Speed of Sound in Dense Medium

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University of Wrocław
(In collaboration with Pok Man Lo)

BRAINSTORMING WORKSHOP: DECIPHERING THE EQUATION OF STATE USING GRAVITATIONAL WAVES FROM ASTROPHYSICAL SOURCES
WARSAW, POLAND
05.08.2024 - 07.08.2024

pQCD Constraint on Equation of State (EoS)

Theoretical Upper Bound on (squared) speed of sound

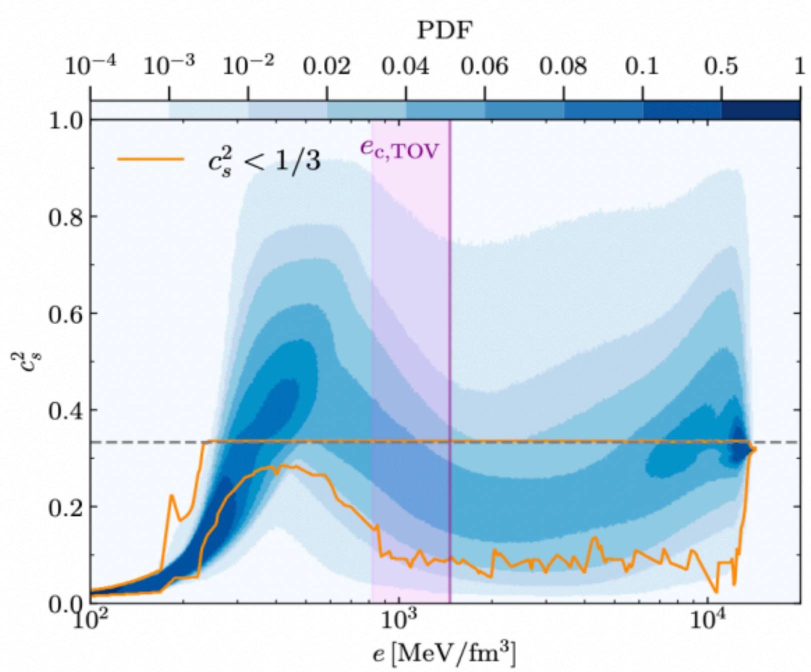

Figure 1. PDF of the sound speed squared as function of the energy density. The purple region marks the 95%-interval of maximum central energy densities, so that the vertical purple line represents an estimate for the largest possible energy density in a neutron star. The orange contour marks the region containing EOSs with $c_s^2 < 1/3$.

On the Sound Speed in Neutron Stars

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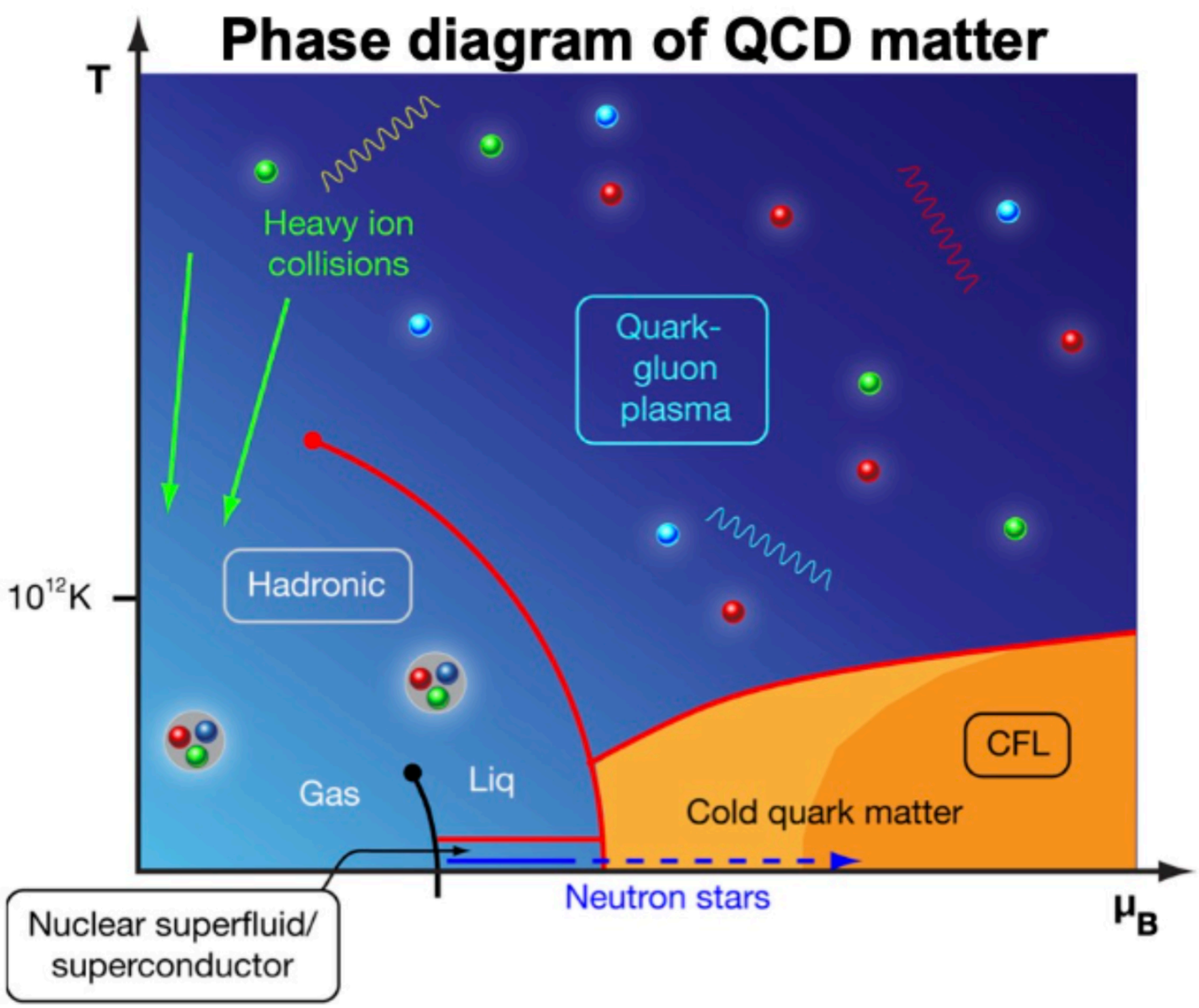
ABSTRACT

Determining the sound speed c_s in compact stars is an important open question with numerous implications on the behaviour of matter at large densities and hence on gravitational-wave emission from neutron stars. To this scope, we construct more than 10^7 equations of state (EOSs) with continuous sound speed and build more than 10^8 nonrotating stellar models consistent not only with nuclear theory and perturbative QCD, but also with astronomical observations. In this way, we find that EOSs with sub-conformal sound speeds, i.e., with $c_s^2 < 1/3$ within the stars, are possible in principle but very unlikely in practice, being only 0.03% of our sample. Hence,

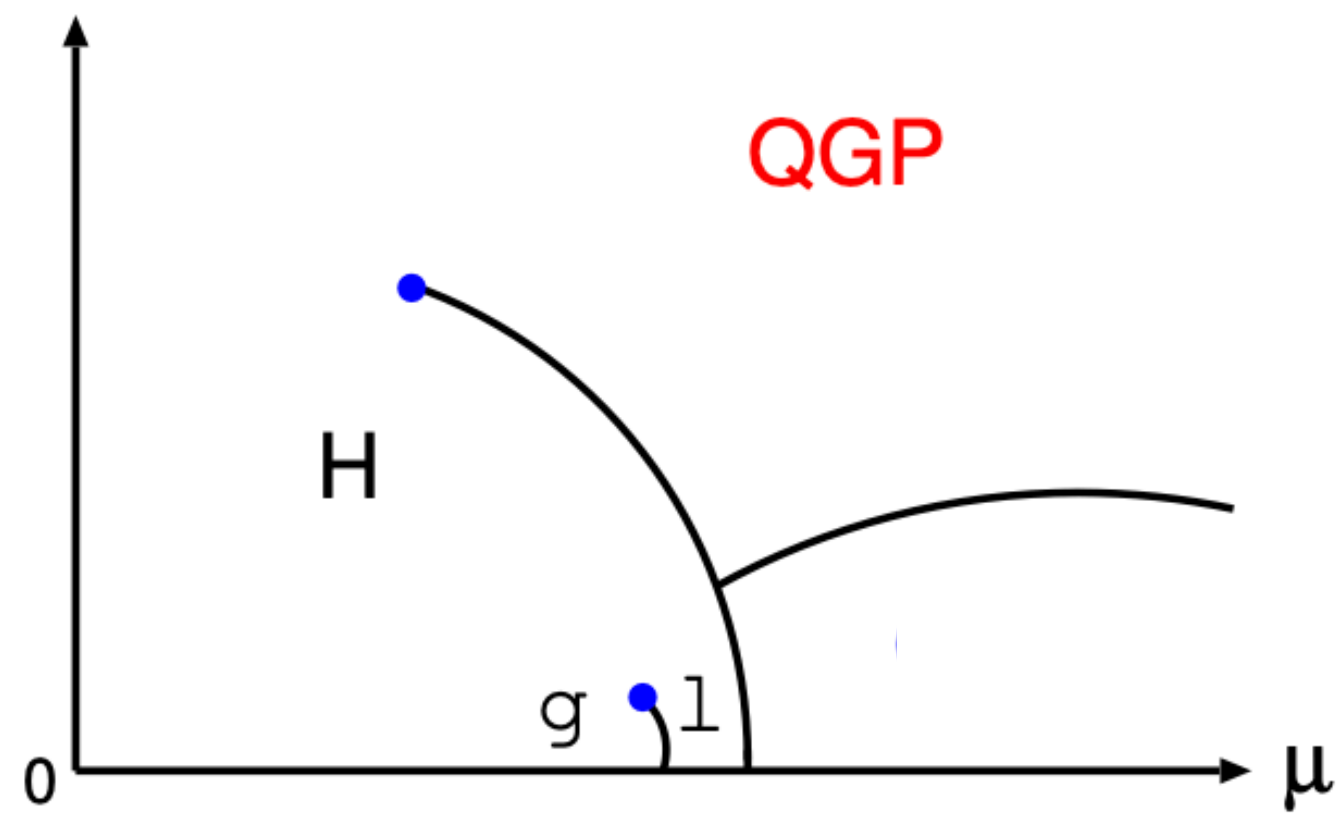
- **Stiffness of EoS**
- **Chiral transition**
- **.....**

QCD Phase Diagram

What we theorise versus what we know (also debatable)...



By Panos Christakoglou



- **Supernova**
- **Heavy Ion Collisions**
- **Compact stars**

- **CEP?**
- **Chiral transition ?**
- **(De)confinement transition?**
- **.....**

Table of Contents

Nambu-Jona-Lasinio Model

Quasi-Particle Model

$$m^* = m - G_s n_s$$
$$\mu^* = \mu - G_v n_v$$

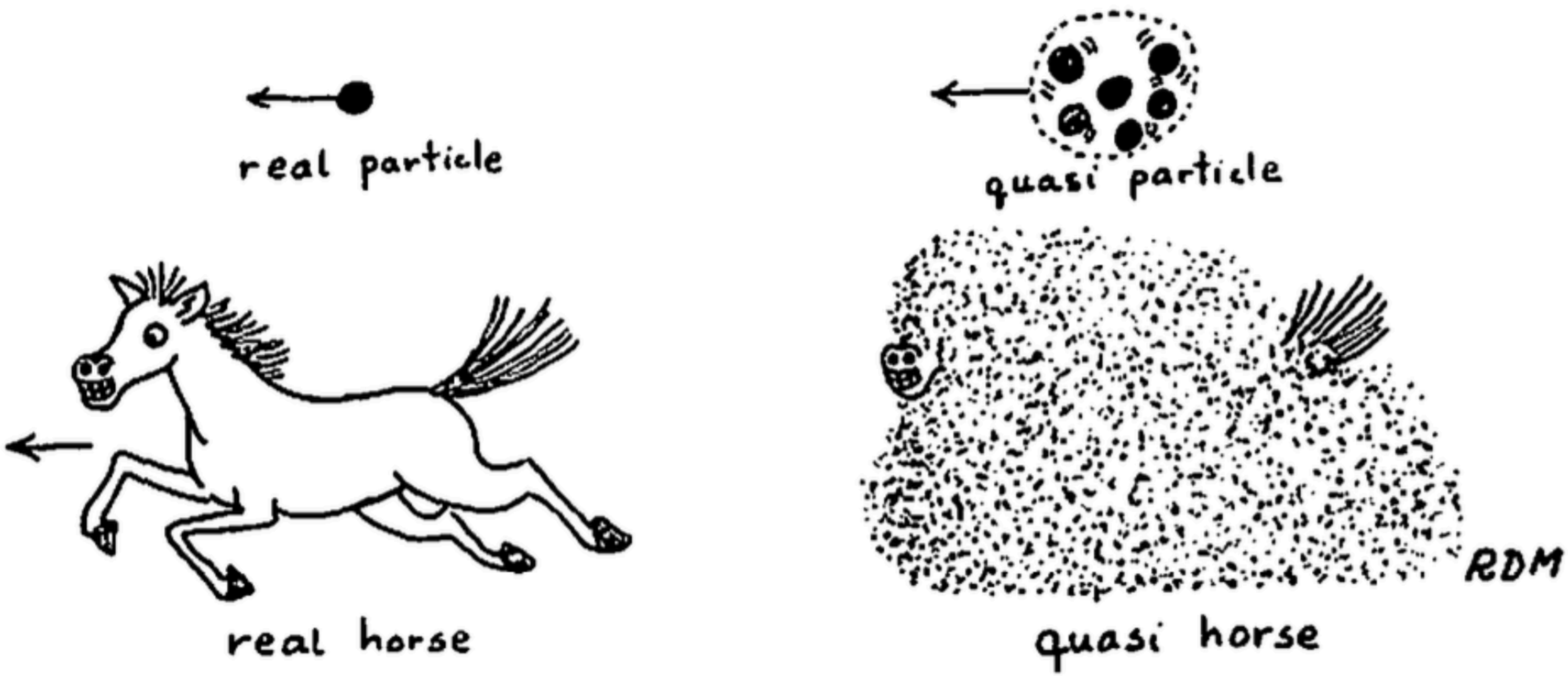
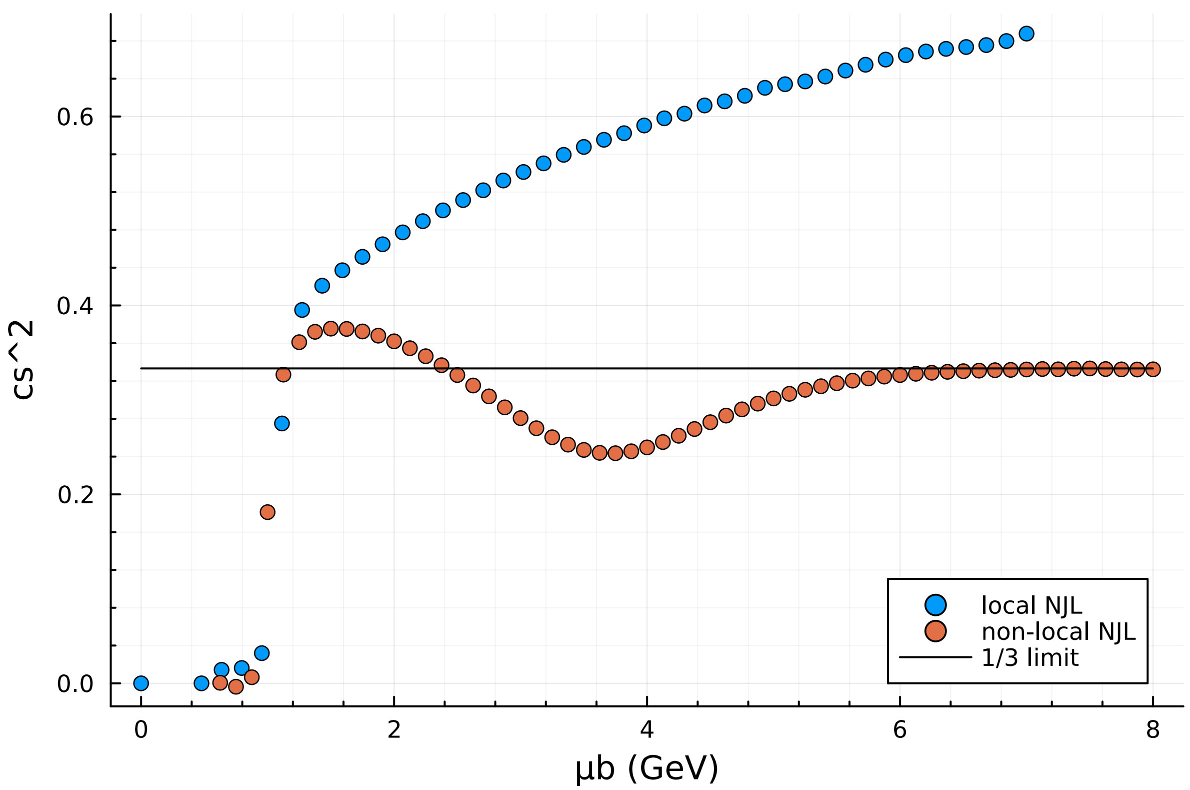


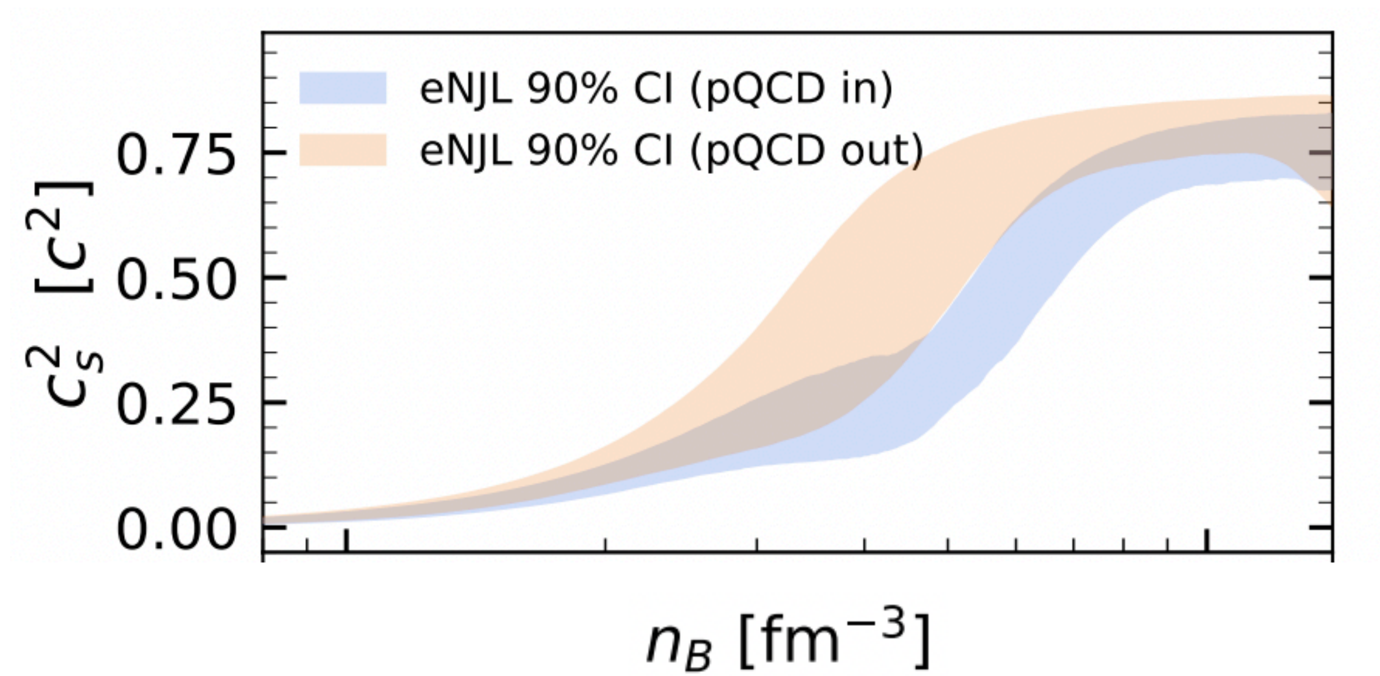
Fig. 0.4 ***Quasi Particle Concept***

Adapted from Richard D. Mattuck, *"A Guide to Feynman Diagrams in the Many-Body Problem"*

Speed of Sound: Standard Nambu-Jona Lasinio (NJL) Model



Dynamical model



Marquez, Malik, et. al. (2024)

(Non-Local) Nambu-Jona-Lasinio Model

$$m^*(p) = m - \gamma(p) G_s n_s$$

$$\mu^*(p) = \mu - \gamma(p) G_v n_v$$

Quasi-Particle Model

Momentum dependence

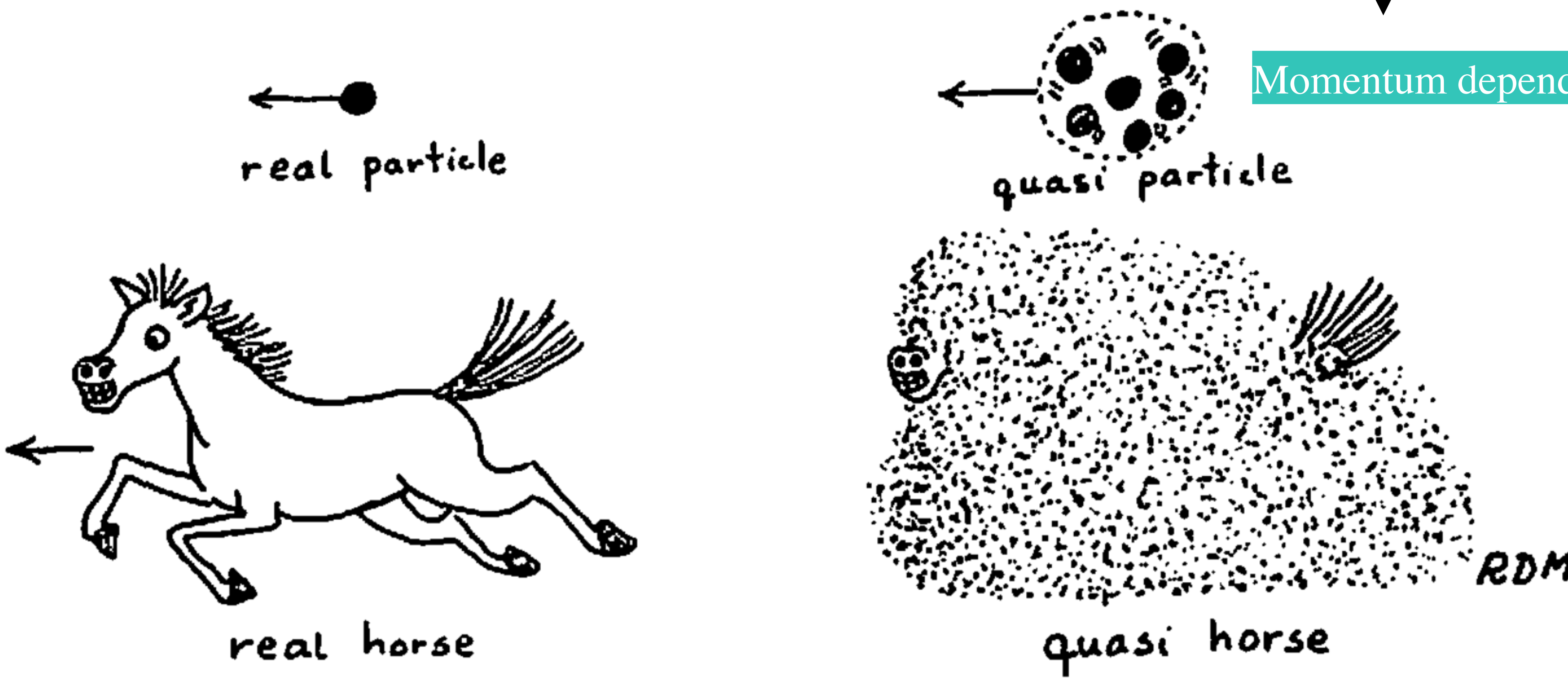


Fig. 0.4 ***Quasi Particle Concept***

Adapted from Richard D. Mattuck, “A Guide to Feynman Diagrams in the Many-Body problem”

(Non-local) Nambu-Jona-Lasinio Model

Quasi-particle mass

$$m^*(p) = m + \gamma(p)\,2G_s N_c N_f \int \frac{d^3q}{(2\pi)^3}\gamma(q)\frac{4m_q^*}{2E_q}[1 - n(E_q - \mu_q^*) - n(E_q + \mu_q^*)]$$

Quasi-particle chemical potential

$$\mu^*(p) = \mu - \gamma(p)\,2G_v N_c N_f \int \frac{d^3q}{(2\pi)^3}\gamma(q)\frac{4}{2}[n(E_q - \mu_q^*) - n(E_q + \mu_q^*)]$$

$$n(x) = \frac{1}{1 + e^{x/T}} \qquad E_q = \sqrt{q^2 + (m_q^*)^2}$$

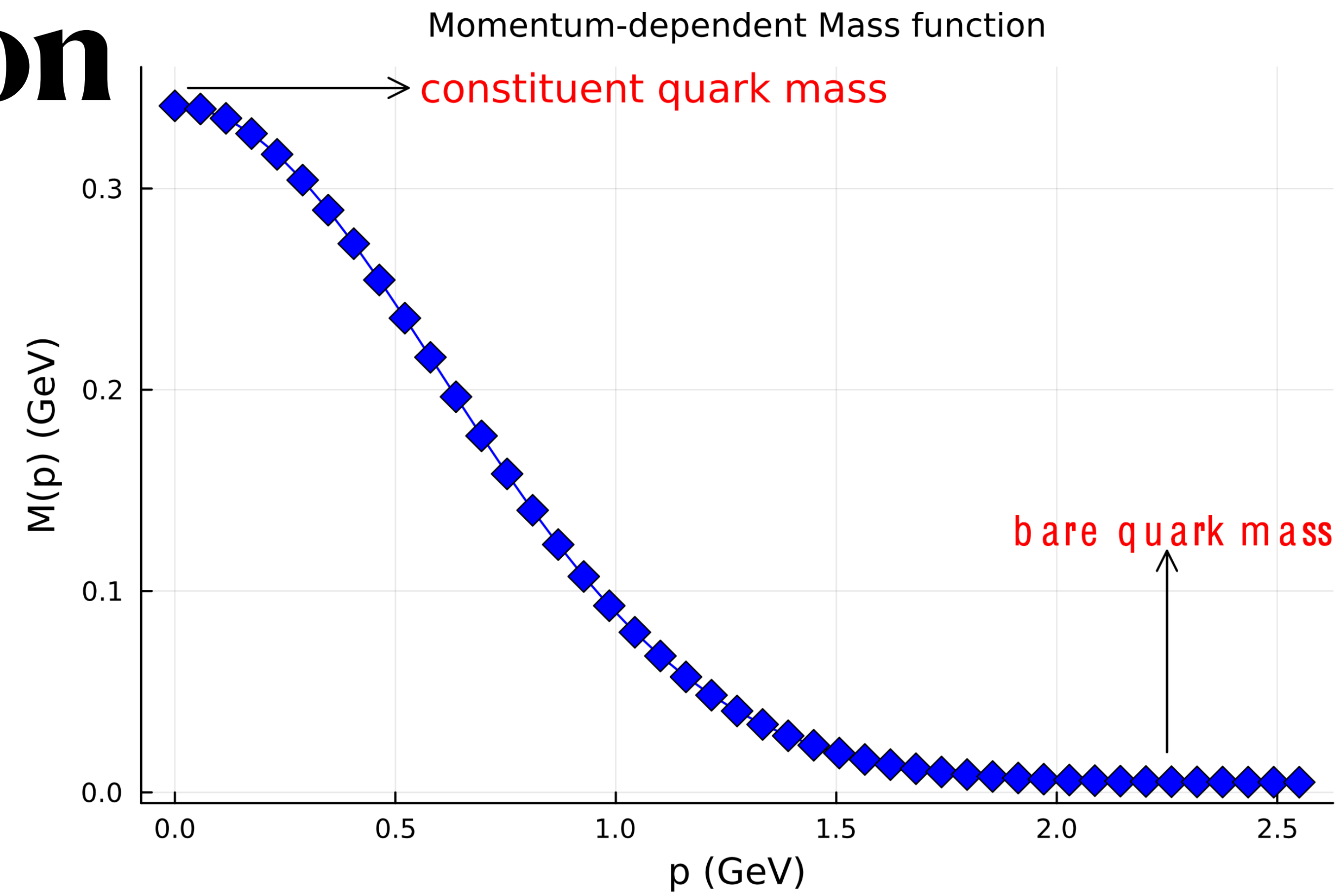
Dynamical interaction

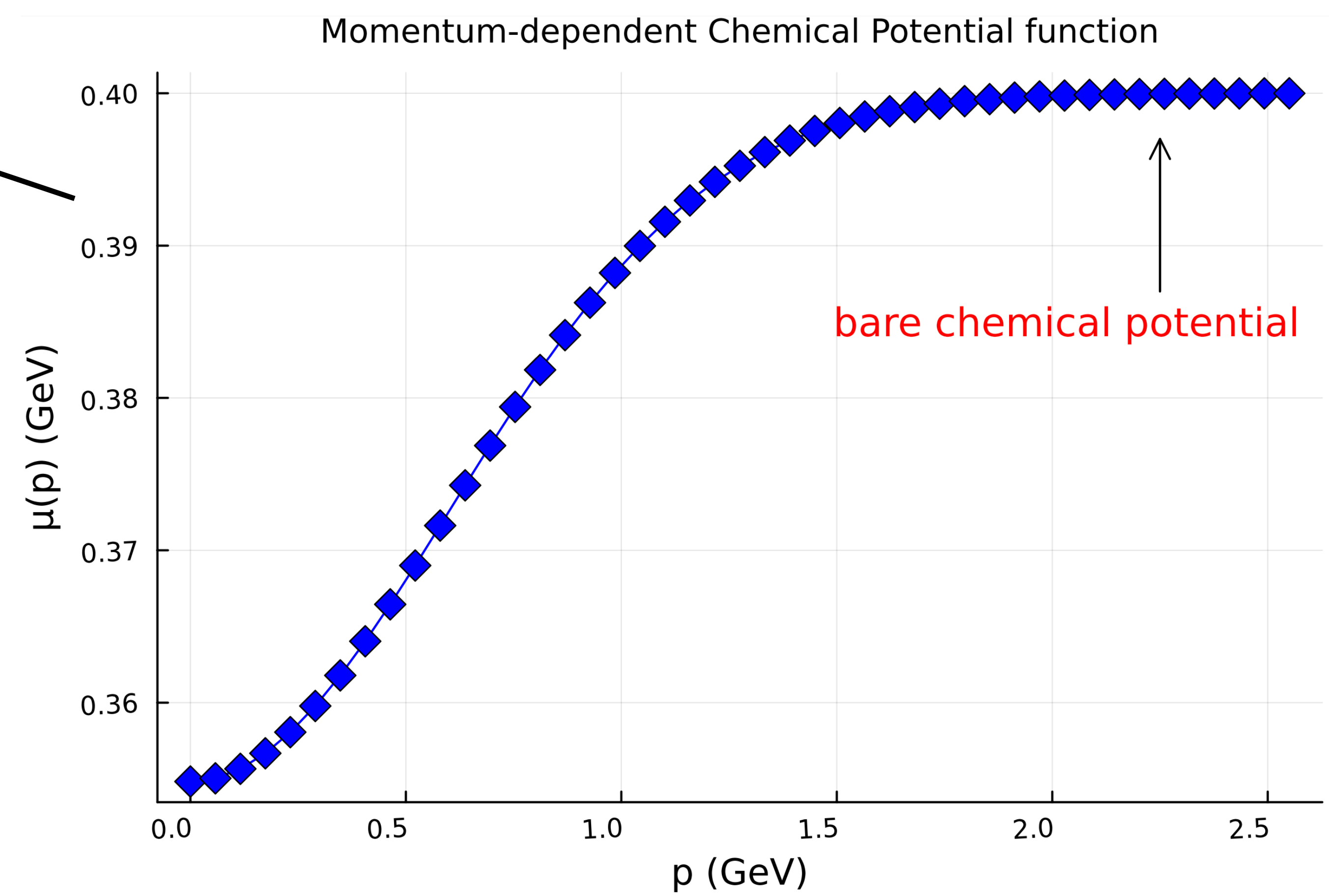
- M(p) saturates asympto--tically to current quark mass
- μ(p) saturates asympto--tically to bare μ

?

- **Change in dispersion** relation -

$$E(p) = \sqrt{p^2 + M(p)^2}$$





Local v/s Dynamical
T → 0

Local Interaction

$$n_v = N_c N_f \frac{1}{\pi^2}\int_0^\infty dq q^2 \Theta(\mu^* - q)$$

$$= N_c N_f \frac{1}{\pi^2}\int_0^{\mu^*} dq q^2$$

$$= N_c N_f \frac{1}{3\pi^2}\mu^*(\mu)^3$$

$$n_v = n_v(\mu^*(\mu))$$

$$\mu^* = \mu - G_v n_v$$

Trivial Fermi surface

Dynamical Interaction

$$n_v = N_c N_f \frac{1}{\pi^2}\int_0^\infty dq q^2 \Theta(\mu_q^* - q)$$

$$= N_c N_f \frac{1}{\pi^2}\int_0^{\mu^*_{p_f(\mu)}} dq q^2$$

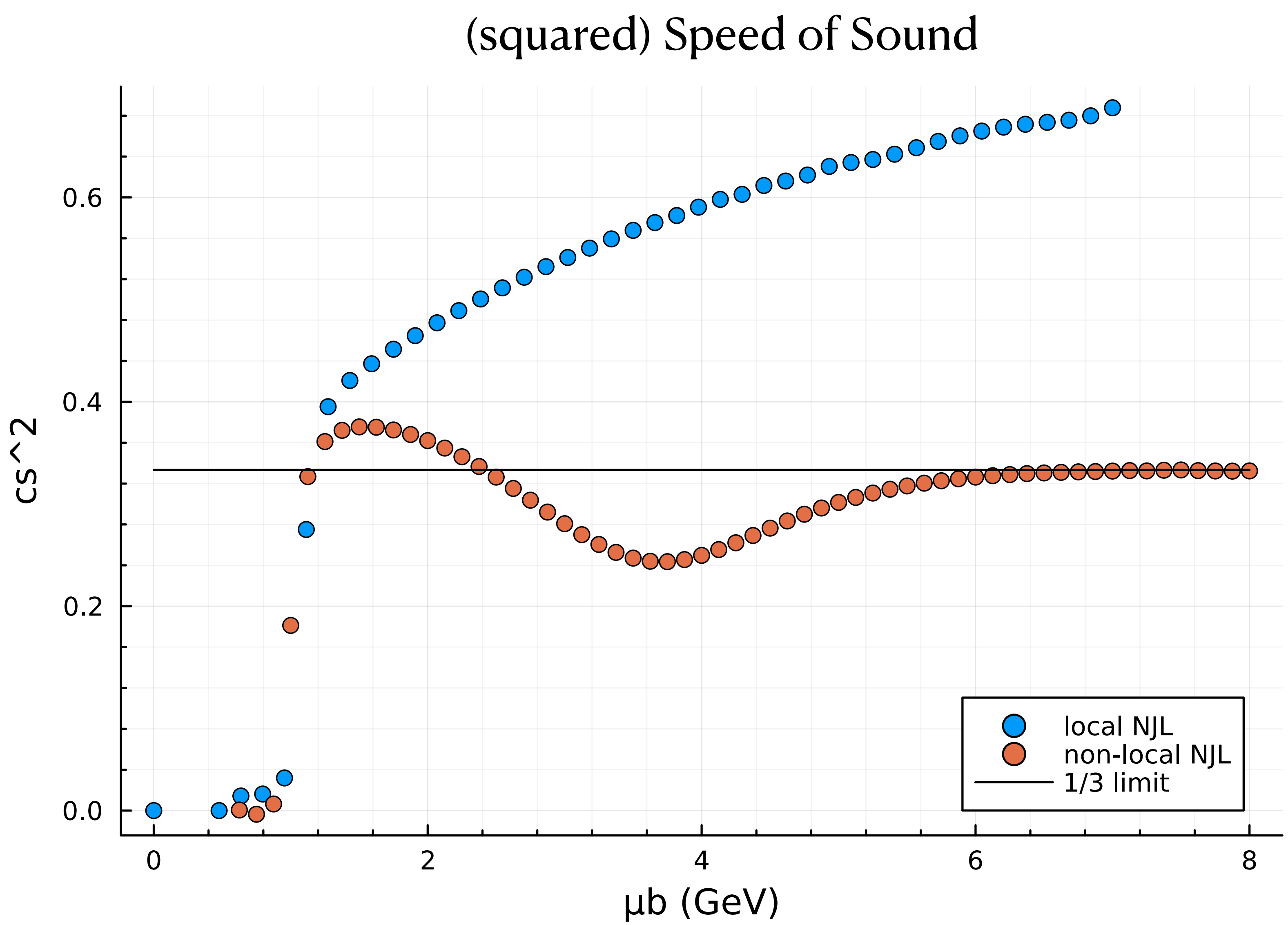
$$n_v = n_v\left[\mu^*_{p_f(\mu)}\right]$$

$$\mu^*(p) = \mu - \gamma(p) G_v n_v$$

NATURAL DENSITY-DEPENDENCE

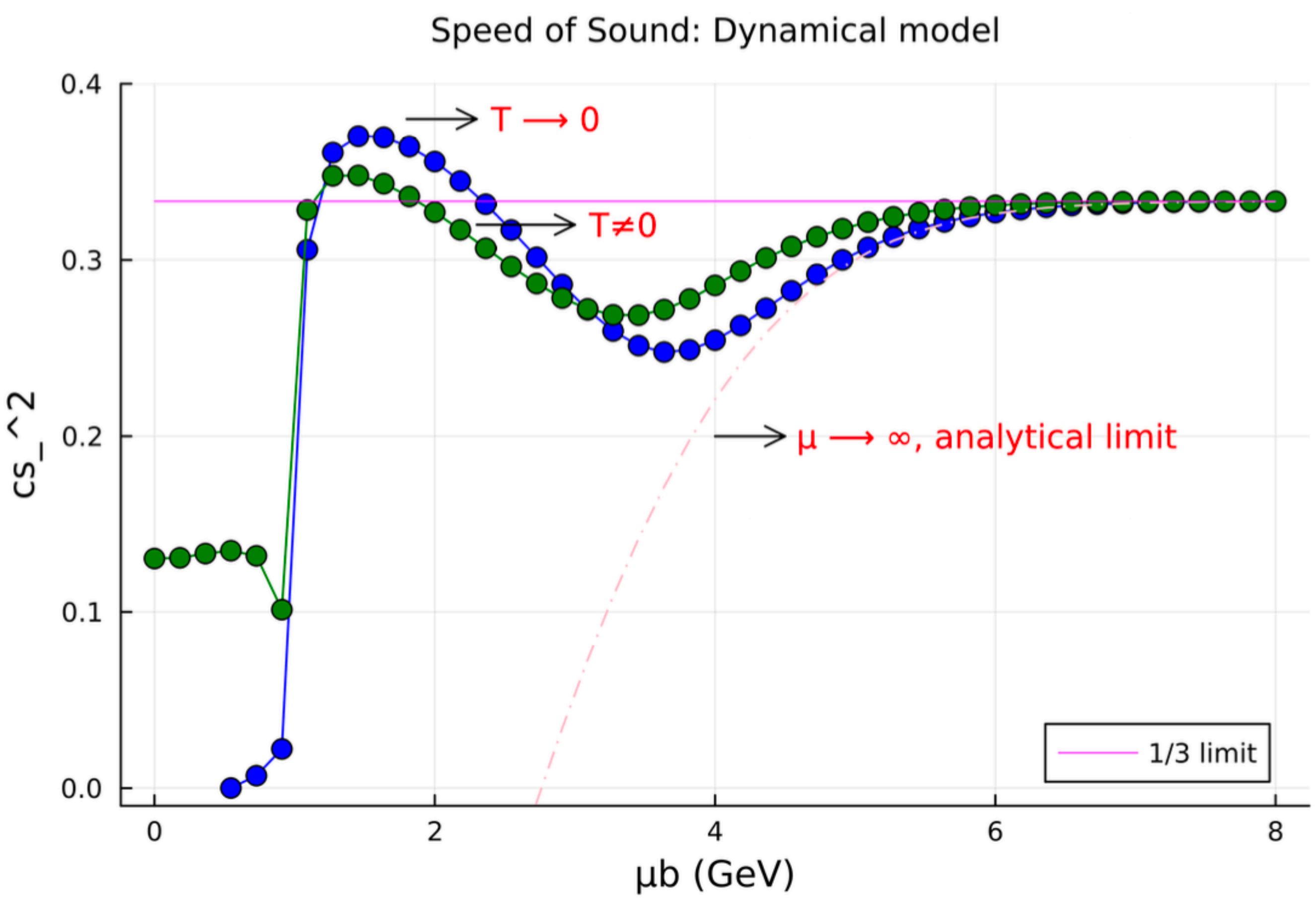
Non-trivial Interacting Fermi surface !

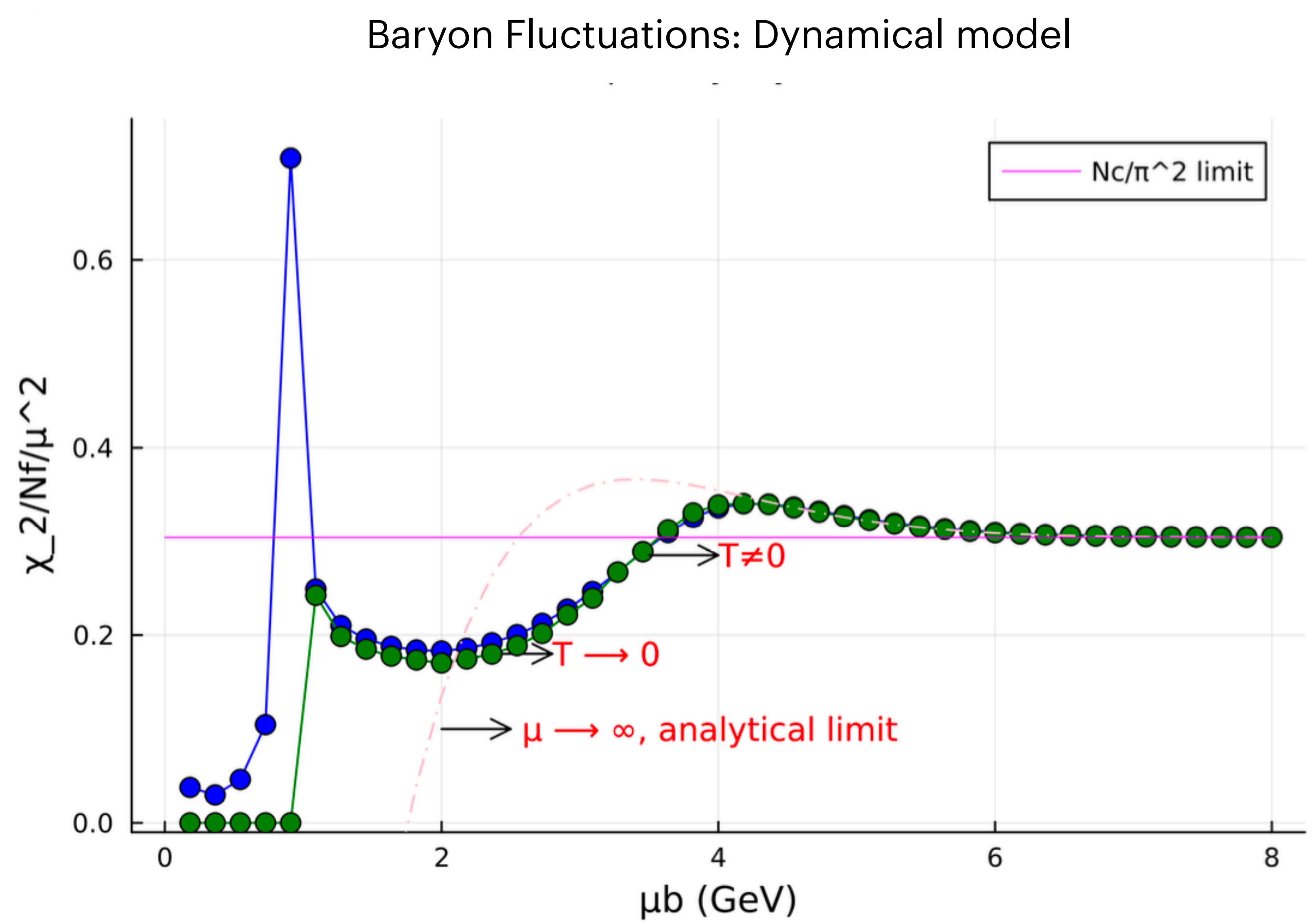
CONSEQUENCE OF DYNAMICAL INTERACTION T ⟶ 0



$$cs^2 = \frac{\frac{n_v}{\mu}}{\frac{dn_v}{d\mu}} \longrightarrow cs^2 = \frac{1}{\mu}\frac{n_v}{\chi_2}$$

SPEED OF SOUND V/S CHIRAL SUSCEPTIBILITY





$$cs^2_{\mu\to\infty} \approx \frac{1}{3}\left[1 - \frac{\omega_\infty}{\mu}\left(1+\frac{2\mu^2}{UV^2}\right)e^{-\frac{\mu^2}{UV^2}}\right]$$

$$\chi 2_{\mu\to\infty} \approx \frac{N_c N_f}{\pi^2}\mu^2\left[1 - \frac{2\omega_\infty}{\mu}\left(1-\frac{\mu^2}{UV^2}\right)e^{-\frac{\mu^2}{UV^2}}\right]$$

Non-local cut-off is inherent gluon interaction scale

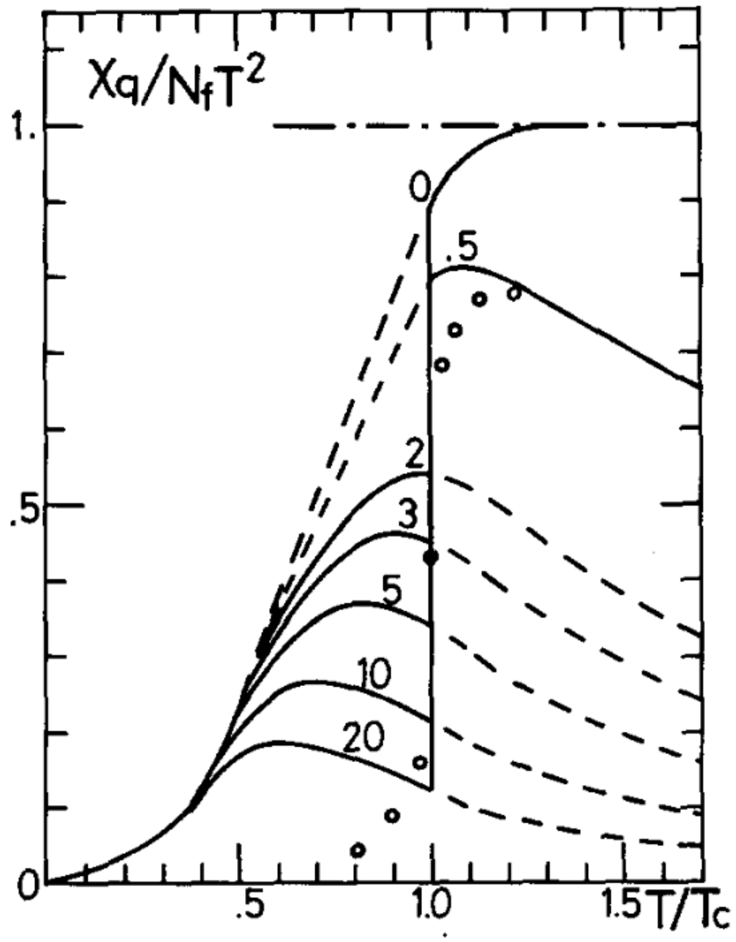


Fig. 1. The temperature dependence of the quark-number susceptibility χ_q in the unit of N_fT^2 with some of the vector coupling $g_V\Lambda^2$: $g_V\Lambda^2=0$, 0.5, 2, 3, 5, 10, 20, which are indicated with the numbers attached to the respective curves. The dash-dotted line shows the free massless case. The small circles are the lattice result on an $8^3\times4$ lattice with the quark mass $m/T=0.2$ [7] compiled in ref. [9].

Physics Letters B 271 (1991) 395–402
North-Holland

PHYSICS LETTERS B

Quark-number susceptibility and fluctuations in the vector channel at high temperatures ☆

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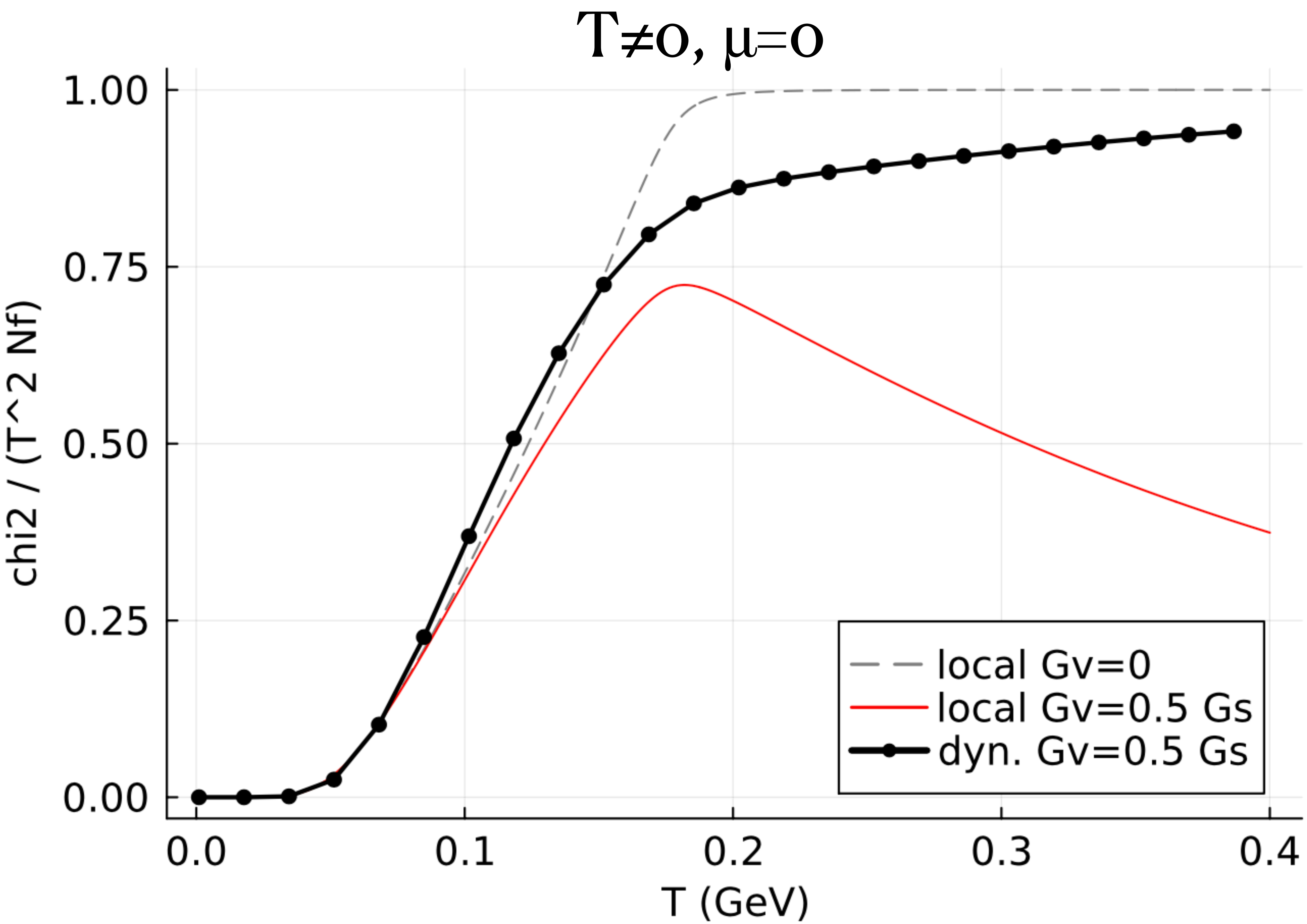
Received 9 July 1991; revised manuscript received 10 September 1991

The quark-number susceptibility χ_q is examined as an observable which may help to reveal the physical picture of the high-temperature phase of QCD. It is emphasized that χ_q is intimately related with the fluctuations in the vector channel of the system. It is shown that the results of the recent lattice simulations of χ_q can be understood in terms of a possible change of the interactions between quark and anti-quarks in the vector channel, and imply that the fluctuations in the vector channel is greatly suppressed in the high-temperature phase in contrast with those in the scalar and pseudo-scalar ones.

BARYON FLUCTUATIONS: DYNAMICAL MODEL



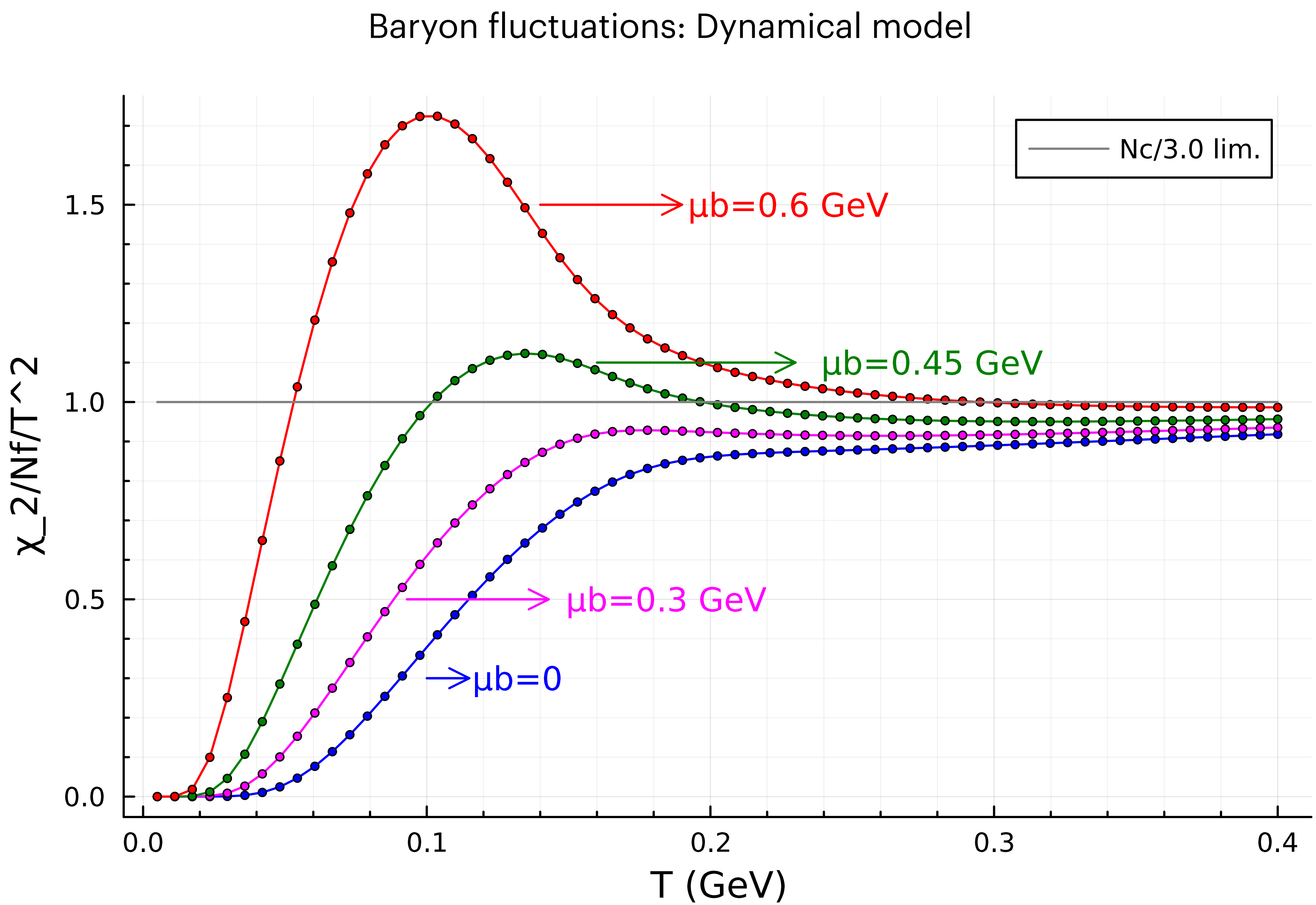
local

$$\chi_2 = \frac{dn_v}{d\mu} \propto \frac{T^2}{1 + CT^2}$$

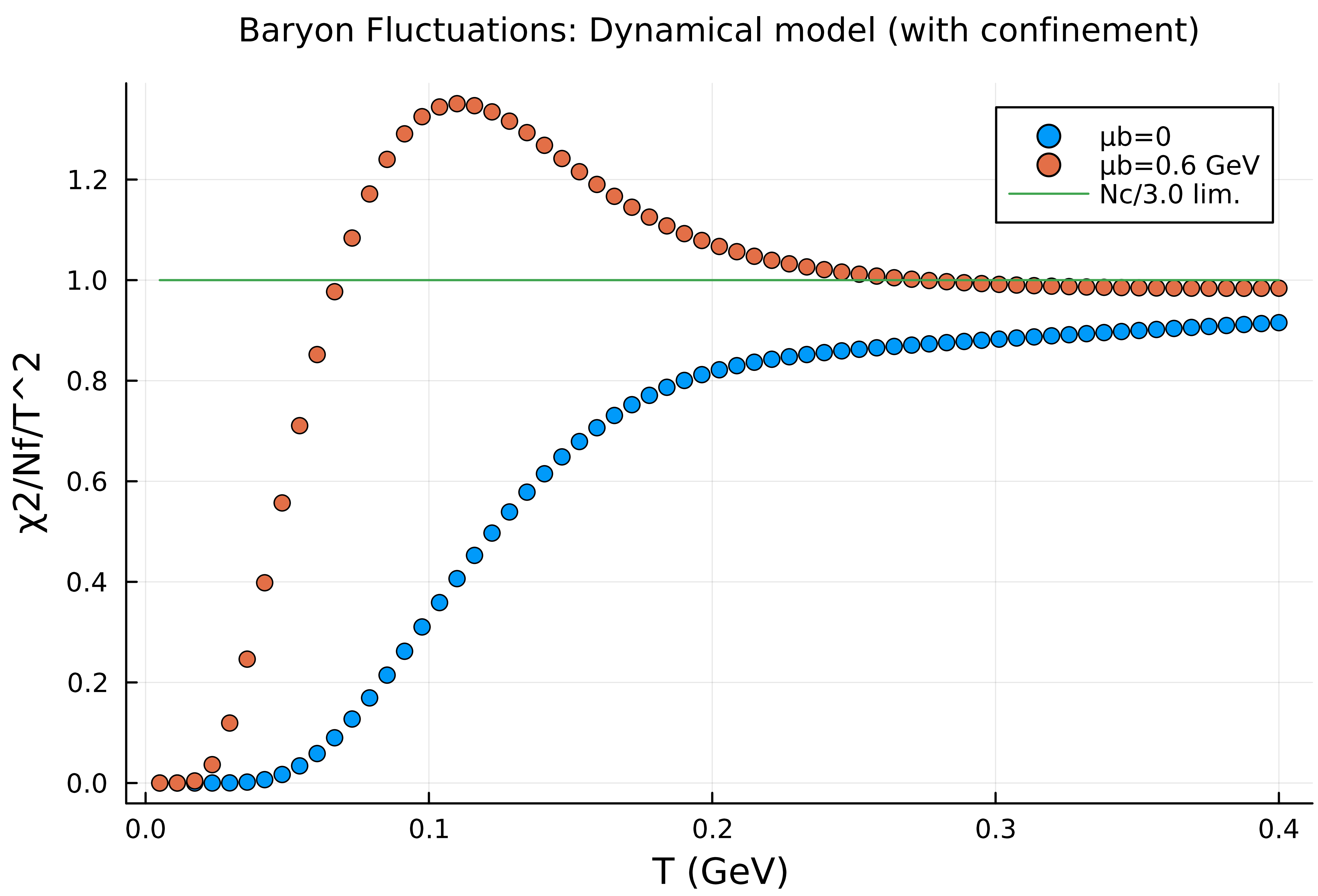
dynamical

$$\chi_2 = \frac{dn_v}{d\mu} \propto T^2$$

CONSEQUENCE OF DYNAMICAL INTERACTION
Finite Temperature



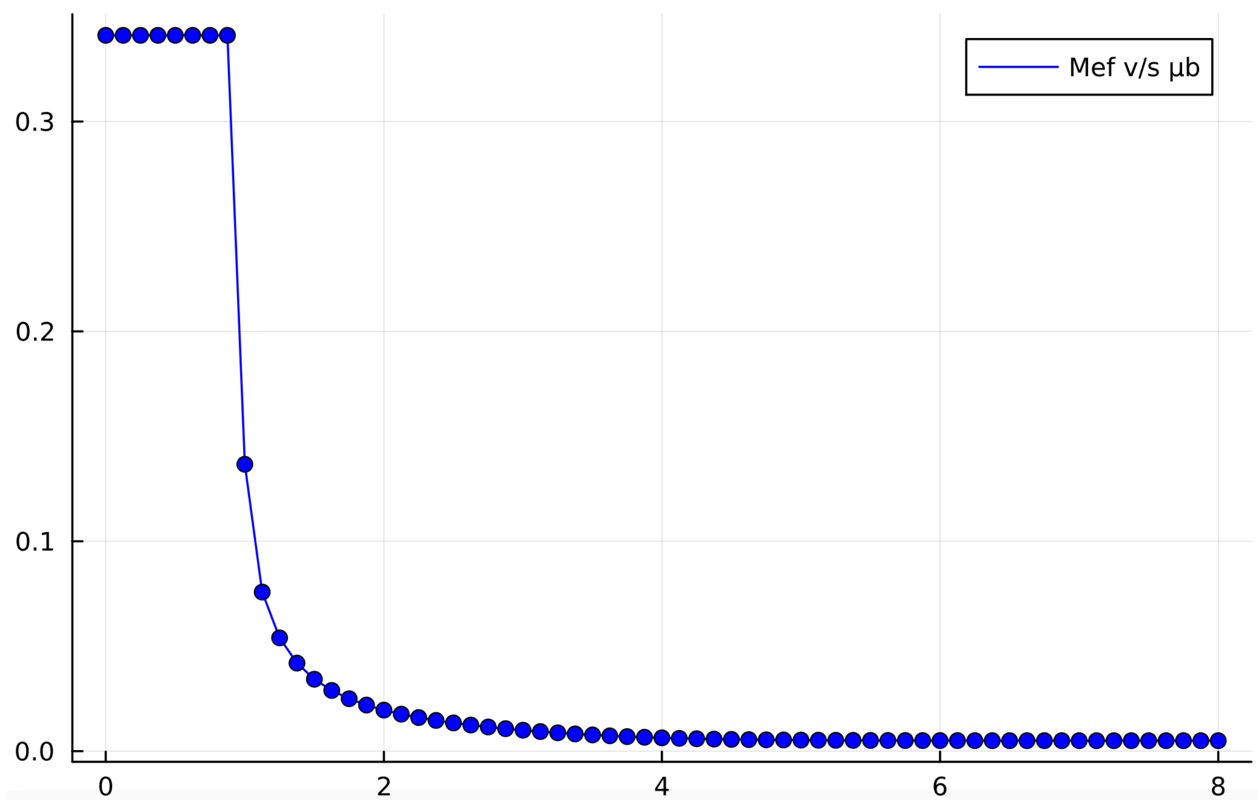
PRELIMINARY



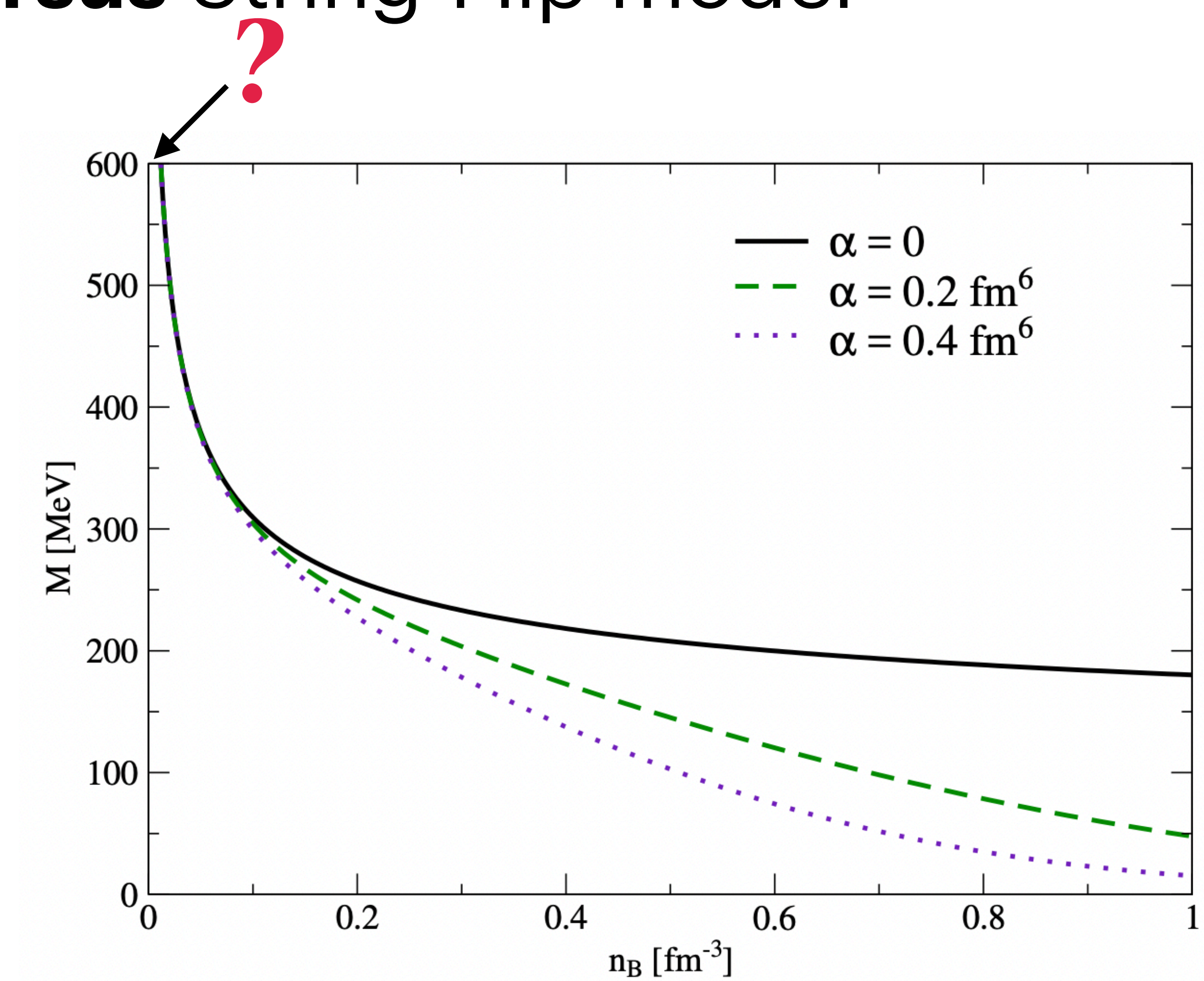
Missing ingredients: mechanism for (de)confinement

(De)confinement

Dynamical model **versus** String-Flip model



Dynamical model



Niels-Uwe F. Bastian, (2021)

Infinite Mass ⟶ Infinite chiral condensate

Coulomb Gauge Model

$$\mu'(p) = \mu + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{1}{2} \left(n(\tilde{E}) - \bar{n}(\tilde{E})\right)$$

$$A(p) = 1 + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{A(q)\,\hat{p}\cdot\hat{q}}{2\tilde{E}(q)} \left(1 - n(\tilde{E}) - \bar{n}(\tilde{E})\right)$$

$$B(p) = m + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{B(q)}{2\tilde{E}(q)} \left(1 - n(\tilde{E}) - \bar{n}(\tilde{E})\right)$$

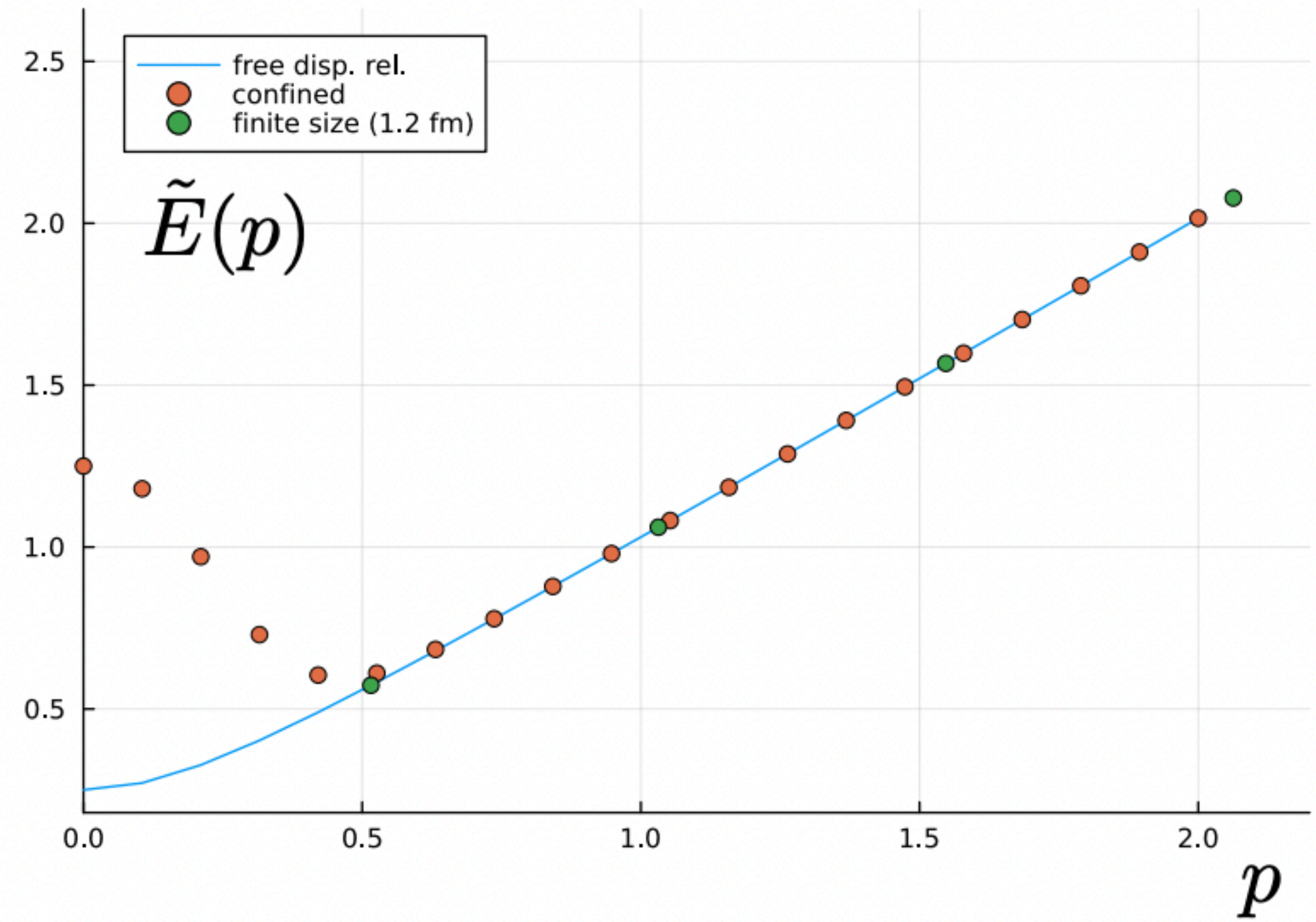
$$\tilde{E}(p) = \sqrt{A(p)^2 p^2 + B(p)^2}$$

$$n(\tilde{E}) = \frac{1}{e^{\beta(\tilde{E}(q) - \mu'(q))} + 1}.$$

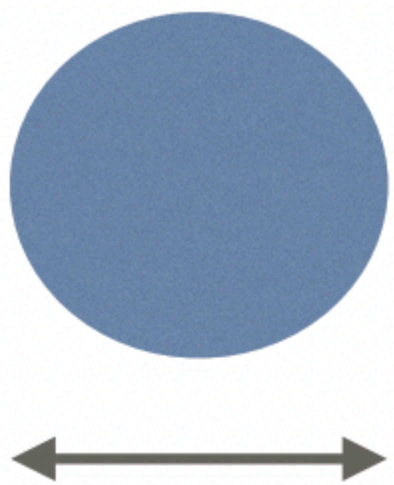
Conf. via:
A -> Infinity
thermal weights -> 0;
non-sense??
Quark Suppression

A-conf

$$\tilde{E}(p) = \sqrt{A(p)^2 p^2 + B(p)^2}$$



Bag Model



as a finite size effect

$$k \to k_n = \frac{n\pi}{L}$$

$$\Lambda_{\mathrm{IR}}$$

proton wavefunction

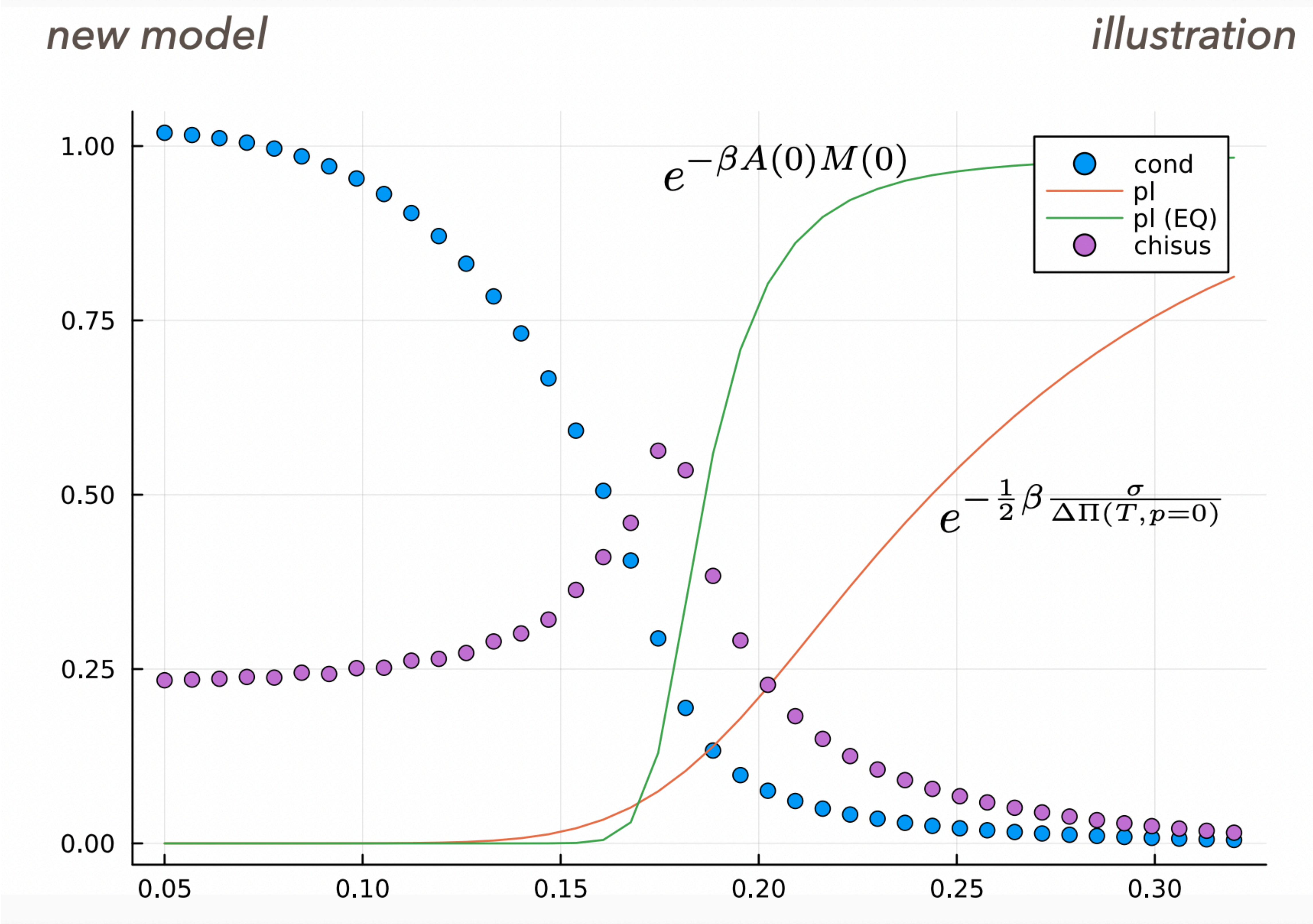
G. Krein EPJA 18 (2003)

Bicudo et. al. PRD 45 5 (1992)

also recently

K. Fukushima, T. Kojo, W. Weise
Phys. Rev. D 102, 096017 (2020)

PRELIMINARY



SUMMARY & CONCLUSIONS

- Inherent gluon scale essential for modelling interacting Fermi surface —> natural density-dependence.
- Non-local cut-off —> essential, quantifiable contributions to cs^2 and χ_2 & not a scale to be removed.
- cs^2 and χ_2 reach asymptotic limit in a dynamical model.
- χ_2 along finite T (zero μ) reaches asymptotic limit in a dynamical model.
- (De)confinement and chiral symmetry essential ingredients at intermediate densities and temperatures.

BACK UP SLIDES

(Non-Local) NJL with Diquark

$$m_p^* = m + \gamma_p 2G_s N_f \int \frac{d^3q}{(2\pi)^3} \gamma_q \frac{4m_q^*}{2E_q} [1 - n(E_q^-) - n(E_q^+) + \frac{E_q^+}{\epsilon_d^+(q)}(1 - 2n(\epsilon_d^+(q))) + \frac{E_q^-}{\epsilon_d^-(q)}(1 - 2n(\epsilon_d^-(q)))]$$

$$\mu_p^* = \mu - \gamma_p 2G_v N_f \int \frac{d^3q}{(2\pi)^3} \gamma_q \frac{4}{2} [n(E_q^-) - n(E_q^+) + \frac{E_q^+}{\epsilon_d^+(q)}(1 - 2n(\epsilon_d^+(q))) - \frac{E_q^-}{\epsilon_d^-(q)}(1 - 2n(\epsilon_d^-(q)))]$$

$$\Delta_p = \gamma_p 2G_d N_f \int \frac{d^3q}{(2\pi)^3} \gamma_q \Delta_q \frac{4}{2} [\frac{1 - 2n(\epsilon_d^+(q))}{\epsilon_d^+(q)} + \frac{1 - 2n(\epsilon_d^-(q))}{\epsilon_d^-(q)}]$$

DIQAURK PAIRING GAP
<qq>

$$E_q = \sqrt{p^2 + (m_q^*)^2}$$

$$E_q^\pm = E_q \pm \mu_q^*$$

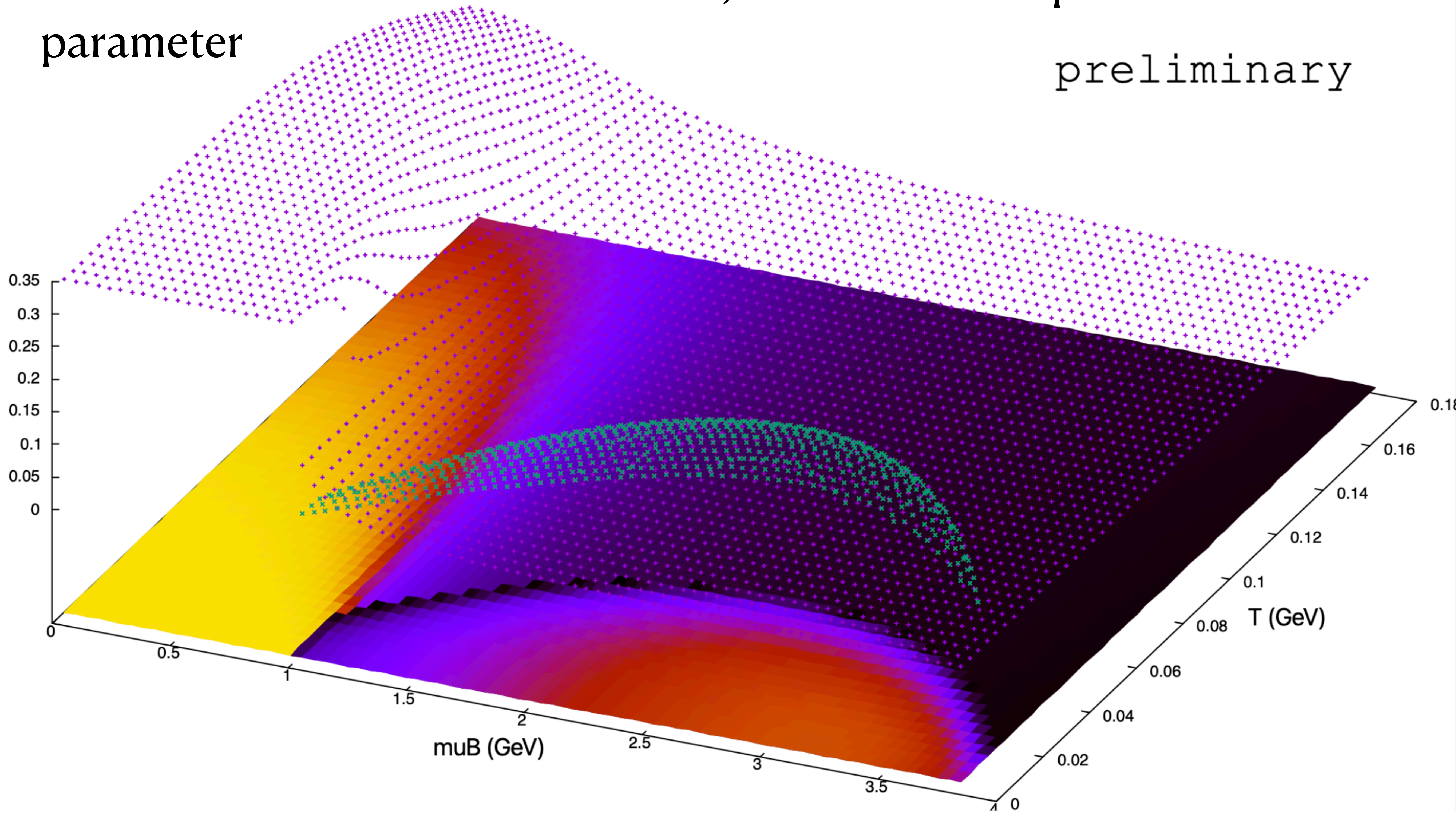
$$\epsilon_d^\pm(q) = \sqrt{(E_q^\pm)^2 + \Delta_q^2}$$

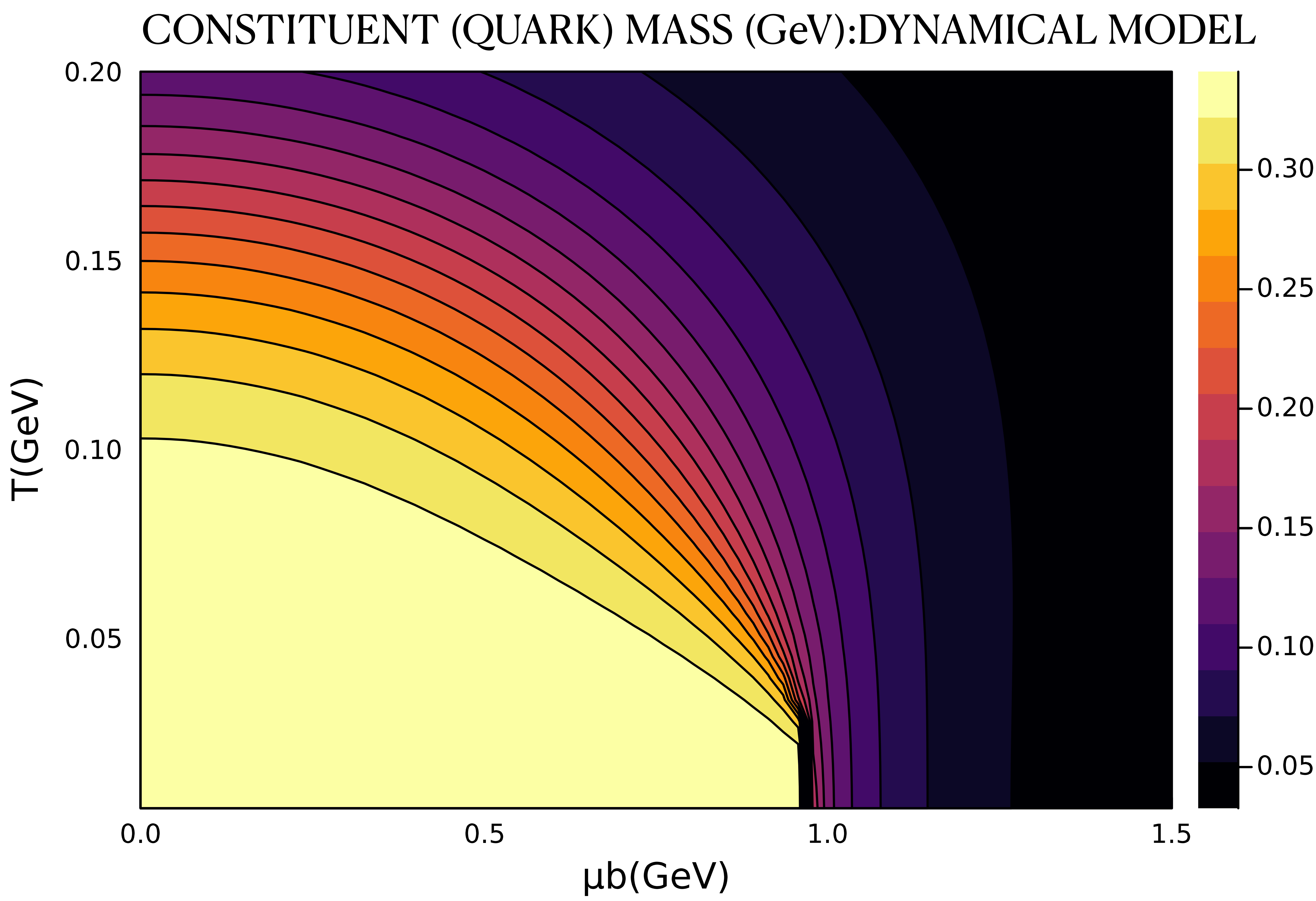
Quasi-particle property modified

Non-locality of constituent mass, chemical potential and diquark pairing gap

QCD Phase diagram

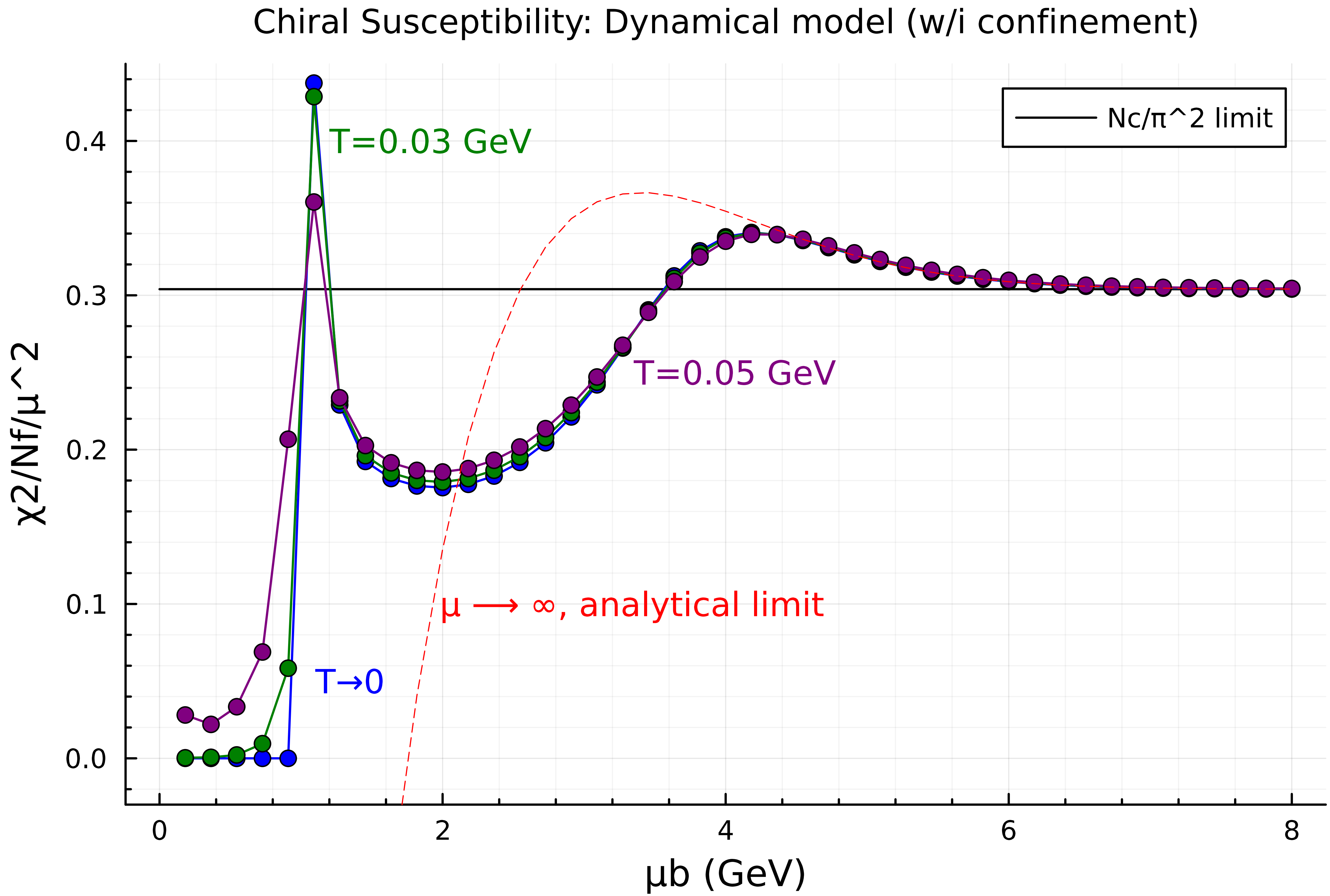
Position of CEP - no confinement & how it moves with confinement included and non-local interaction, it's the contour plot of the order parameter





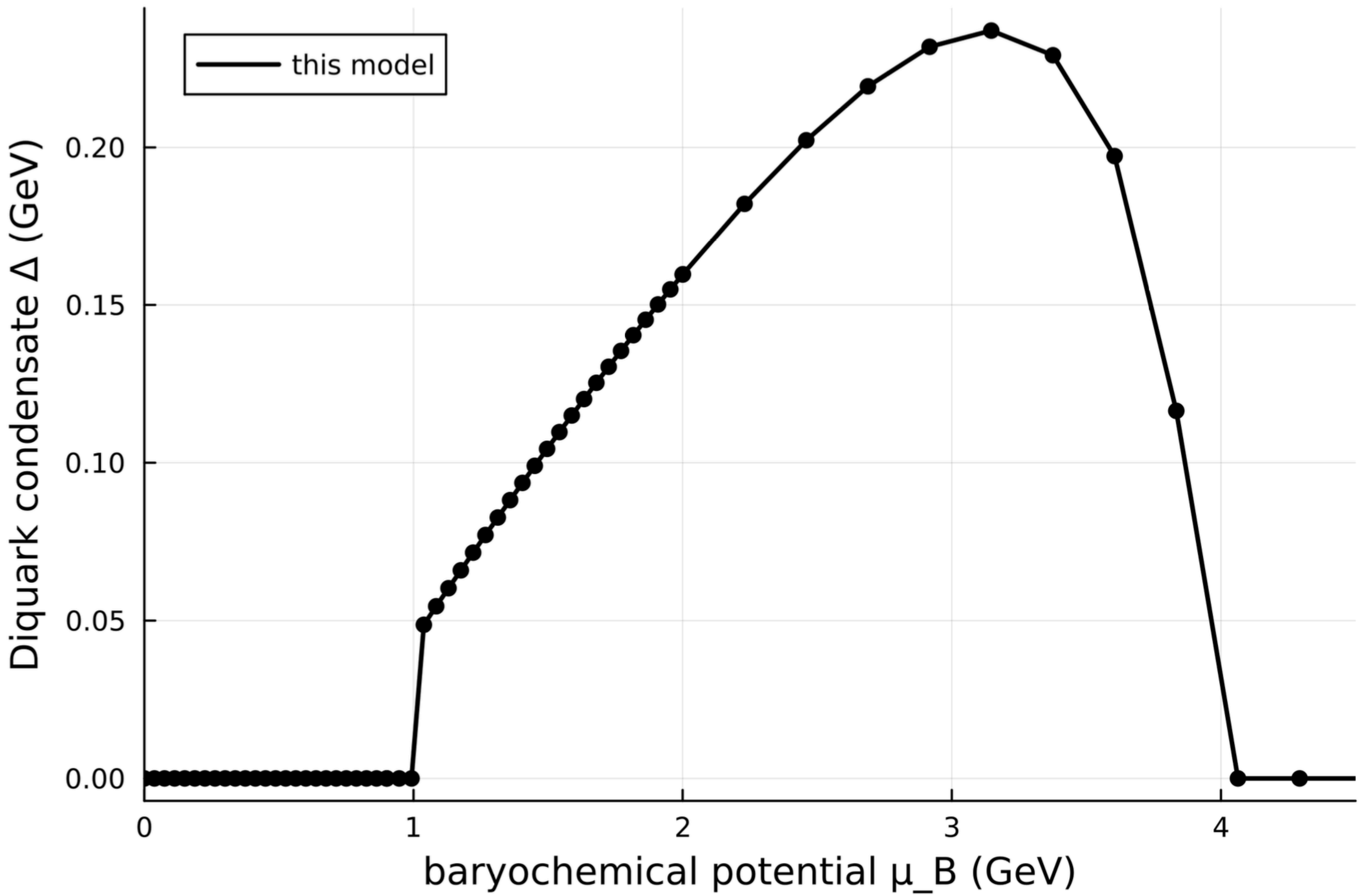
CONSTITUENT (QUARK) MASS (GeV):DYNAMICAL MODEL
T(GeV)
0.20
0.15
0.10
0.05
μb(GeV)
0.0
0.5
1.0
1.5
0.30
0.25
0.20
0.15
0.10
0.05

Verify this

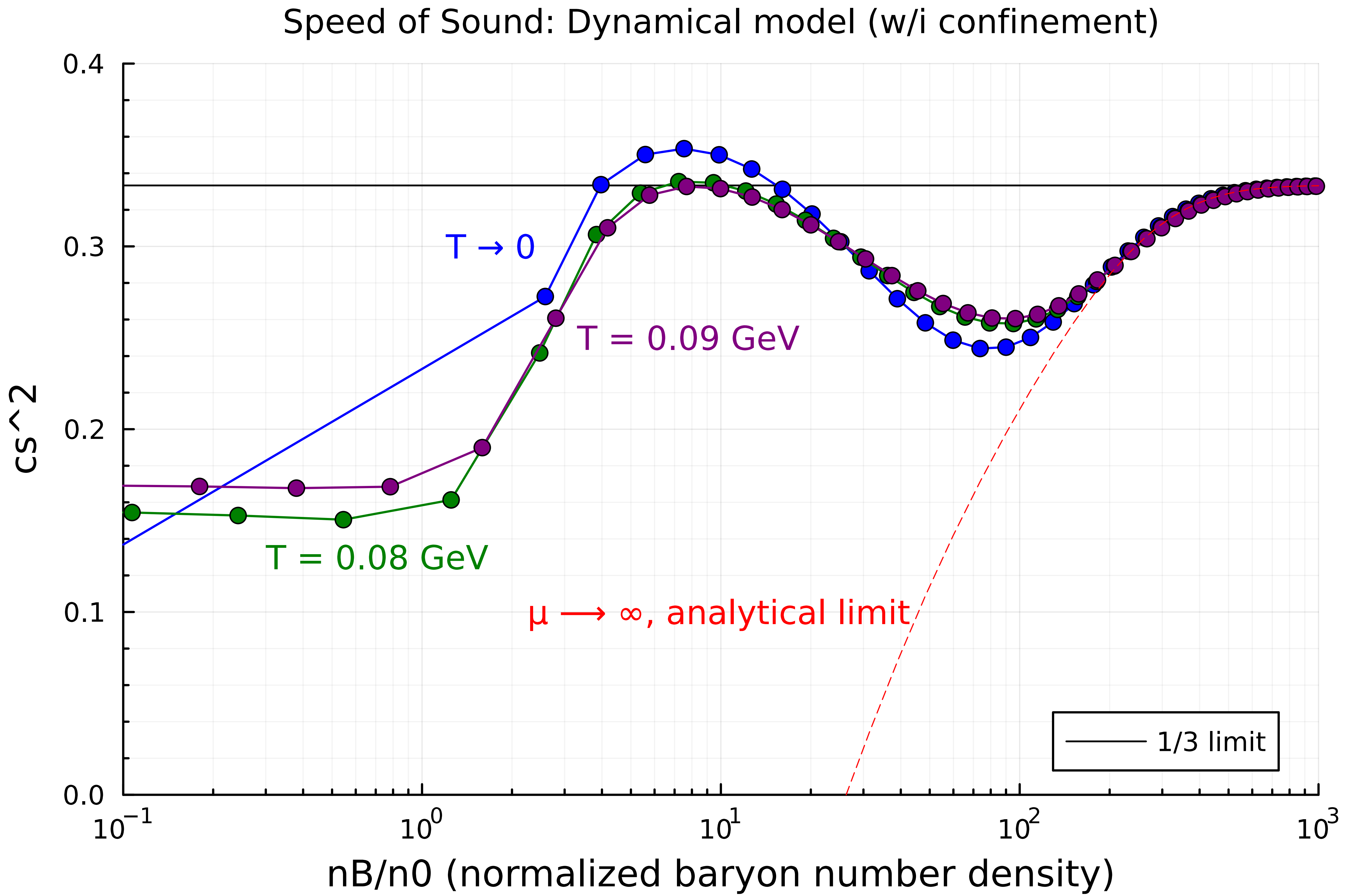


Chiral Susceptibility: Dynamical model (w/i confinement)
Nc/π^2 limit
T=0.03 GeV
T=0.05 GeV
μ ⟶ ∞, analytical limit
T→0
χ2/Nf/μ^2
μb (GeV)
0.0
0.1
0.2
0.3
0.4
0
2
4
6
8

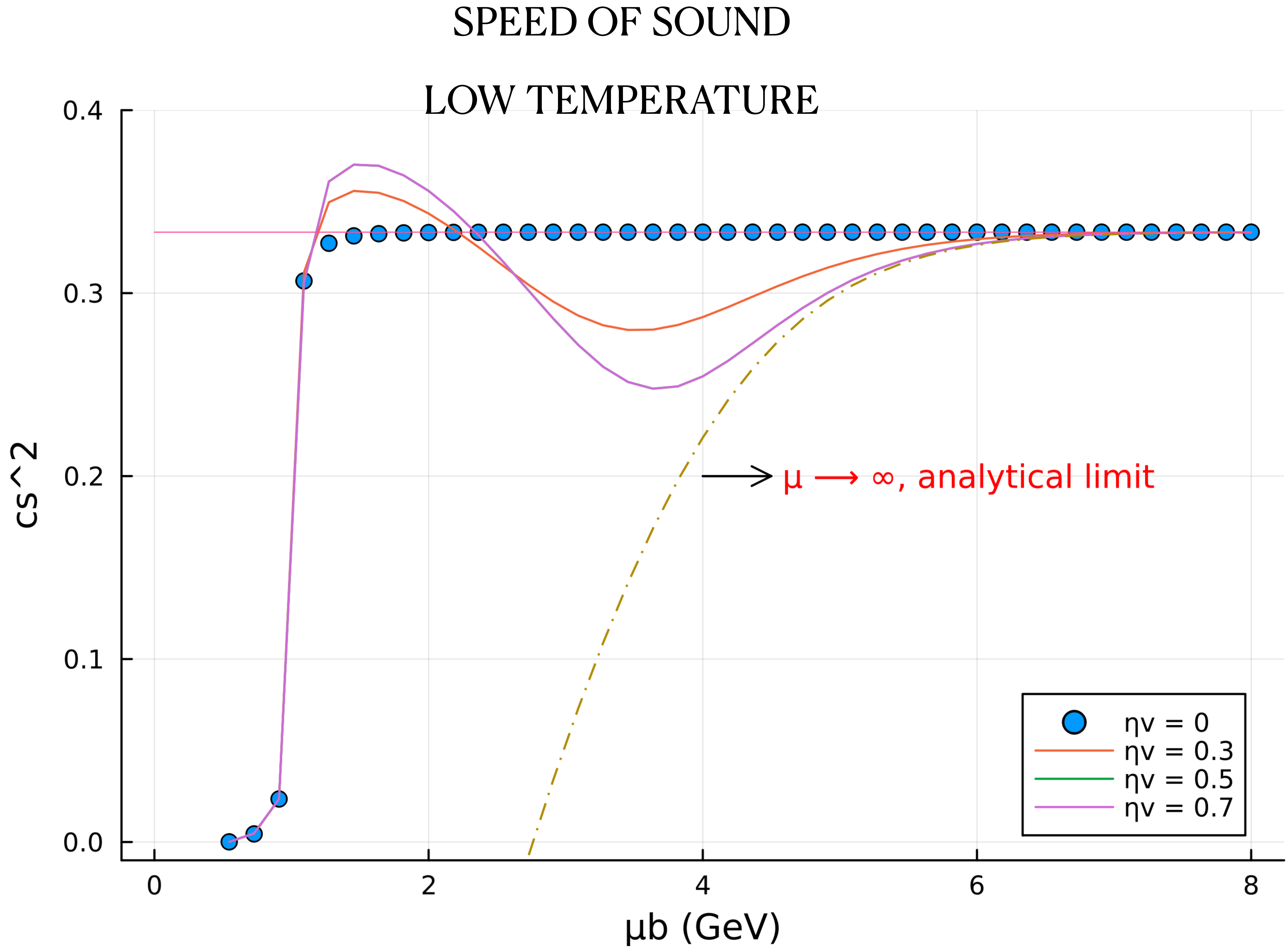
Diquark Gap

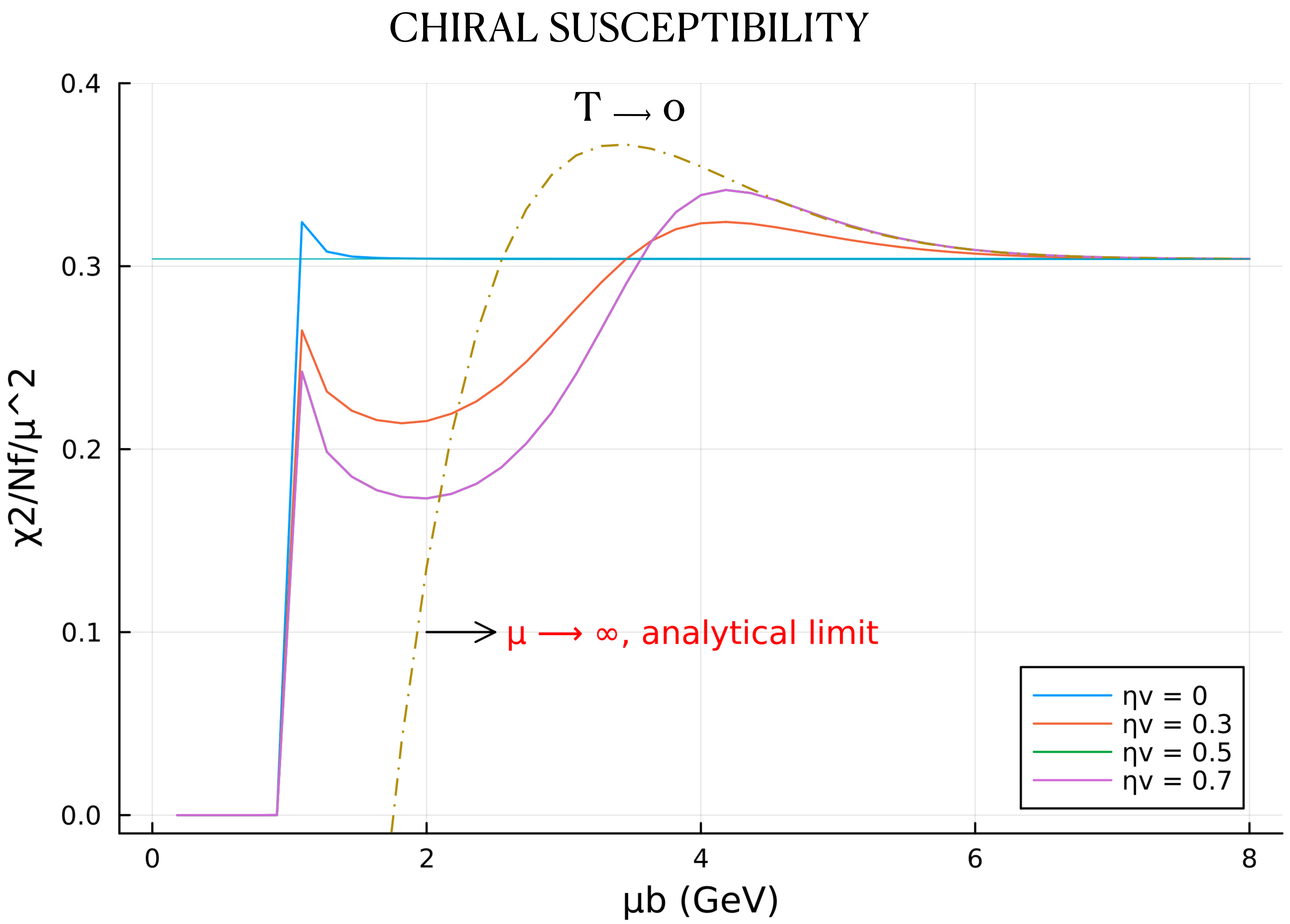


this model
Diquark condensate Δ (GeV)
0.00
0.05
0.10
0.15
0.20
0
1
2
3
4
baryochemical potential μ_B (GeV)



Speed of Sound: Dynamical model (w/i confinement)
cs^2
nB/n0 (normalized baryon number density)
T → 0
T = 0.09 GeV
T = 0.08 GeV
μ ⟶ ∞, analytical limit
1/3 limit
0.0
0.1
0.2
0.3
0.4
10^−1
10^0
10^1
10^2
10^3


SPEED OF SOUND
LOW TEMPERATURE
0.4
0.3
0.2
0.1
0.0
cs^2
0
2
4
6
8
μb (GeV)
μ ⟶ ∞, analytical limit
ηv = 0
ηv = 0.3
ηv = 0.5
ηv = 0.7


CHIRAL SUSCEPTIBILITY
T ⟶ 0
μ ⟶ ∞, analytical limit
ηv = 0
ηv = 0.3
ηv = 0.5
ηv = 0.7
χ2/Nf/μ^2
μb (GeV)
0.0
0.1
0.2
0.3
0.4
0
2
4
6
8

$$n(x) = \frac{1}{1 + \exp(x/T)}$$

$$\sigma = f(\sigma, \omega, \Delta)$$

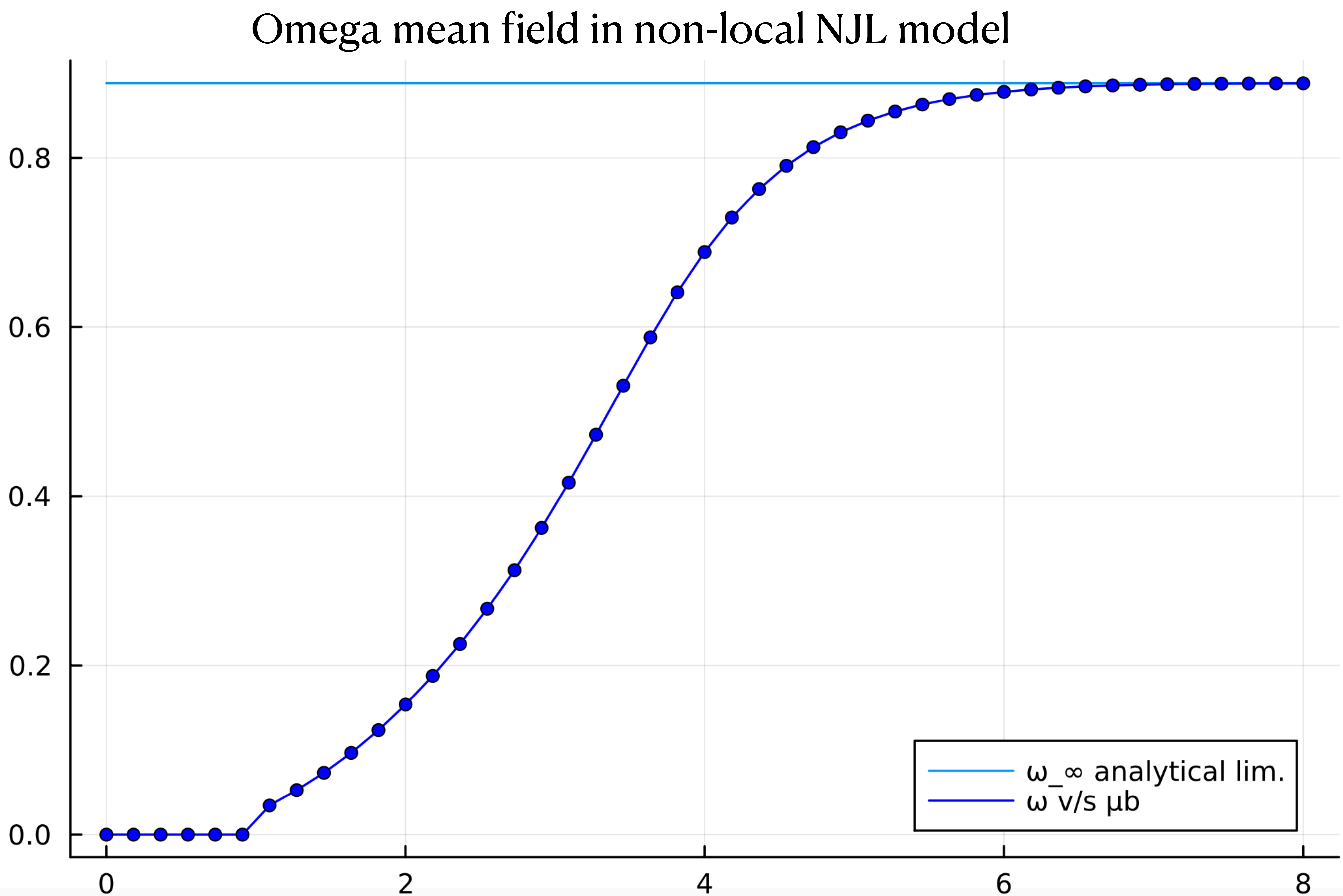
$$\sigma = \frac{4 * 2\,G_s}{2\pi^2} \int dq\, q^2\, \gamma(q) \frac{M_q}{2E_q} \left[1 - n(E_q^+) - n(E_q^-) + \frac{E_q^+}{\epsilon_q^+}\left(1 - 2n(\epsilon_q^+)\right) + \frac{E_q^-}{\epsilon_q^-}\left(1 - 2n(\epsilon_q^-)\right)\right]$$

$$\omega = f(\sigma, \omega, \Delta)$$

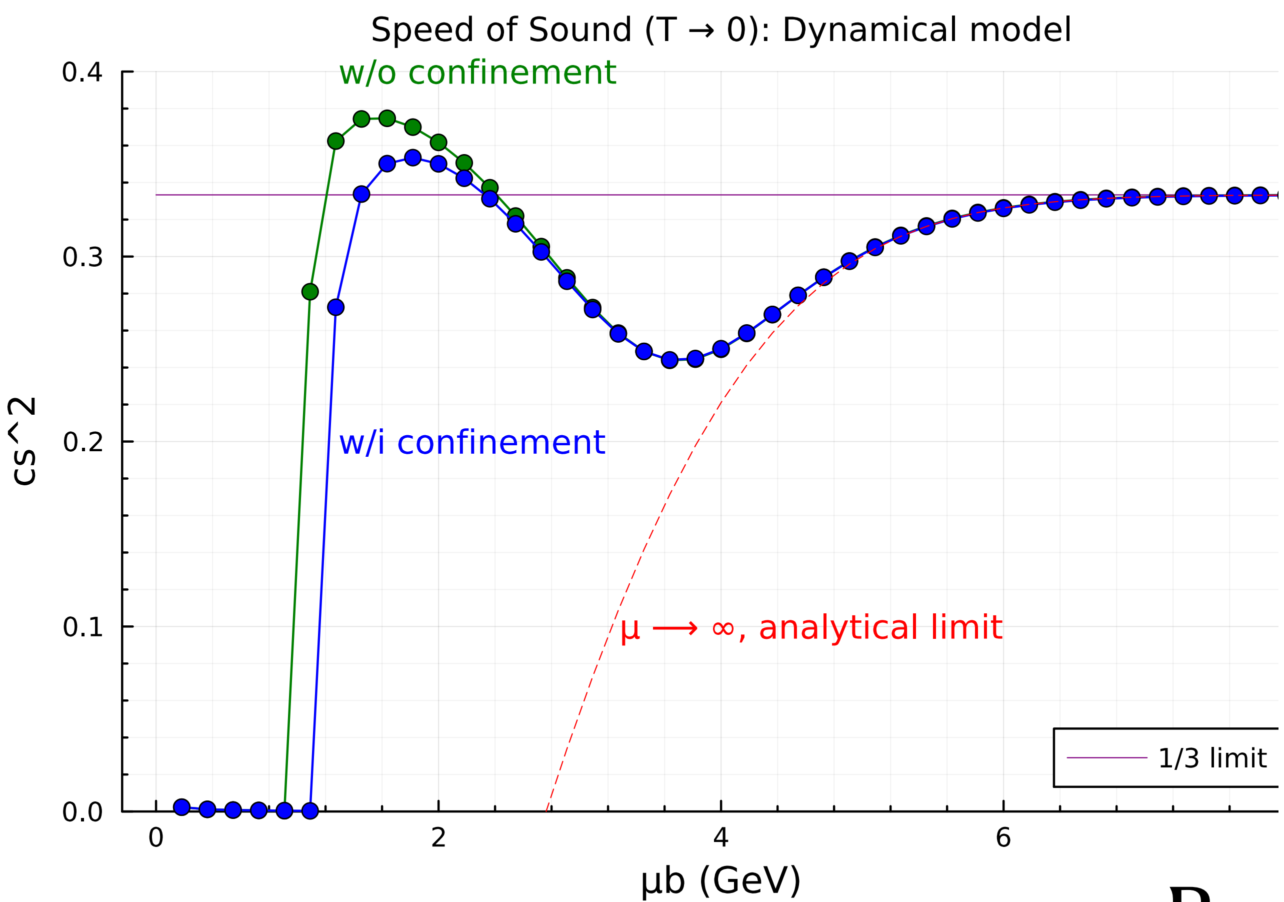
$$\omega = \frac{4 * 2\,G_v}{2 * 2\pi^2} \int dq\, q^2\, \gamma(q) \left[n(E_q^-) - n(E_q^+) + \frac{E_q^+}{\epsilon_q^+}\left(1 - 2n(\epsilon_q^+)\right) - \frac{E_q^-}{\epsilon_q^-}\left(1 - 2n(\epsilon_q^-)\right)\right]$$

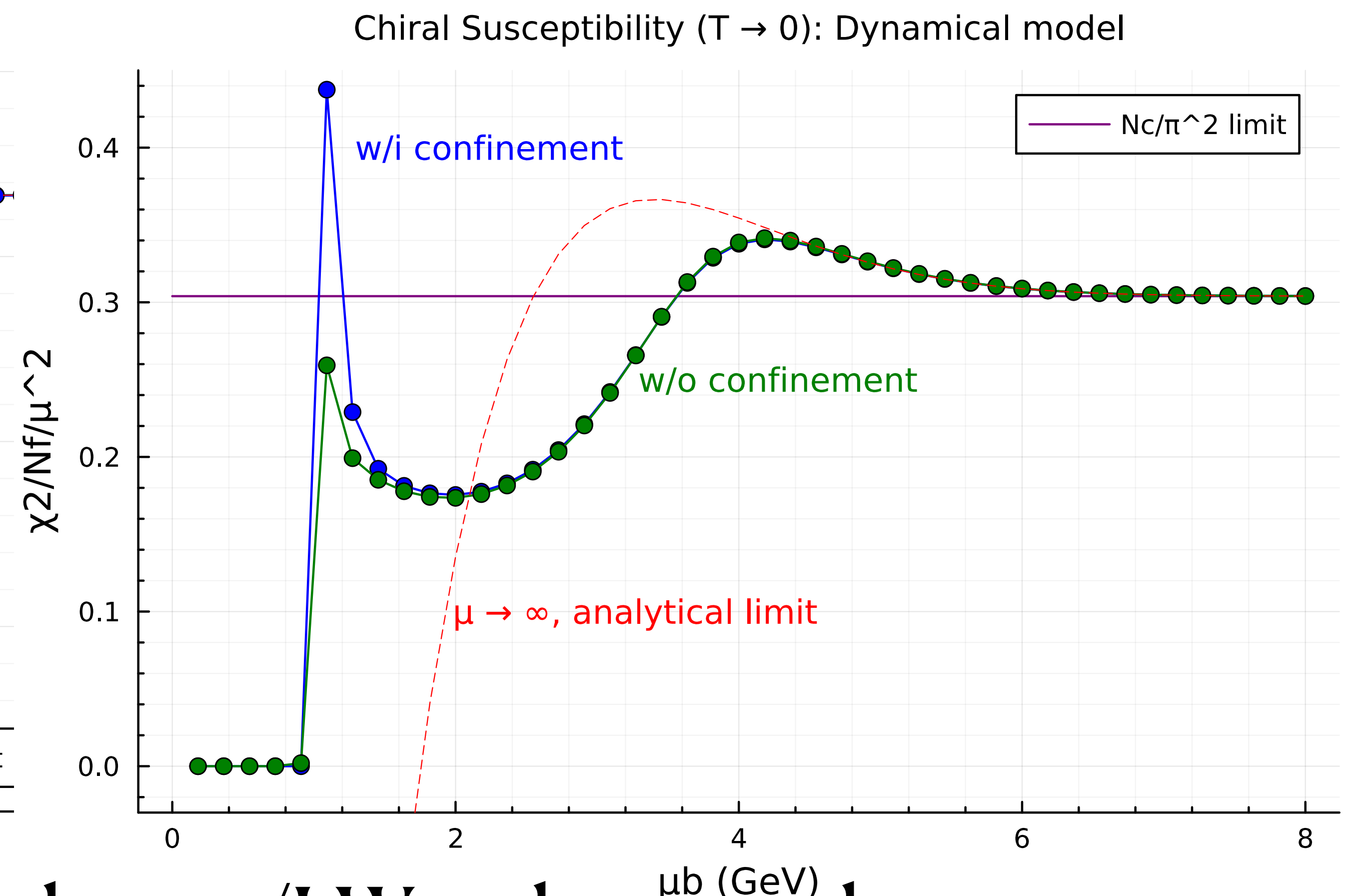
$$\Delta = f(\sigma, \omega, \Delta)$$

$$\Delta = \frac{4 * 2\,G_d}{2 * 2\pi^2} \int dq\, q^2\, \gamma(q) \Delta \left[\frac{1}{\epsilon_q^+}\left(1 - 2n(\epsilon_q^+)\right) + \frac{1}{\epsilon_q^-}\left(1 - 2n(\epsilon_q^-)\right)\right]$$


Omega mean field in non-local NJL model
0.0
0.2
0.4
0.6
0.8
0
2
4
6
8
ω_∞ analytical lim.
ω v/s μb

SPEED OF SOUND V/S CHIRAL SUSCEPTIBILITY





Replace w/I Wroclaw graphs

$$cs^2_{\mu\to\infty} \approx \frac{1}{3}\left[1 - \frac{\omega_\infty}{\mu}\left(1+\frac{2\mu^2}{UV^2}\right)e^{-\frac{\mu^2}{UV^2}}\right]$$

$$\chi 2_{\mu\to\infty} \approx \frac{N_c N_f}{\pi^2}\mu^2\left[1-\frac{2\omega_\infty}{\mu}\left(1-\frac{\mu^2}{UV^2}\right)e^{-\frac{\mu^2}{UV^2}}\right]$$

Non-local cut-off is inherent gluon interaction scale