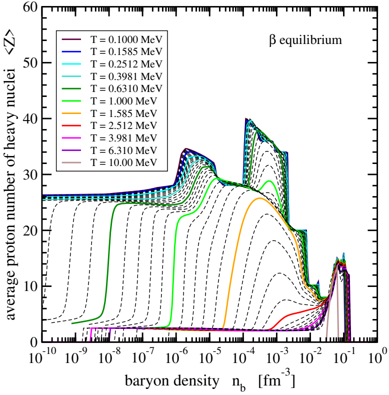
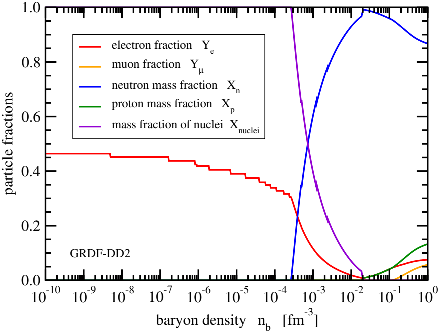
- **average neutron $\langle N \rangle$ and charge number $\langle Z \rangle$ of heavy nuclei**

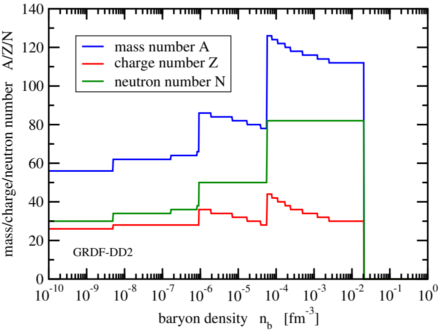




Compact Star Matter Equation of State – Low Densities

- temperature $T = 0$, β equilibrium
- sequence of ions in background of electrons, phase transitions
- free neutrons above neutron drip density





Conclusions

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- extension of relativistic mean-field approach (see arXiv:2408.00425)
  - density dependent minimal nucleon-meson couplings
  - additional tensor couplings (ω and ρ)
- new parameterisation of effective interaction
  - careful selection of observables (only nuclei)
  - self-consistent determination of uncertainties

$\Rightarrow$ improved description of nuclei

- modified equation of state (EOS) of nuclear matter
  - small K & L, but stiff EoS
- neutron stars
  - $M_{\mathrm{max}} > 2M_{\mathrm{sol}}$, radii with DDT smaller than with DD2
- work in progress: revision of cluster description/mass shifts, EoS tables with new parameterisation DDT

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Thank You for Your Attention!

Backup Slides

Field Equations - Nucleons

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- **modified Dirac equation** for nucleons

$$H\Psi_\eta = \left[\vec{\alpha}\cdot\vec{p} + \beta\left(M_\eta - S\right) + V_\eta + i\vec{\gamma}\cdot\frac{\vec{r}}{r}T_\eta\right] = E_\eta\Psi_\eta$$

  - scalar potential $S = \Gamma_\sigma \sigma \Rightarrow$ effective mass $M_\eta^* = M_\eta - S$
  - vector potential $V_\eta = \Gamma_\omega \omega_0 + g_\eta \Gamma_\rho \rho_0 + \frac{1+g_\eta}{2}\Gamma_\gamma A_0 + V^{(R)}$
    with $g_\eta = \pm 1$ for $\eta = p/n$ and rearrangement contribution
    $V^{(R)} = \frac{d\Gamma_\omega}{dn_b} n_\omega \omega_0 + \frac{d\Gamma_\rho}{dn_b} n_\rho \rho_0 - \frac{d\Gamma_\sigma}{dn_b} n_\sigma \sigma$
  - tensor potential $T_\eta = -\frac{\Gamma_{T\omega}}{M_p}\frac{\vec{r}}{r}\cdot\vec{\nabla}\omega_0 - \frac{\Gamma_{T\rho}}{M_p}\frac{\vec{r}}{r}\cdot\vec{\nabla}\rho_0$

    $\Rightarrow$ effective at surface of nuclei,
    vanishes in homogeneous nuclear matter

Field Equations III

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- **Klein-Gordon/Poisson equations** for mesons/photon

$$
\begin{aligned}
-\Delta\sigma + M_\sigma^2 \sigma &= \Gamma_\sigma n_\sigma \\
-\Delta\omega_0 + M_\omega^2 \omega_0 &= \Gamma_\omega n_\omega + \frac{\Gamma_{T\omega}}{M_p} \vec{\nabla} \cdot \vec{\jmath}_\omega^{(t)} \\
-\Delta\rho_0 + M_\rho^2 \rho_0 &= \Gamma_\rho n_\rho + \frac{\Gamma_{T\rho}}{M_p} \vec{\nabla} \cdot \vec{\jmath}_\rho^{(t)} \\
-\Delta A_0 &= \Gamma_\gamma n_\gamma
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Field Equations - Mesons & Photon

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\end{aligned}$$

- source densities and currents
  - $n_\sigma = n_p^{(s)} + n_n^{(s)}, n_\omega = n_p^{(v)} + n_n^{(v)}, n_\rho = n_p^{(v)} - n_n^{(v)}, n_\gamma = n_p^{(v)}$
    with scalar and vector densities $n_\eta^{(s)} = \langle \Psi_\eta | \Psi_\eta \rangle, n_\eta^{(v)} = \langle \Psi_\eta | \gamma^0 | \Psi_\eta \rangle$
  - $\vec{\jmath}_\omega = \vec{\jmath}_p + \vec{\jmath}_n, \vec{\jmath}_\rho = \vec{\jmath}_p - \vec{\jmath}_n$
    with tensor currents $\vec{\jmath}_\eta^{(t)} = \langle \Psi_\eta | i\vec{\alpha} | \Psi_\eta \rangle$

Description of Correlations at Low Densities I

TECHNISCHE UNIVERSITÄT DARMSTADT

- **finite temperature, exact limit ⇒ virial equation of state (VEOS)**
  (E. Beth and G. Uhlenbeck, Physica 3(1936) 729, Physica 4 (1937) 915;
  C. J. Horowitz and A. Schwenk, NPA 776 (2006) 55)
  - expansion of pressure in powers of fugacities $z_i = \exp(\mu_i/T)$

$$p = TV \left(\sum_i \frac{g_i}{\lambda_i^3} z_i + \sum_{ij} \frac{b_{ij}}{\lambda_i^{3/2} \lambda_j^{3/2}} z_i z_j + \ldots \right)$$ with thermal wavelength $\lambda_i = [2\pi/(m_i T)]^{1/2}$

and virial coefficients $g_i, b_{ij}, \ldots \Rightarrow$ limitation $n_i \lambda_i^{-3} \ll 1$

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    and virial coefficients $g_i, b_{ij}, \ldots \Rightarrow$ limitation $n_i \lambda_i^{-3} \ll 1$
  - only two-body correlations relevant at lowest densities, encoded in

    $$b_{ij} = \frac{1+\delta_{ij}}{2} \frac{\lambda_i^{3/2}\lambda_j^{3/2}}{\lambda_{ij}^3} \int dE \, \exp\left(-\frac{E}{T}\right) D_{ij}(E) \pm \delta_{ij} \frac{g_i}{2^{5/2}} \qquad \lambda_{ij} = \{2\pi/[(m_i+m_j)T]\}^{1/2}$$

    with 'density of states' $D_{ij}(E) = \sum_k g_k^{(ij)} \delta(E - E_k^{(ik)}) + \sum_l \frac{g_l^{(ij)}}{\pi} \frac{d\delta_l^{(ij)}}{dE}$

    $\Rightarrow$ contribution from bound states and continuum,
    depends only on experimental data: binding energies $E_k^{(ik)}$, phase shifts $\delta_l^{(ij)}$
    (not independent! Levinson theorem)

Description of Correlations at Low Densities II

TECHNISCHE UNIVERSITÄT DARMSTADT

- **simplification of VEOS**
  $\Rightarrow$ **nuclear statistical equilibrium (NSE)** $\Rightarrow$ chemical picture
  - consider nucleons and all nuclei (ground and excited states)
  - no contributions from continuum, no explicit interaction

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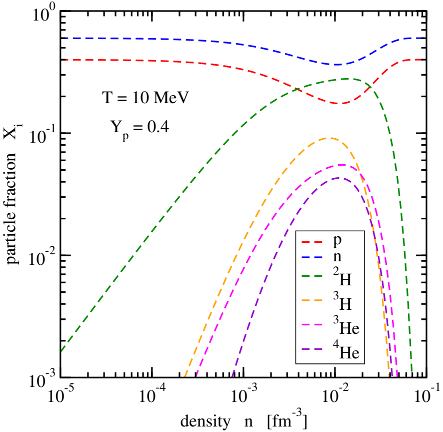
  $\Rightarrow$ **nuclear statistical equilibrium (NSE)** $\Rightarrow$ chemical picture
  - consider nucleons and all nuclei (ground and excited states)
  - no contributions from continuum, no explicit interaction
- **extension of VEOS**
  $\Rightarrow$ **generalized (cluster) Beth-Uhlenbeck approach** $\Rightarrow$ physical picture
  (G. Röpke, L. Münchow, and H. Schulz, NPA 379 (1982) 536,
  M. Schmidt, G. Röpke, and H. Schulz, Ann. Phys. 202 (1990) 57,
  G. Röpke, N.-U. Bastian et al., NPA 897 (2013) 70)
  - quantum statistical description with thermodynamic Green's functions
  - part of interaction included in self-energies of quasiparticles
  - modified second virial coefficient
    $\Rightarrow$ dependence on particle-pair momentum,
    correction factor in continuum contribution

  $\Rightarrow$ suppression of cluster formation with increasing density

Light Clusters and Continuum Correlations

- particle mass fractions
  $X_i = A_i \frac{n_i}{n} \qquad n = n_b = \sum_i A_i n_i$
- low densities: two-body correlations most important
- high densities: dissolution of clusters
  $\Rightarrow$ Mott effect



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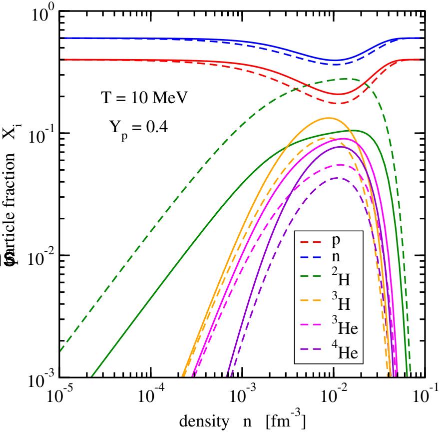
- low densities: two-body correlations most important
- high densities: dissolution of clusters
  $\Rightarrow$ Mott effect
- effect of NN continuum correlations
  - dashed lines: without continuum
  - full lines: with continuum

  $\Rightarrow$ reduction of deuteron fraction, redistribution of other particles
- correct low-density limit



Light Clusters in Heavy-Ion Collisions

TECHNISCHE UNIVERSITÄT DARMSTADT

emission of light nuclei

- determination of density and temperature of source

  S. Kowalski et al. PRC 75 (2007) 014601

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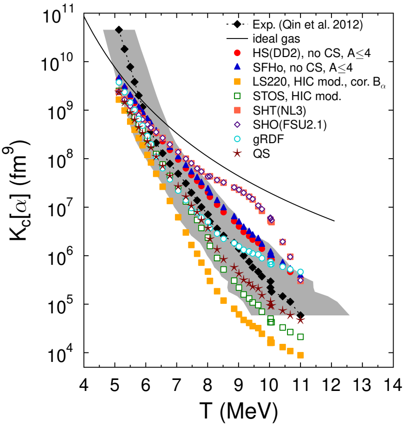

M. Hempel, K. Hagel, J. Natowitz, G. Röpke, S. Typel, PRC C 91 (2015) 045805

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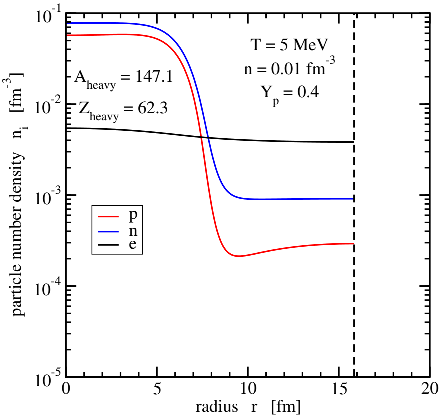
more recent data from INDRA collaboration
see R. Bougault et al., J. Phys. G 47 (2020) 025103
and analysis, e.g., in
H. Pais et al., Phys. Rev. Lett 125 (2020) 012701
H. Pais et al., J. Phys. G 47 (2020) 105204
T. Custodio et al., Eur. Phys. J. A 56 (2020) 295

Formation of Heavy Clusters

TECHNISCHE UNIVERSITÄT DARMSTADT

- **nuclear matter**
  - liquid-gas phase transition
  - separation of phases
  - no surface or Coulomb effects
- **heavy nuclei in stellar matter**
  - relativistic density functional with nucleons, light nuclei, electrons (for charge neutrality)
  - spherical Wigner-Seitz cell
  - extended Thomas-Fermi approximation
  - self-consistent calculation



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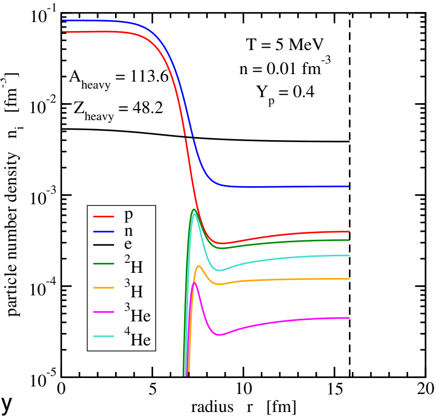
  - self-consistent calculation
  - increased probability of finding light clusters at nuclear surface
  - effective binding energy from energy difference to homogeneous matter

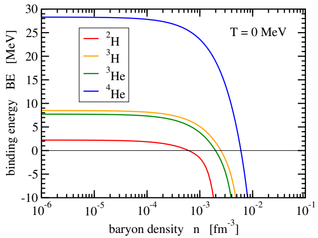


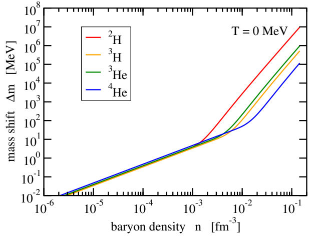
Mass Shifts at High Densities

- **choice of density dependence of cluster mass shifts**
  - low densities: linear in n as given by parameterisation of Gerd Röpke
  - higher densities (above Mott density): steeper function ($\propto n^3$, artificial) to avoid reappearance of clusters

$\Rightarrow$ no clusters above saturation density by construction

$\Rightarrow$ transition to mixture of nucleons as quasiparticles





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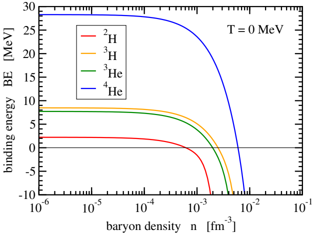
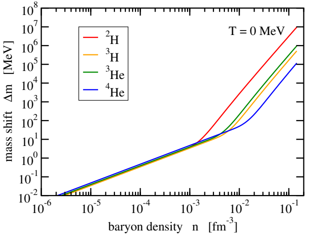
- **representation of short-range correlations (SRC) above saturation density in energy density functionals?**





Correlations and Mass Shifts at High Densities

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- **clusters as effective many-body correlations**
  - internal motion of nucleons in cluster
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- **zero temperature**
  - condensation of bosonic clusters
    $\Rightarrow$ condition on chemical potentials $\mu_d = \mu_n + \mu_p$
    $\Rightarrow \Delta m_d = S_d^{\mathrm{meson}} - m_d + \sqrt{k_n^2 + (m_n - S_n^{\mathrm{meson}})^2} + \sqrt{k_p^2 + (m_p - S_p^{\mathrm{meson}})^2}$
    with Fermi momenta k_n and k_p of neutrons and protons
    $\Rightarrow$ density dependence of mass shift Δm_d
    for given deuteron mass fraction $X_d = 2n_d/n$
  - revision of functional form of cluster mass shifts
    (S. Typel, Eur. Phys. J. ST 229 (2020) 3433)

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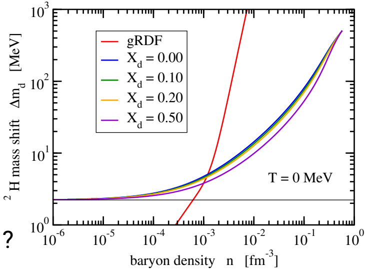
  - comparison of deuteron mass shifts of original GRDF model with condensation condition for fixed X_d
    $\Rightarrow$ parametrisation of Δm_d for transition ?



Deuteron Mass Shift and Mass Fraction I

TECHNISCHE UNIVERSITÄT DARMSTADT

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  (S. Burrello, S. Typel, EPJ A 58 (2022) 120)
  - extrapolation of deuteron mass shift Δm_d to high densities
  - correct low-density limit
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    $\Rightarrow$ rescaling of meson couplings to recover energy at saturation

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    $\Rightarrow$ rescaling of meson couplings to recover energy at saturation
  - dependence on scaling χ of coupling strength of mesons
    to nucleons inside deuteron
    - $\chi = 1$ full strength as in nuclear medium
    - $\chi < 1$ reduced strength
  - restrictions on allowed deuteron mass fraction X_d
    - positive effective masses of nucleons required

Deuteron Mass Shift and Mass Fraction II

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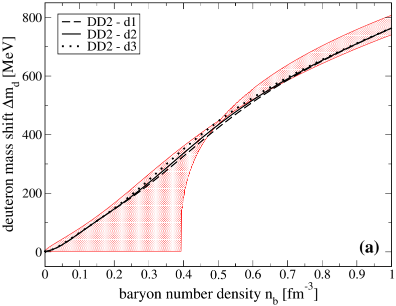
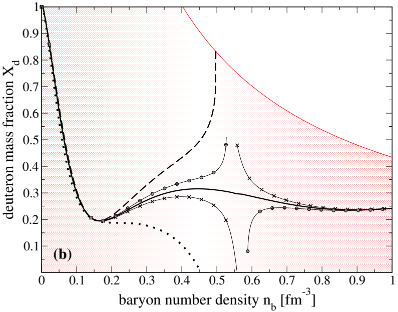
- *case 1:* determination of Δm_d from given $X_d \Rightarrow$ straightforward

Deuteron Mass Shift and Mass Fraction II

- *case 1:* determination of Δm_d from given X_d $\Rightarrow$ straightforward
- *case 2:* calculation of X_d from given Δm_d (relevant case)
  - *example:* symmetric nuclear matter, $\chi = 1$, heuristic form
  - sensitivity to Δm_d $\Rightarrow$ fine-tuning required for $\chi > 1/\sqrt{2}$

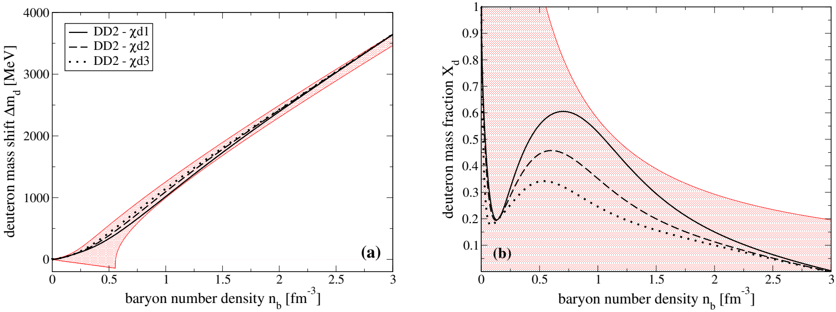




Deuteron Mass Shift and Mass Fraction III

TECHNISCHE UNIVERSITÄT DARMSTADT

- *case 1:* determination of Δm_d from given $X_d \Rightarrow$ straightforward
- *case 2:* calculation of X_d from given Δm_d (relevant case)
  - *example:* symmetric nuclear matter, $\chi = 1/\sqrt{2}$, heuristic form
    $\Rightarrow$ smooth variation of deuteron mass fraction X_d



Asymmetric Nuclear Matter with Quasi-Deuterons

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- density dependence of **symmetry energy E_{sym}**
  - quantitfies isospin dependence of energy
  - parabolic approximation
    (comparison of neutron matter with symmetric nuclear matter)
- comparison of models
  - GRDF with original
    DD2 parameterisation
    (without deuterons, red line)
  - GRDF with quasi-deuterons
    - dependence of mass-shift parameterisation
    - stiffening of E_{sym} at high n_b
    - correct low-density limit
      (half deuteron binding energy for $n_b \to 0$)