Infinite nuclear matter within the Hartree approximation for nucleons and pions

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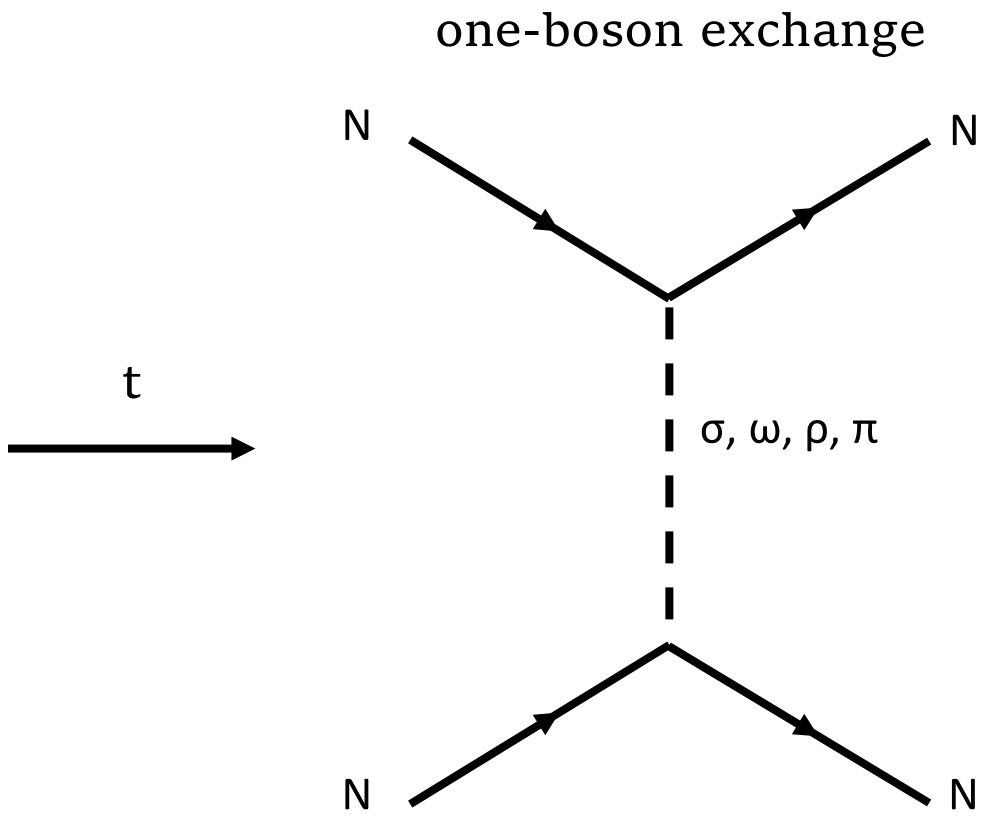
INFINITE NUCLEAR MATTER

SATURATION DENSITY n_0	$0.155 \pm 0.05\ \text{fm}^{-3}$ ($\approx 2 \times 10^{14}\ \text{g/cm}^3$)
BINDING ENERGY $\frac{B}{A}$	-16 ± 1.0 MeV
COMPRESSION MODULUS $K = 9n_B \frac{\partial p}{\partial n_B}$	250 ± 50 MeV
SYMMETRY ENERGY e_{sym}	32.0 ± 2.0 MeV



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L.-J. Guo, et al.. Insights into Neutron Star Equation of State by Machine Learning. *The Astrophysical Journal*, 2024.

WALECKA MODEL

$$\mathcal{L} = \underbrace{\bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi}_{\mathcal{L}_{Dirac}} + \underbrace{\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2)}_{\mathcal{L}_\sigma} - \underbrace{\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu}_{\mathcal{L}_\omega} - \underbrace{\frac{1}{4}\rho^a_{\mu\nu}\rho^{a\mu\nu} + \frac{1}{2}m_\rho^2\rho^a_\mu\rho^{a\mu}}_{\mathcal{L}_\rho}$$

$$\underbrace{-\frac{1}{3}g_3\sigma^3 - \frac{1}{4}g_4\sigma^4 + \frac{1}{4}c_4(\omega_\mu\omega^\mu)^2}_{\mathcal{L}_{non-lin}} + \underbrace{g_\sigma\sigma\bar{\psi}\psi - g_\omega\omega_\mu\bar{\psi}\gamma^\mu\psi - g_\rho\bar{\psi}\gamma^\mu\tau^a\rho^a_\mu\psi}_{\mathcal{L}_{int}}$$

! Infinite # of d.o.f →

a) no-sea approx.

b) mean-field approx.:

1. $\sigma \to \langle\sigma\rangle \equiv \sigma_0$
2. $\omega_\mu \to \langle\omega_\mu\rangle \equiv \delta_{\mu 0}\omega_0$
3. $\rho^a_\mu \to \langle\rho^a_\mu\rangle \equiv \delta^{a3}\delta_{\mu 0}\rho^3_0$

$$\begin{cases} m^* \equiv m - g_\sigma\sigma_0 \\ E^*(\boldsymbol{p}) \equiv E(\boldsymbol{p}) - g_\omega\omega_0 \\ \mu_n^* \equiv \mu_n - g_\omega\omega_0 + g_\rho\rho_0 \\ \mu_p^* \equiv \mu_p - g_\omega\omega_0 - g_\rho\rho_0 \end{cases}$$

d.o.f: g_σ, g_3, g_4, g_ω, c_4, g_ρ

EOS: $P(\varepsilon)$

energy-momentum tensor →

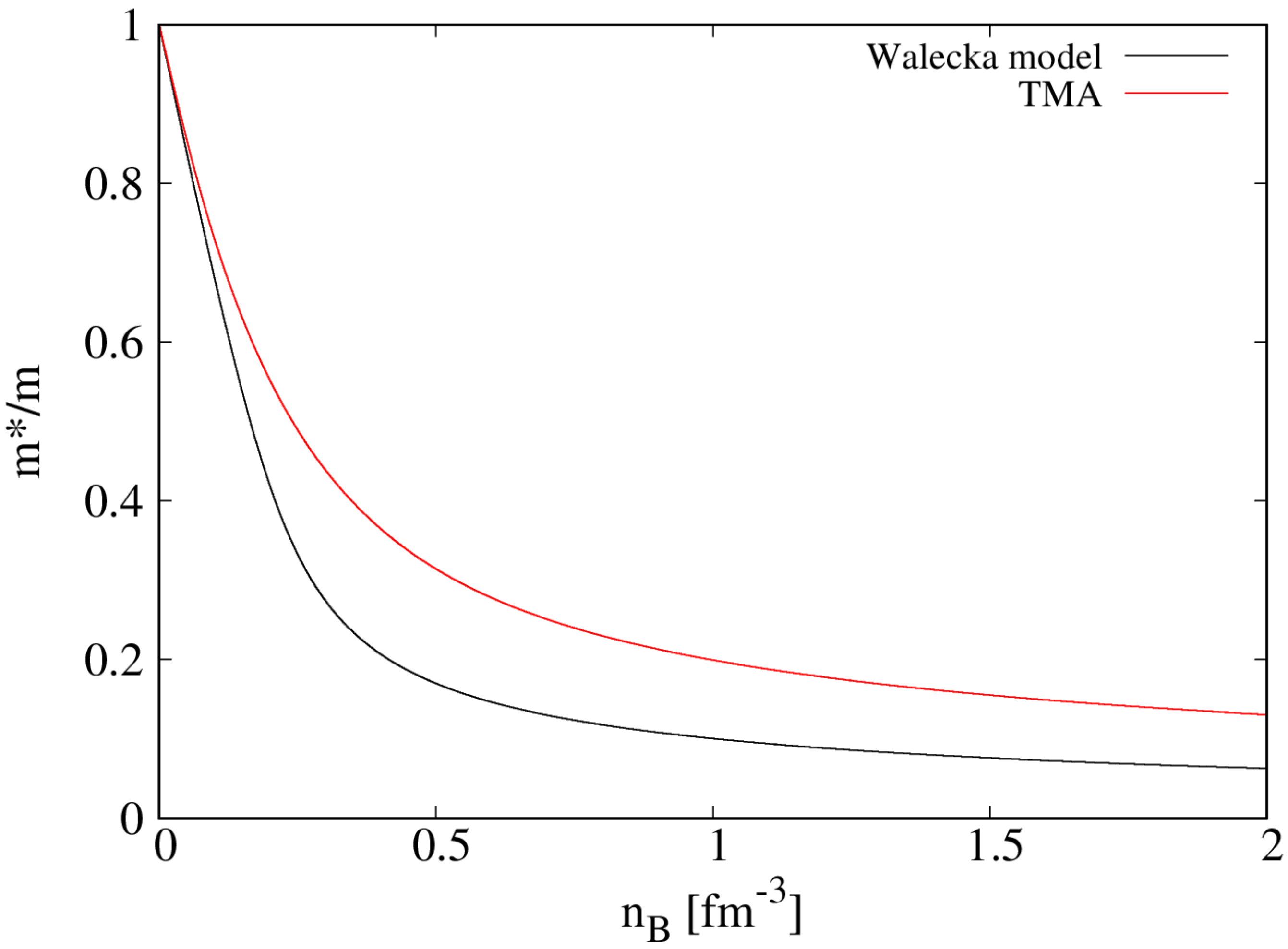
$$T^{\mu\nu}_{MFA} = i\bar{\psi}\gamma^\mu\partial^\nu\psi - g^{\mu\nu}\left(\frac{1}{2}m_\omega^2\omega_0^2 - \frac{1}{2}m_\sigma^2\sigma_0^2\right)$$

perfect fluid →

$$\langle T^{\mu\nu}\rangle = diag(\varepsilon, P, P, P)$$

WALECKA MODEL, PROPERTIES

	Walecka	TMA
n_0 [fm^{-3}]	0.148	0.147
$E/\mathrm{n_B} - m$ [MeV]	−15.76	−16.03
K [MeV]	544.84	317.12
E_{sym} [MeV]	20.35	31.61



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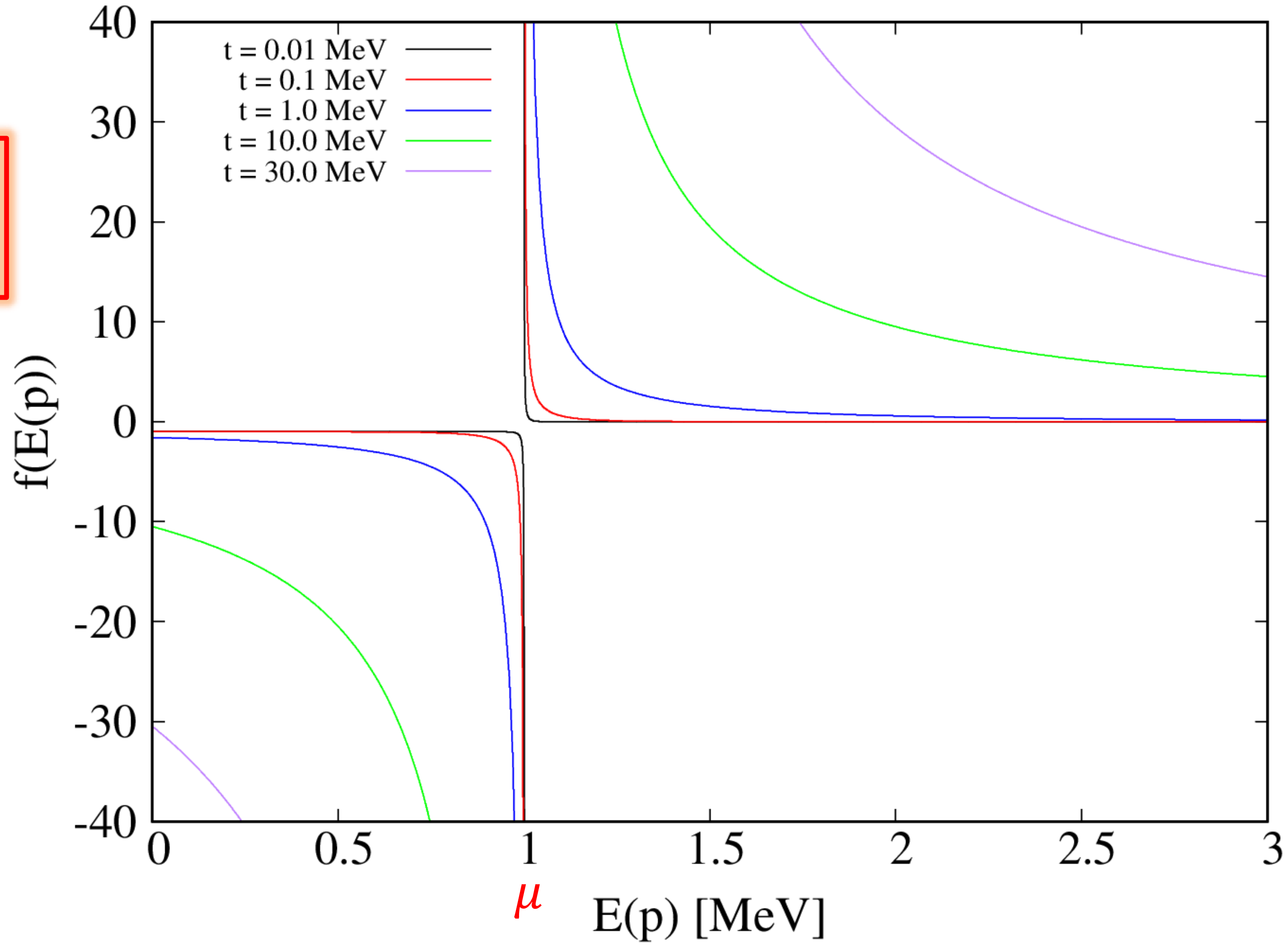
Ilona Bednarek. Relativistic mean field models of neutron stars. Katowice: Wydawnictwo Uniwersytetu Śląskiego, 2007
H. Toki, D. Hirata, Y. Sugahara, K. Sumiyoshi, and I. Tanihata, Relativistic many body approach for unstable nuclei and supernova, *Nucl. Phys. A*, 1995

BOSE-EINSTEIN STATISTICS

$$f(\boldsymbol{p};\{\mu,T\}) = \left(\exp\left(\frac{E(\boldsymbol{p})-\mu}{k_B T}\right)-1\right)^{-1}$$

$$n \geq 0 \quad \Rightarrow \quad E(\boldsymbol{p})-\mu \geq 0$$



A. L. Fetter, J. D. Walecka, Quantum Theory of Many-Particle Systems, *McGraw-Hill Book Company,* 1971

PIONS, 1ST WAY – GROUND STATE APPROACH

$$\pi = \{\pi^-, \pi^+, \pi^0\} \qquad \mathcal{L}_\pi = \frac{1}{2}\left(\partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi} - m_\pi^2 \boldsymbol{\pi}^2\right) + \mathcal{L}_{\pi NN}$$

$$\mathcal{L}_{PV} = -\frac{f_\pi}{\mathrm{m}_\pi} \bar{\psi} \gamma^5 \gamma^\mu \partial_\mu \boldsymbol{\pi} \cdot \boldsymbol{\tau} \psi$$

$$\mathcal{L}_{PS} =- -i g_\pi \bar{\psi} \gamma^5 \boldsymbol{\pi} \cdot \boldsymbol{\tau} \psi$$

$$\begin{cases} \left[\gamma_\mu \partial^\mu - m + g_\sigma \sigma_0 - g_\omega \gamma^0 \omega_0 - \frac{f_\pi}{m_{pi}} \gamma^5 \gamma^\mu \partial_\mu \boldsymbol{\pi} \cdot \boldsymbol{\tau}\right] \psi(x) = 0 \\ (\Box + m_\sigma^2)\langle \sigma(x) \rangle = g_\sigma \langle \bar{\psi}(x) \psi(x) \rangle \\ (\Box + m_\omega^2)\langle \omega_0(x) \rangle = g_\omega \langle \psi^\dagger(x) \psi(x) \rangle \\ (\Box + m_\pi^2)\langle \boldsymbol{\pi}(x) \rangle = \frac{f_\pi}{m_\pi} \partial_\mu \langle \bar{\psi}(x) \gamma^5 \gamma^\mu \boldsymbol{\tau} \psi(x) \rangle \end{cases}$$

Parity transformation

$P: \quad \boldsymbol{x} \to -\boldsymbol{x}$

→

pseudoscalar

↓

$$\langle \boldsymbol{\pi}(x) \rangle \to -\langle \boldsymbol{\pi}(-x) \rangle$$

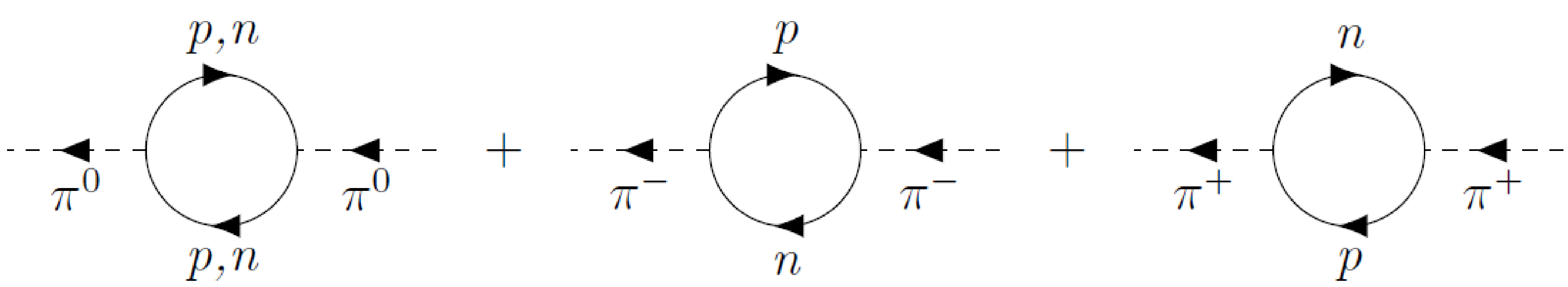
$$\langle \boldsymbol{\pi}(x) \rangle = 0$$

B. Banerjee, N. K. Glendenning, M. Gyulassy, Pion Condensation in a Relativistic Field Theory Consistent with Bulk Properties of Nuclear Matter, *Nucl. Phys. A*, 1981

T. Matsui, B. D. Serot, The Pion Propagator in Relativistic Quantum Field Theories of the Nuclear Many-Body Problem, *Ann. of Phys.*, 1982

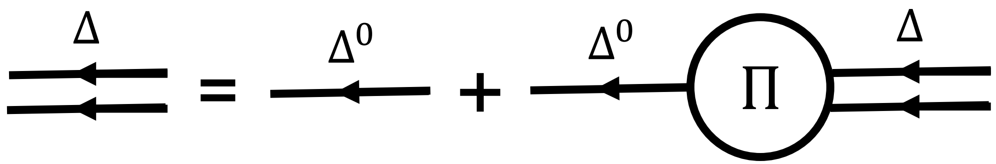
PIONS, 2ND WAY – PROPAGATOR APPROACH

$$n, p \rightarrow n, p + \pi^0 \quad , \quad n \rightarrow p + \pi^- \quad , \quad p \rightarrow n + \pi^+$$



Dyson's equation:

$$i\Delta(p) = i\Delta^0(p) + i\Delta^0(p)\Pi(\mathrm{p})\Delta(\mathrm{p})$$



$$i\Delta^0_{ab}(p) = \frac{\delta_{ab}}{p^2 - m_\pi^2 + i\epsilon}$$

$$i\Delta^0_{ab}(p) = \frac{\delta_{ab}}{p^2 - m_\pi^2 - \mathrm{Re}\Pi(p)}$$

T. Herbert, K. Wehrberger, F. Beck, The Pion Propagator in the Walecka Model with the Delta Baryon, *Nucl. Phys. A,* 1992

PION CONDENSATE

$$n, p \to n, p + \pi^0 \quad , \quad n \to p + \pi^- \quad , \quad p \to n + \pi^+$$

$$\mu_\pi = \mu_{n,p} - \mu_{n,p} = 0$$

$$\mu_\pi = |\mu_n - \mu_p|$$

Pion condensate,
threshold conditions

$$E_\pi(p) = \mu_\pi = 0$$

$$E_\pi(p) = \mu_\pi \leq |\mu_n - \mu_p|$$

$$i\Delta^0_{ab}(p) = \frac{\delta_{ab}}{p^2 - m_\pi^2 - \mathrm{Re}\Pi(p)}$$

$$p^2 = E^2(p) - \boldsymbol{p}^2$$

$$E(\boldsymbol{p}) = \boldsymbol{p}^2 + m_\pi^2 + \mathrm{Re}\Pi(E(p) = 0, \boldsymbol{p}) = 0$$

$$E(\boldsymbol{p}) = \boldsymbol{p}^2 + m_\pi^2 + \mathrm{Re}\Pi\big(E(p) = |\mu_n - \mu_p|, \boldsymbol{p}\big) = \left(\mu_n - \mu_p\right)^2$$

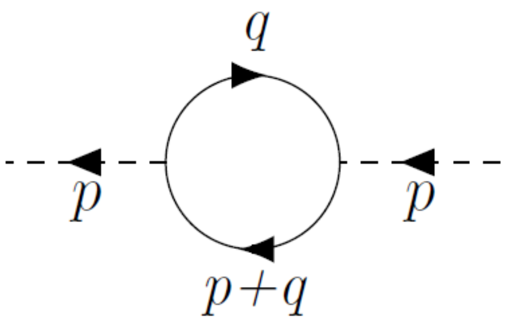
T. Shammsunnahar, S. Saha, K. Kabir, L. M. Nath, Pion Condensation in Symmetric Nuclear Matter, *J. Phys. G: Nucl. Part. Phys.*, 1991

PION SELF-ENERGY

Vertices:

$$\Gamma_{PV} = -i\frac{f_\pi}{m_\pi}\gamma^5\gamma^\mu q_\mu$$

$$\Gamma_{PS} = -ig_\pi\gamma^5$$



$$\Pi(p) = -i\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}[i\Gamma(q)iG(p+q)i\Gamma(-q)iG(q)]$$

Baryon propagator

$$G(q) = (\gamma^\mu q_\mu + m)\left[\frac{1}{q^2-m^2+i\epsilon}+\frac{i\pi}{E(\boldsymbol{p})}\delta(p^0-E(\boldsymbol{p}))\theta(q_F-|\boldsymbol{q}|)\right] \equiv \mathrm{G_F(q)} + \mathrm{G_D(q)}$$ ⬅ DF repr.

$$= (\gamma^\mu q_\mu + m)\left(\frac{\theta(|\boldsymbol{q}|-q_F)}{q^0-E(\boldsymbol{q})+i\epsilon}+\frac{\theta(q_F-|\boldsymbol{q}|)}{q^0-E(\boldsymbol{q})-i\epsilon}\right)-(\gamma^\mu\tilde{q}_\mu+m)\frac{1}{q^0+E(\boldsymbol{q})-i\epsilon} \equiv G_P(q)+G_H(q)+G_A(q)$$ ⬅ PHA repr.

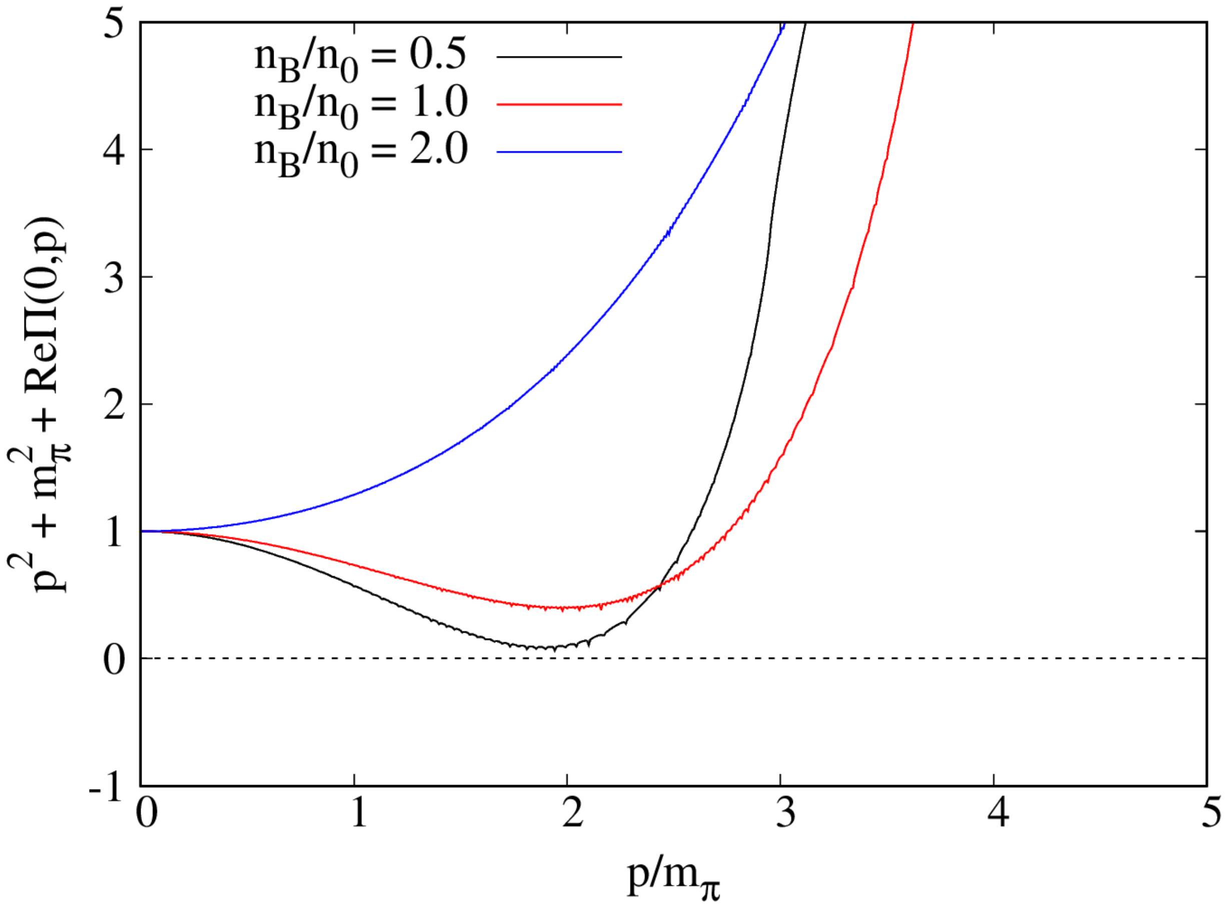
$$\tilde{q}^\mu = (-E(\boldsymbol{q}),\boldsymbol{q})$$

M. Nakano, et al., Effective Meson Masses in Nuclear Matter Based on a cutoff Field Theory, *Phys. Rev. C,* 1997

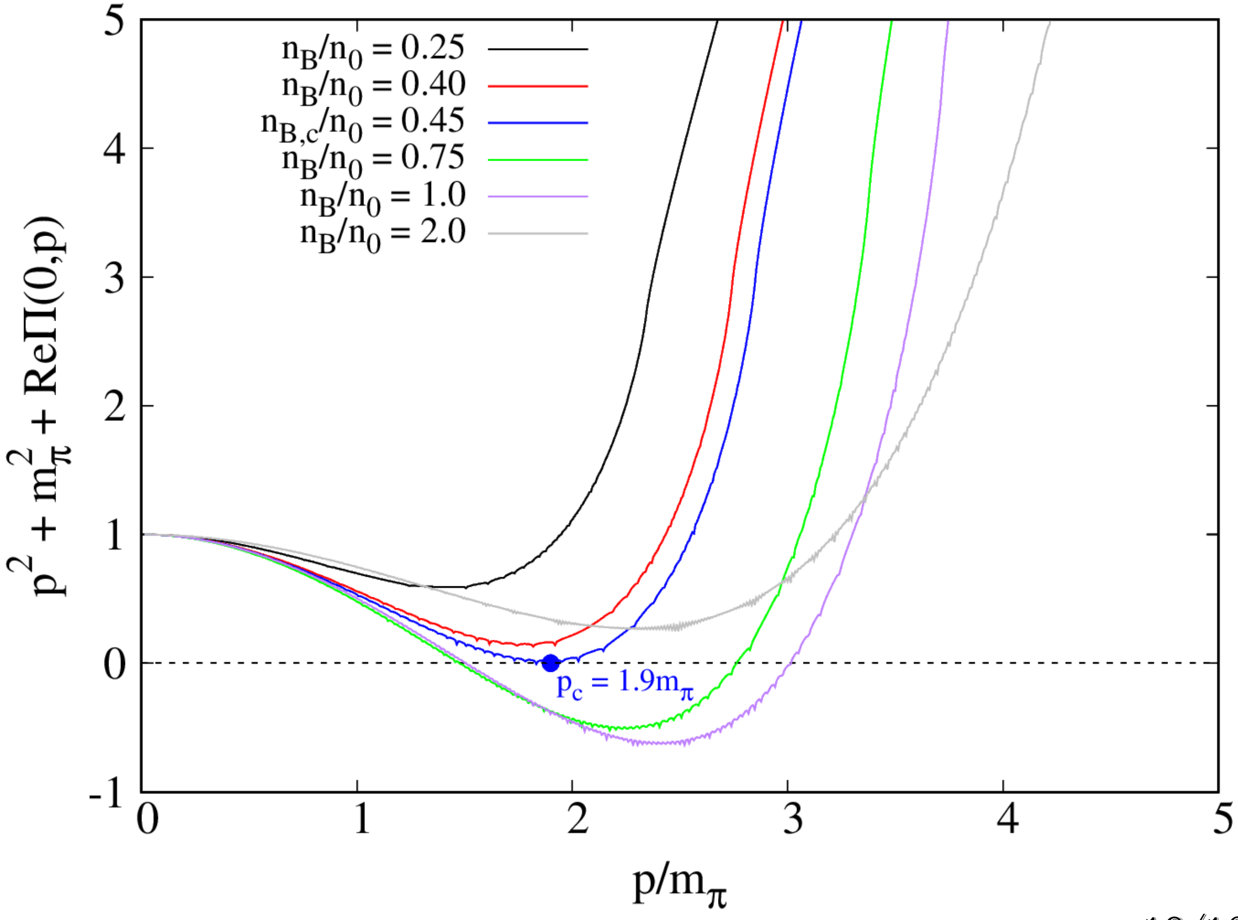
PION CONDENSATE, SYMMETRIC CASE, PV

$$E(\boldsymbol{p}) = \boldsymbol{p}^2 + m_\pi^2 + \mathrm{Re}\Pi(E(p) = 0, \boldsymbol{p}) = 0$$

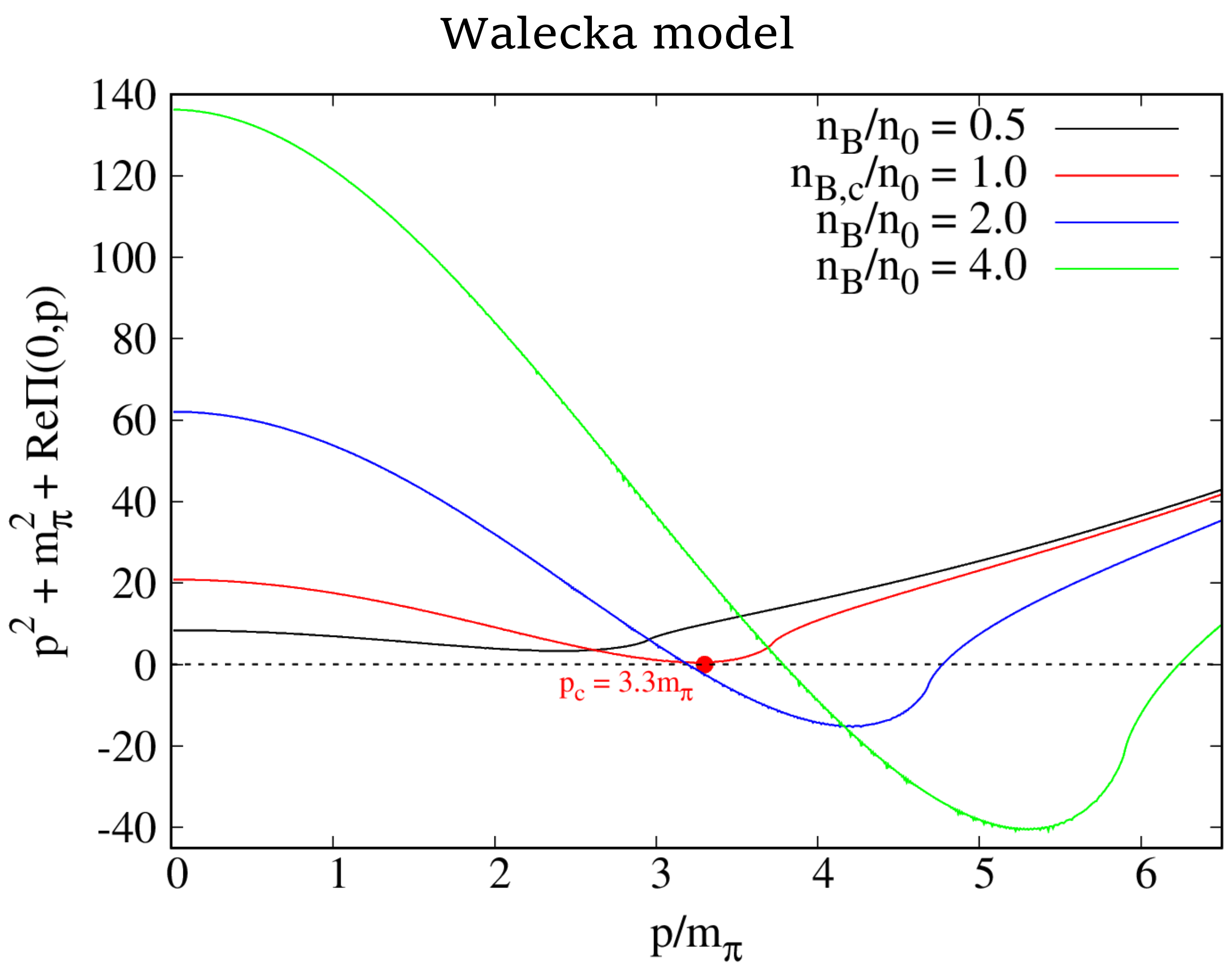
Walecka model

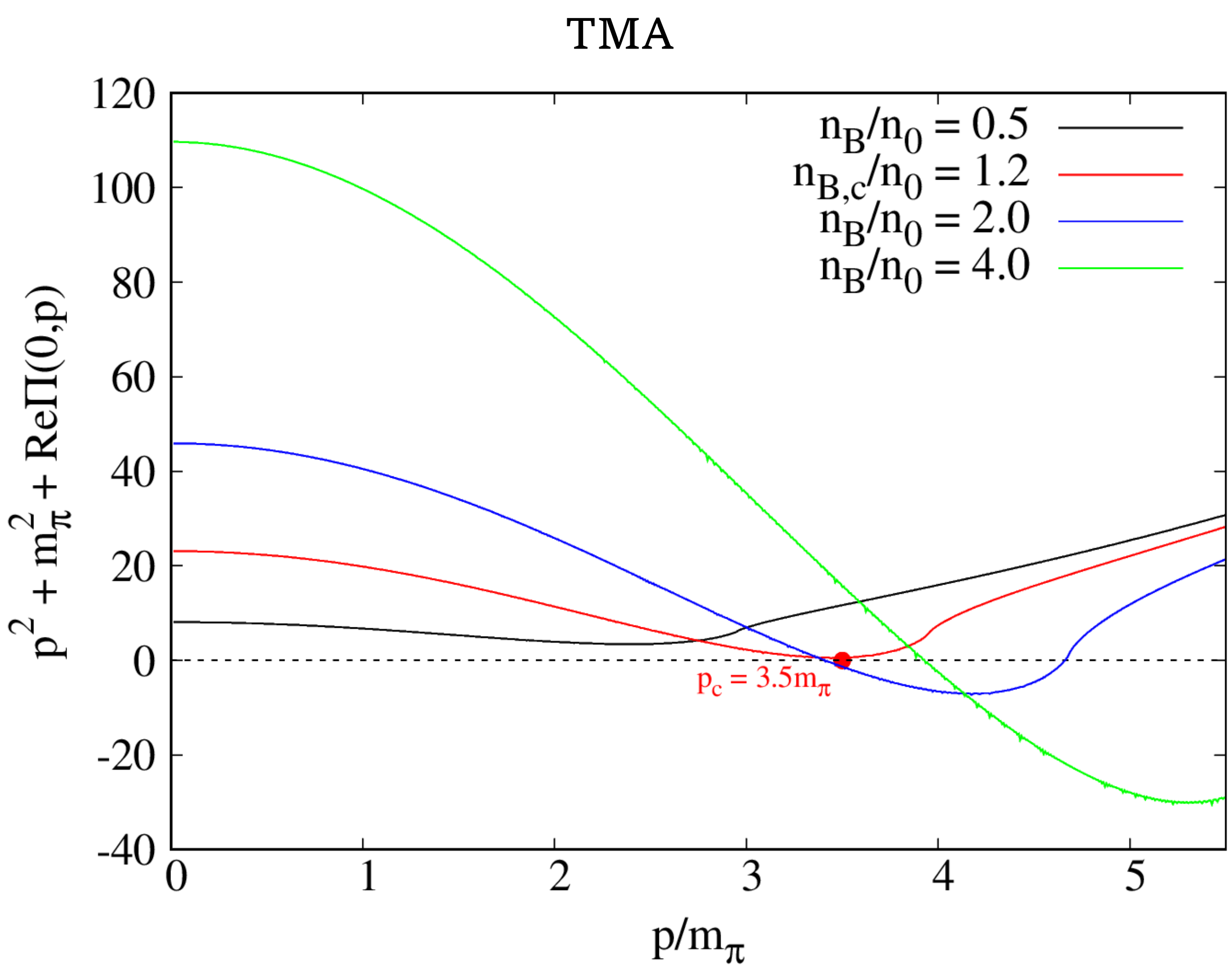


TMA



PION CONDENSATE, SYMMETRIC CASE, PS

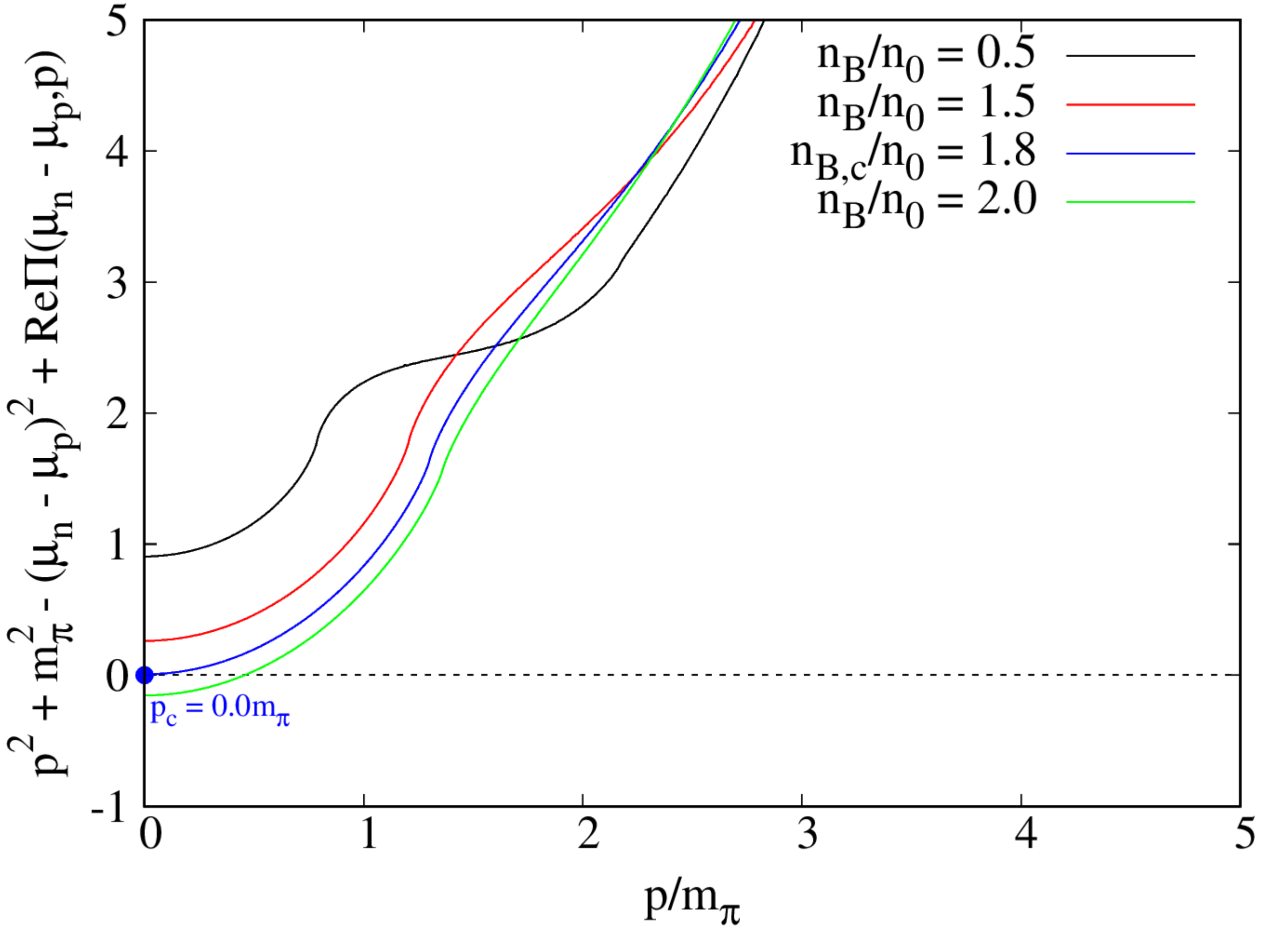




PION CONDENSATE, BETA-EQ., PV

$$E(\boldsymbol{p}) = \boldsymbol{p}^2 + m_\pi^2 + \mathrm{Re}\Pi\big(E(p) = |\mu_n - \mu_p|, \boldsymbol{p}\big) = \big(\mu_n - \mu_p\big)^2$$

Walecka model



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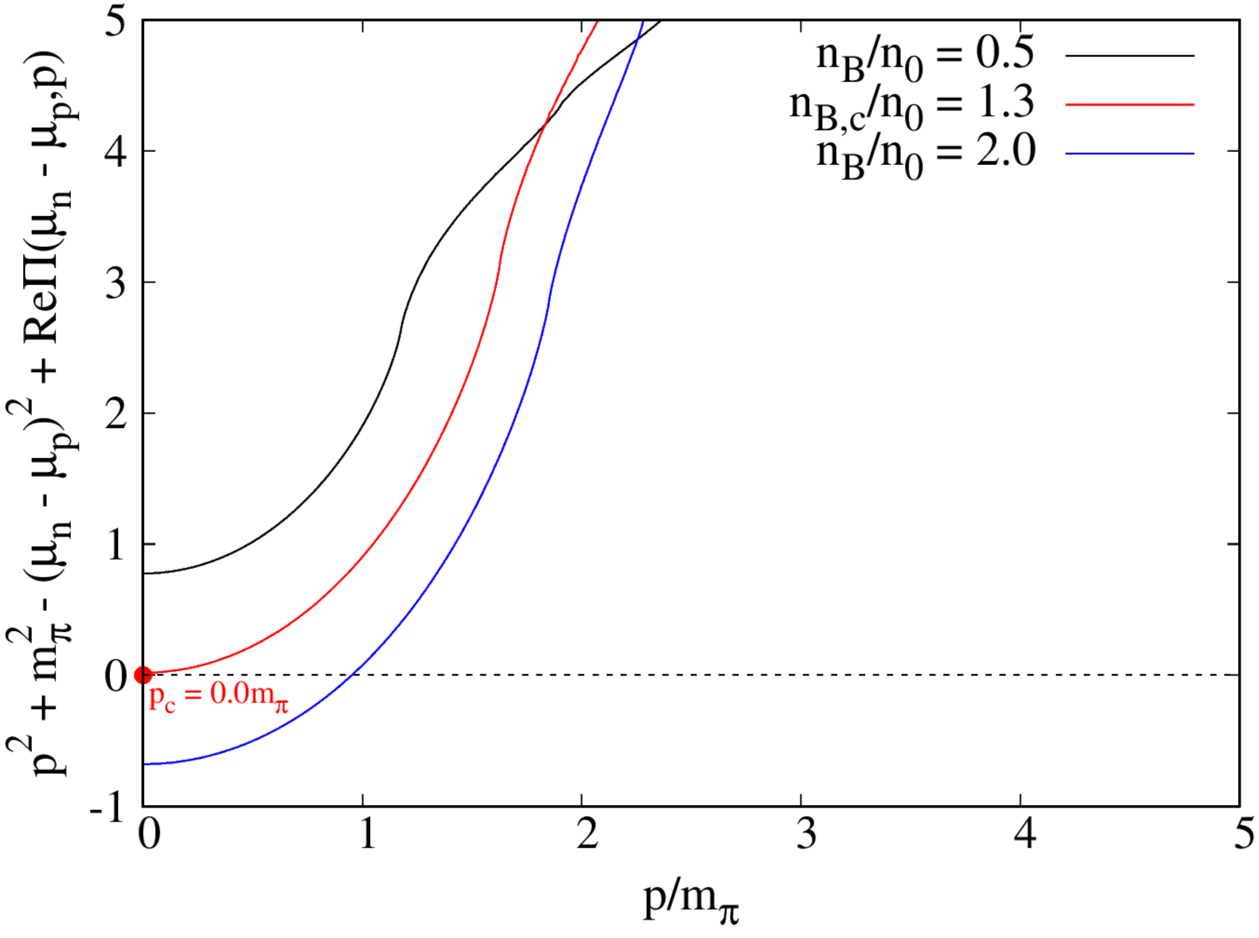
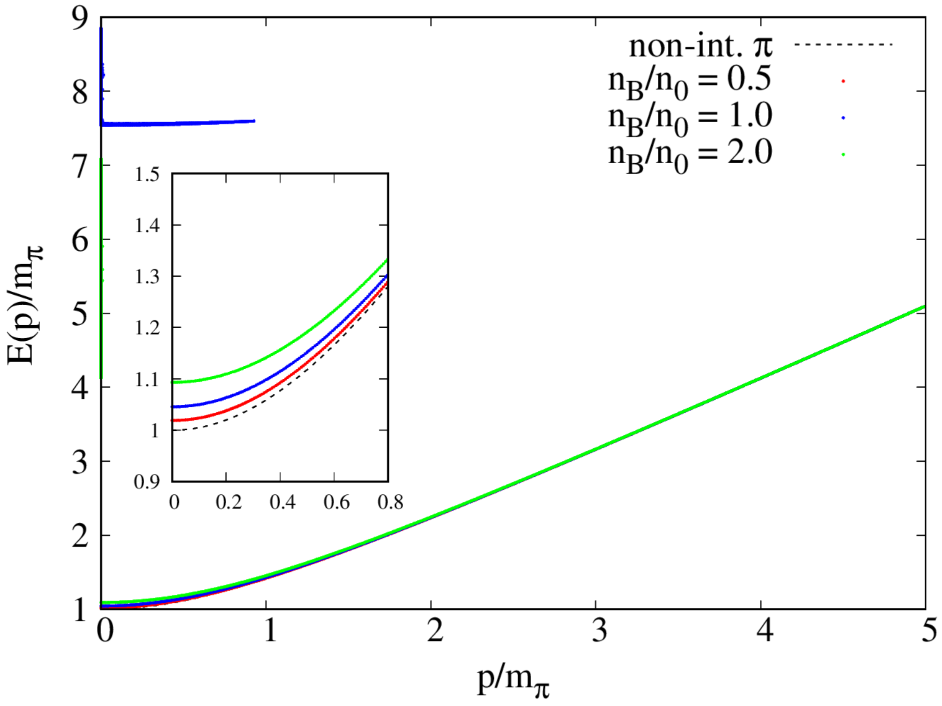
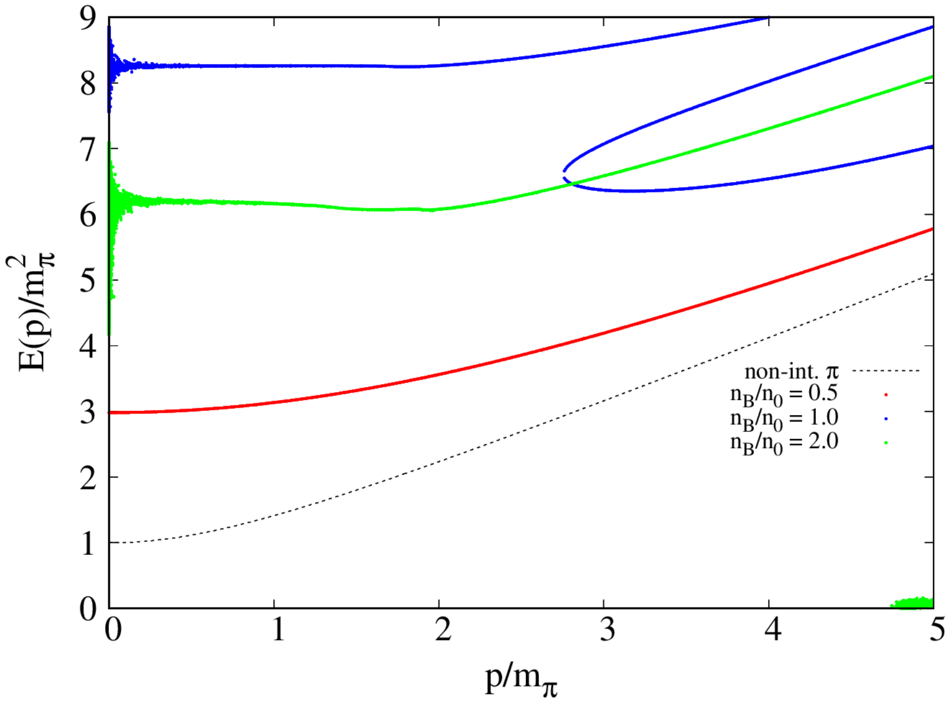
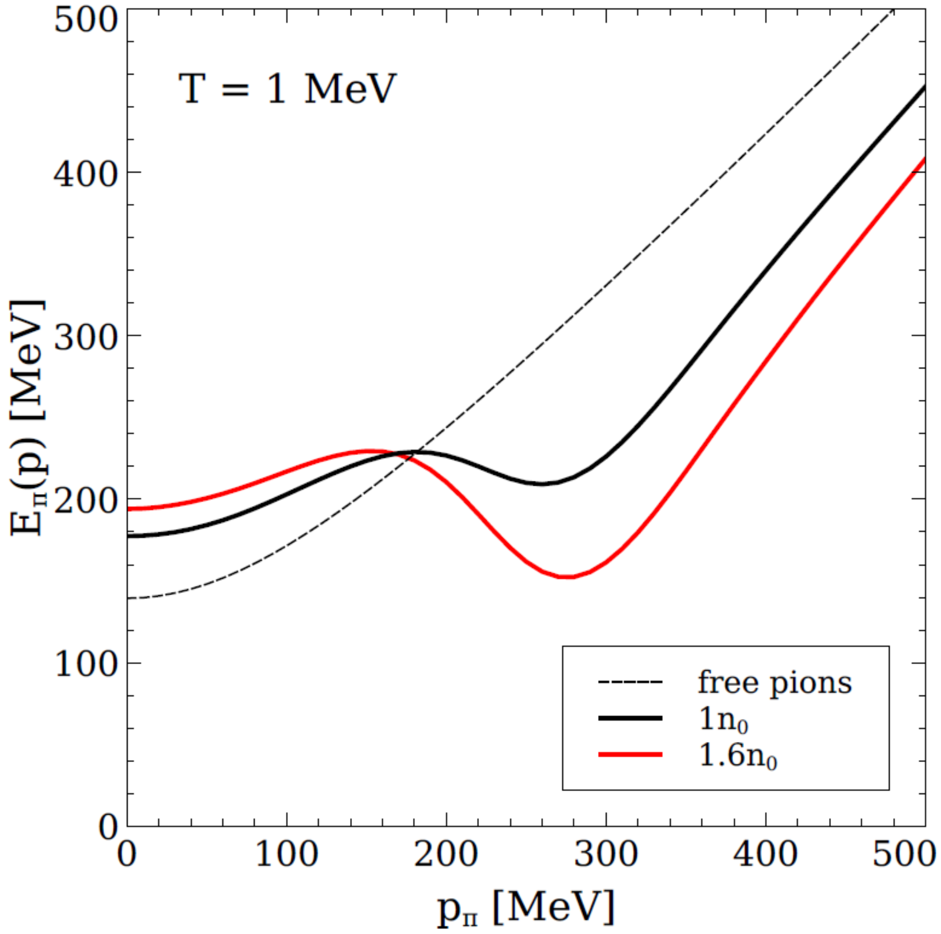


PION DISPERSION RELATION

$$E(\boldsymbol{p}) - \boldsymbol{p}^2 + m_\pi^2 + \mathrm{Re}\Pi(E(\boldsymbol{p}),\boldsymbol{p}) = 0$$

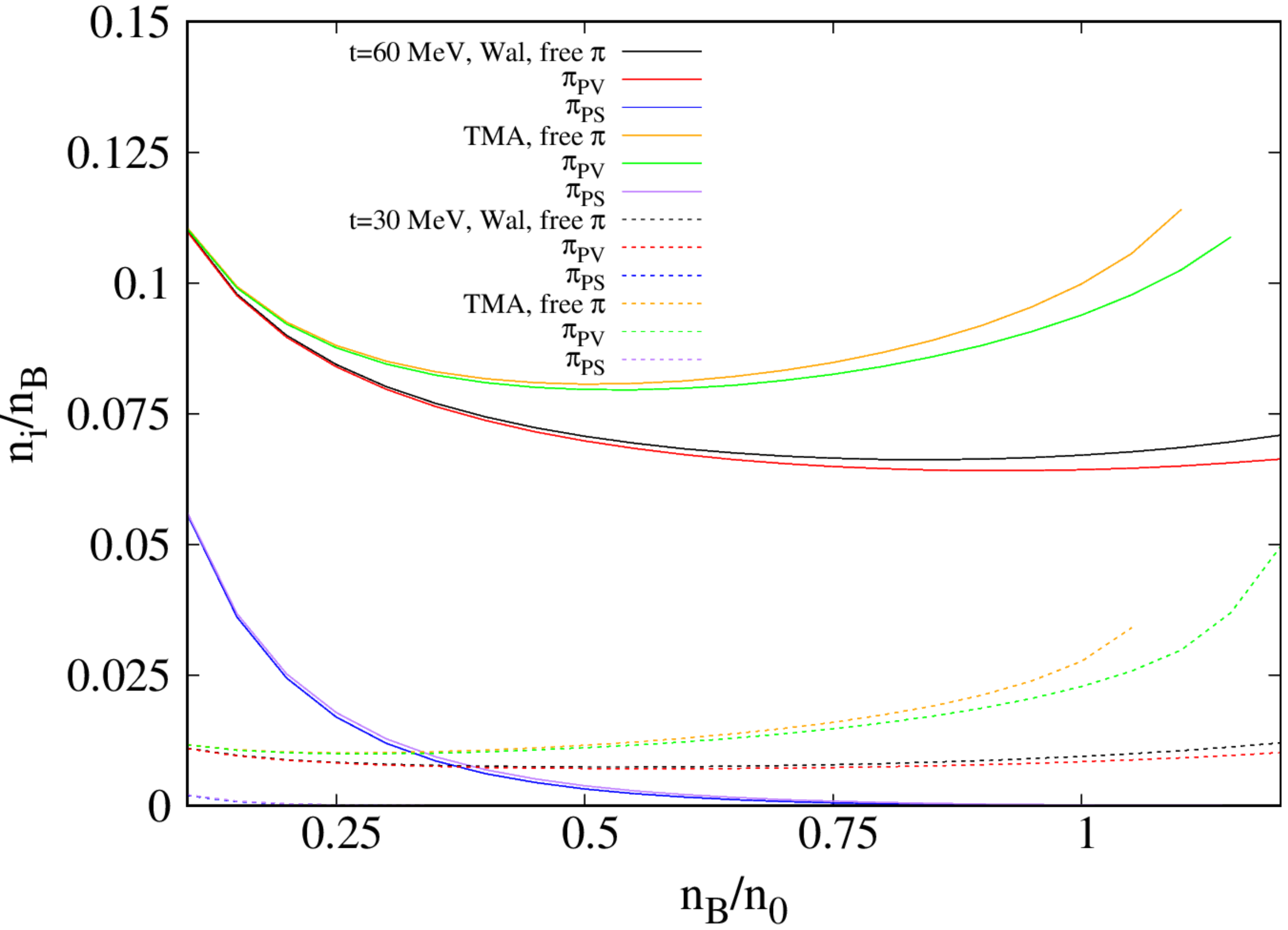




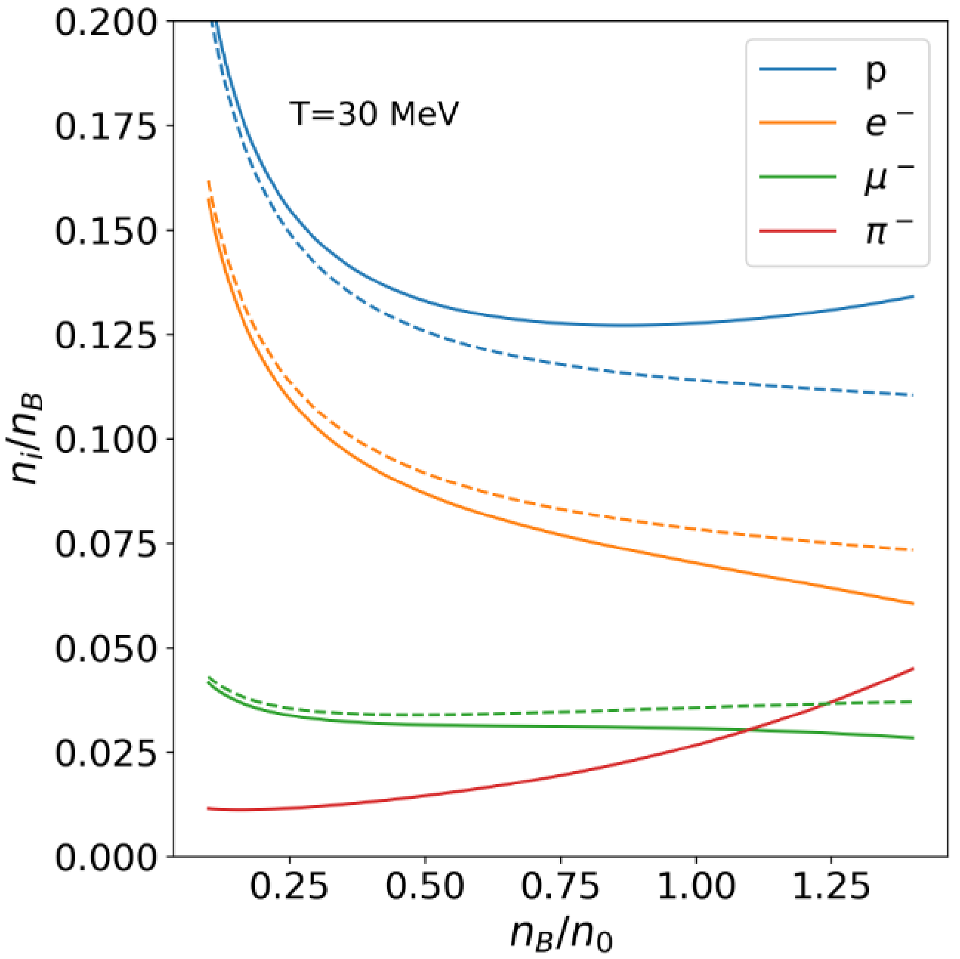


S. P. Harris, B. Fore, S. Reddy, Bulk Viscosity Of Nuclear Matter With Pions In The Neutrino-trapped Regime, *arXiv*, 2024

PION ABUNDANCE IN BETA EQ.



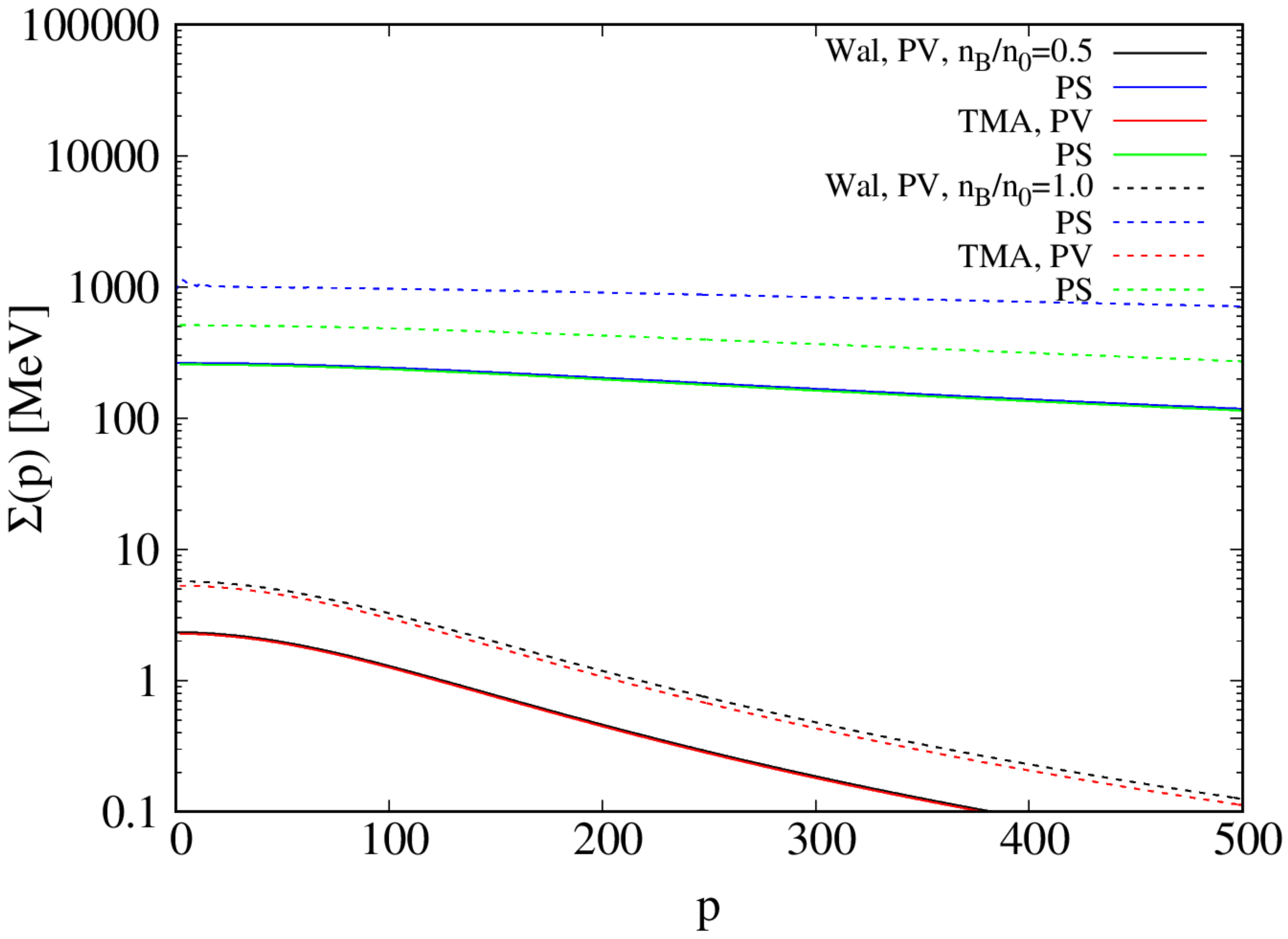
$$n_\pi = \frac{1}{(2\pi)^3}\int d^3p\left(\exp\left(\frac{E_\pi(\boldsymbol{p})-\mu_\pi}{k_BT}\right)-1\right)^{-1}$$

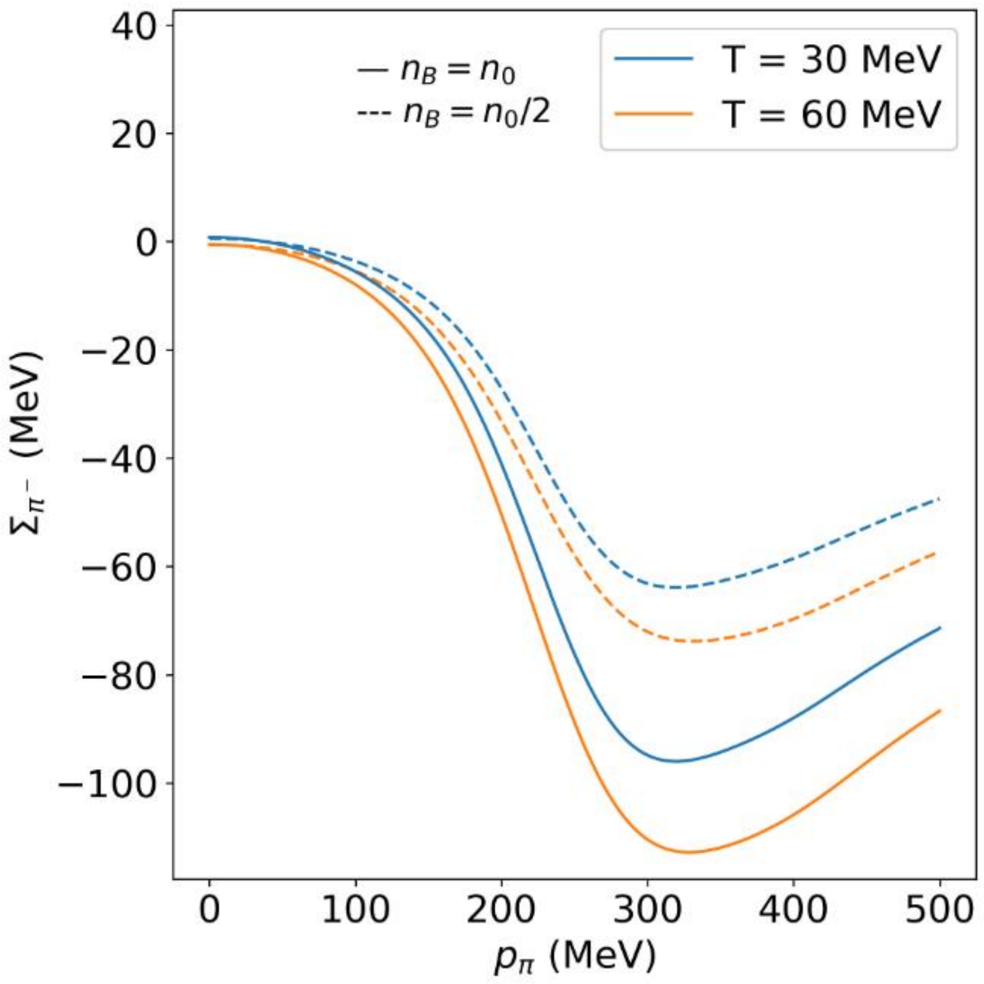


B. Fore, S. Reddy, Pions in Hot Dense Matter and Their Astrophysical Implications, *Phys. Rev. C*, 2020

PION SELF-ENERGY, T=30 MeV

$$\sqrt{\boldsymbol{p}^2 + m_\pi^2 + \mathrm{Re}\Pi(E(p), \boldsymbol{p})} = \sqrt{\boldsymbol{p}^2 + m_\pi^2} + \Sigma(p) \quad \Rightarrow \quad E_{int}(p) - E_{free}(p) = \Sigma(p)$$





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